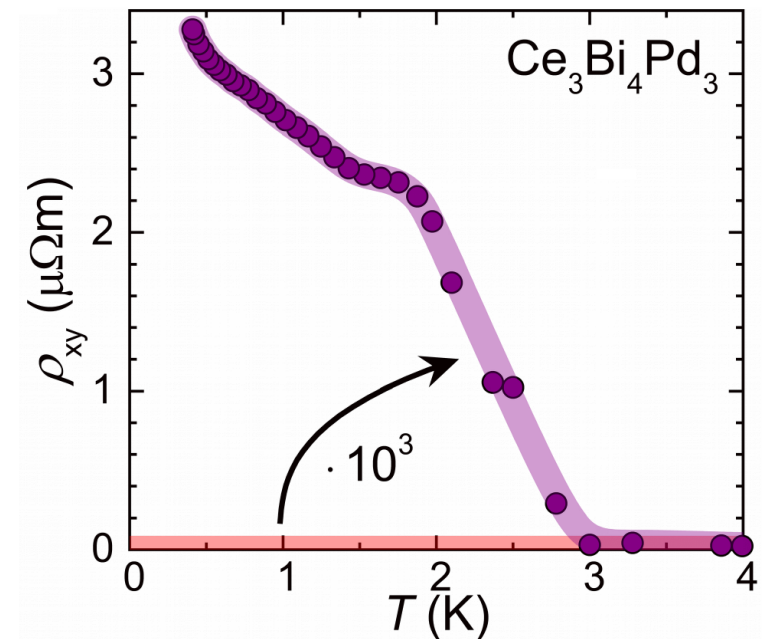
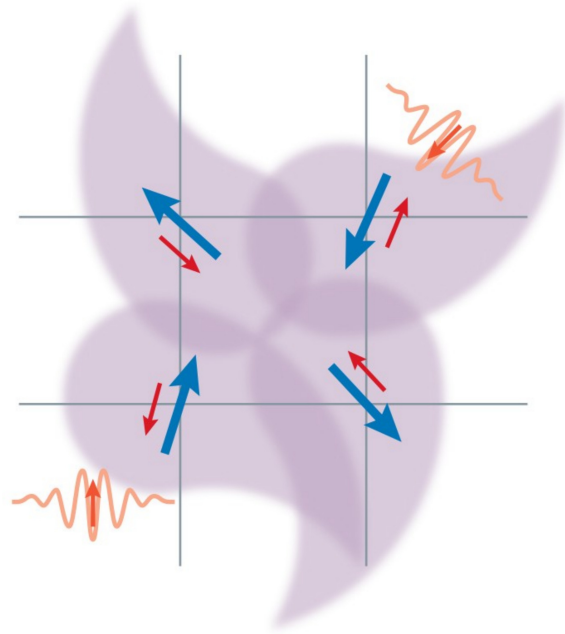
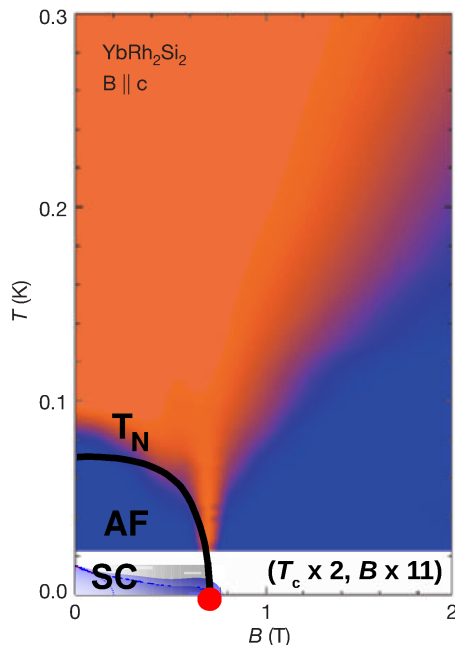


# Heavy fermion systems

## From quantum criticality to electronic topology

exosup2022 : School on Exotic Superconductivity  
13-25 June 2022 Cargèse, Corse (France)

**Silke Paschen**  
Vienna University of Technology

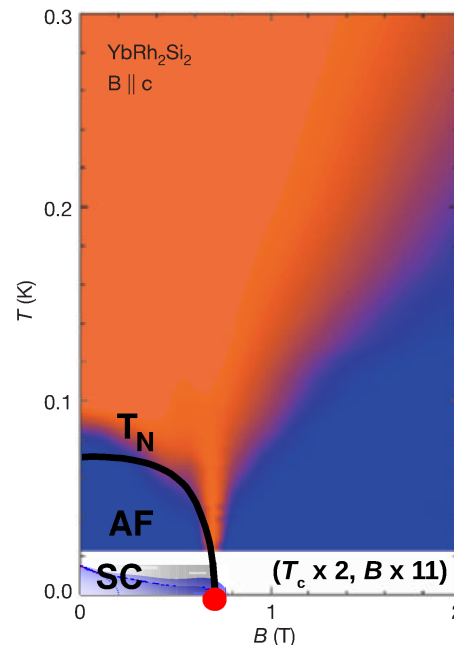


# Heavy fermion systems

## From quantum criticality to electronic topology

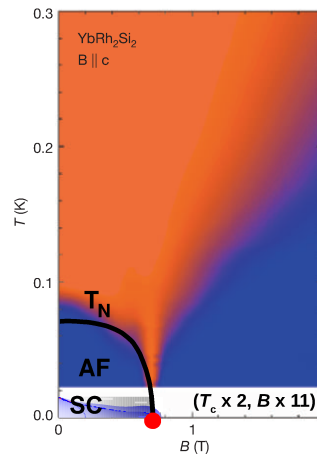
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# Heavy fermion systems

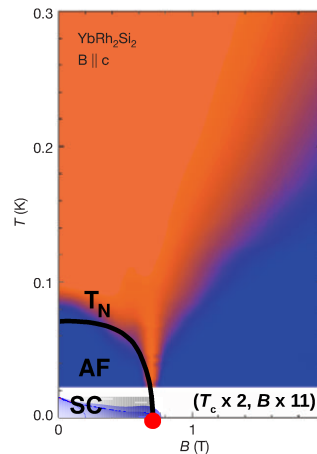
## From quantum criticality to electronic topology



- **Tunable correlation strength**
- **Phase diagrams governed by quantum fluctuations**
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- **Beyond order-parameter quantum criticality**
- **Global phase diagram of heavy fermion compounds**
- **Emergent phases: Unconventional superconductivity**

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# Heavy Fermi liquids

$$\rho = \rho_0 + AT^2$$

$$C/T = \gamma$$

$$A \sim (m^*)^2$$

$$\gamma \sim m^*$$

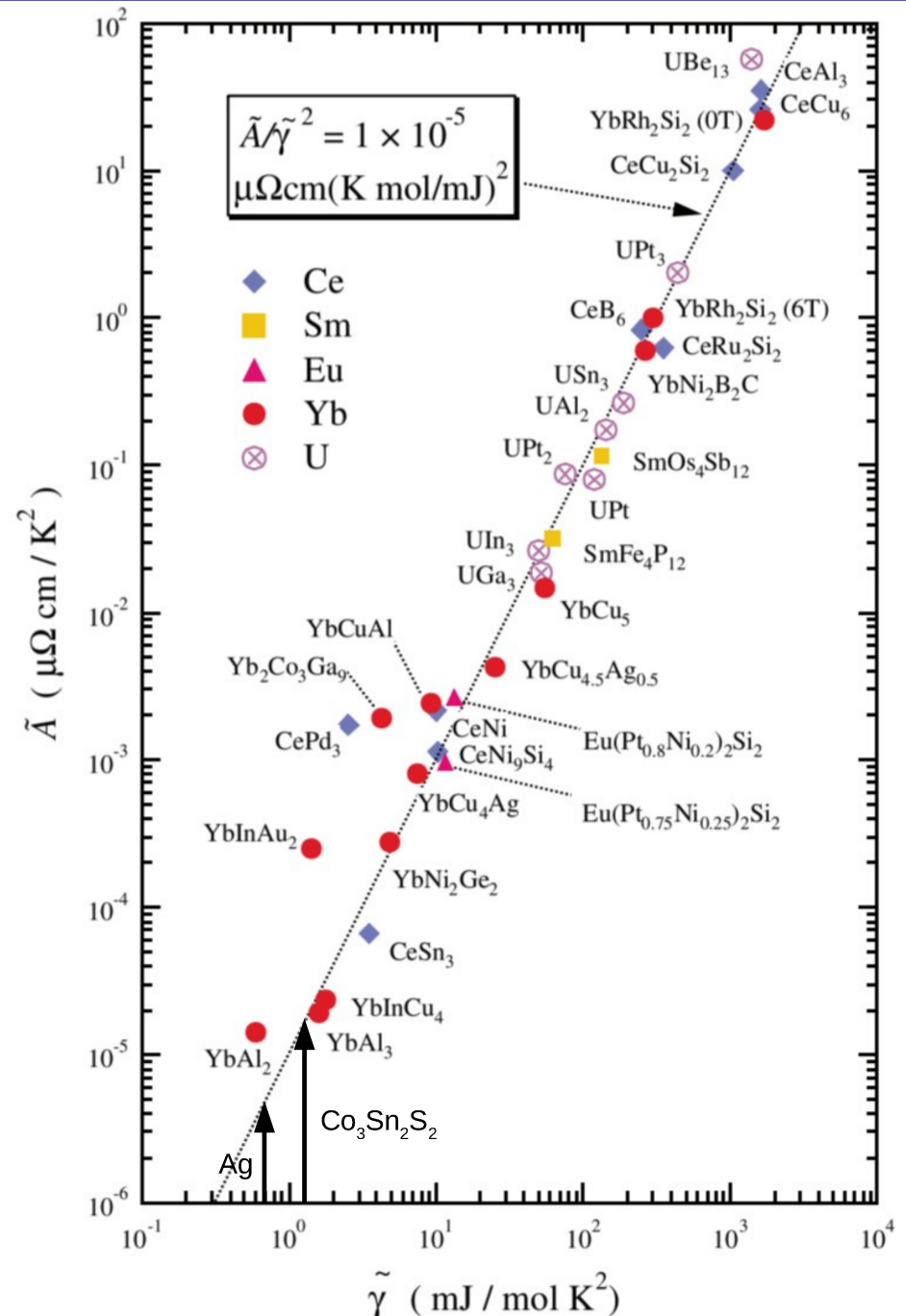
$$m^* = m(1 + \frac{1}{3}F_1^S)$$

With orbital degeneracy:

$$\tilde{A} = \frac{A}{\frac{1}{2}N(N-1)}$$

$$\tilde{\gamma} = \frac{\gamma}{\frac{1}{2}N(N-1)}$$

(Tsuji et al., Phys. Rev. Lett. 94 (2005) 057201) →



# Heavy Fermi liquids: Tuning

$$\rho = \rho_0 + AT^2$$

$$C/T = \gamma$$

$$A \sim (m^*)^2$$

$$\gamma \sim m^*$$

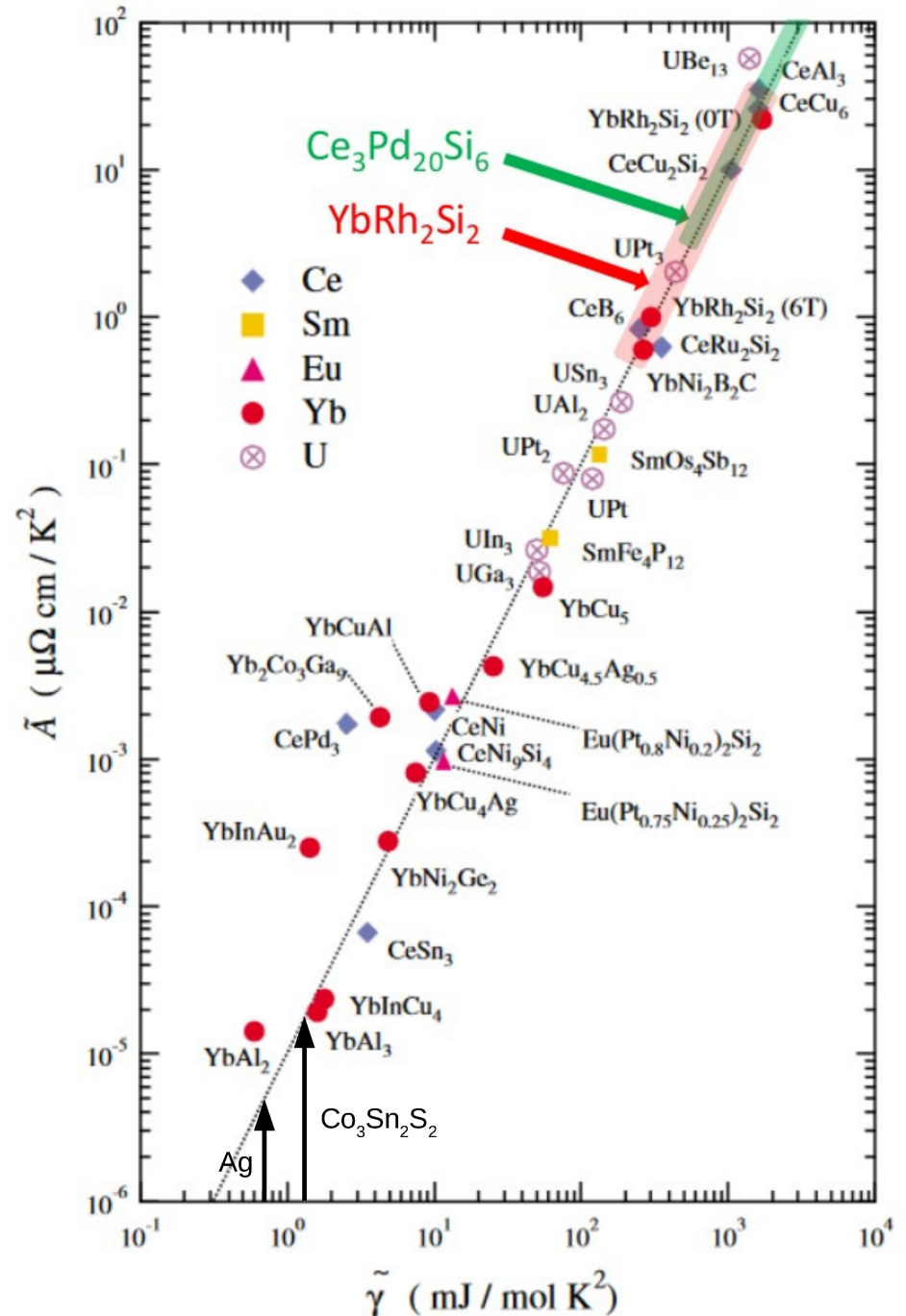
$$m^* = m(1 + \frac{1}{3}F_1^S)$$

With orbital degeneracy:

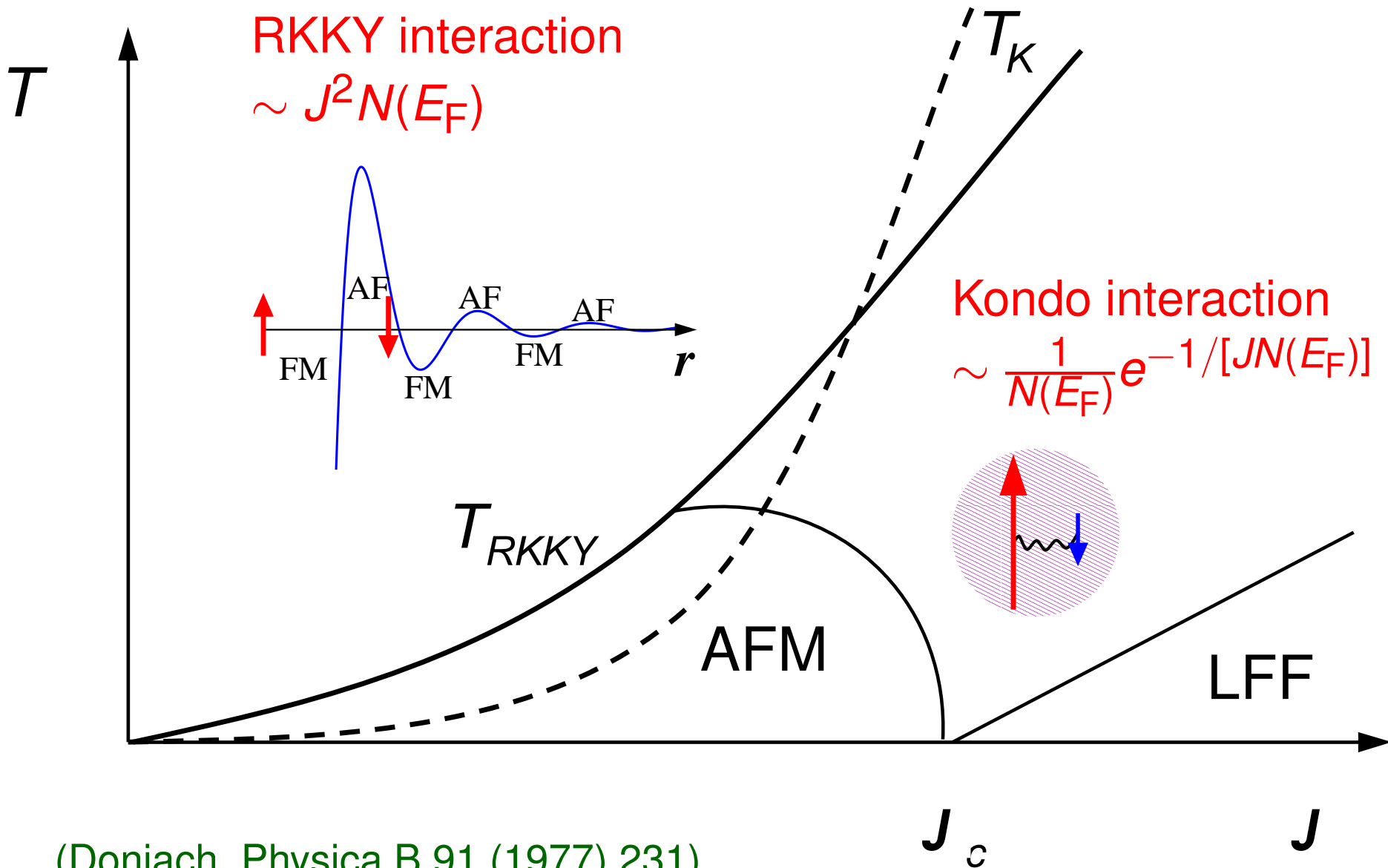
$$\tilde{A} = \frac{A}{\frac{1}{2}N(N-1)}$$

$$\tilde{\gamma} = \frac{\gamma}{\frac{1}{2}N(N-1)}$$

(SP & Si, Nat. Rev. Phys. 3 (2021) 9) →

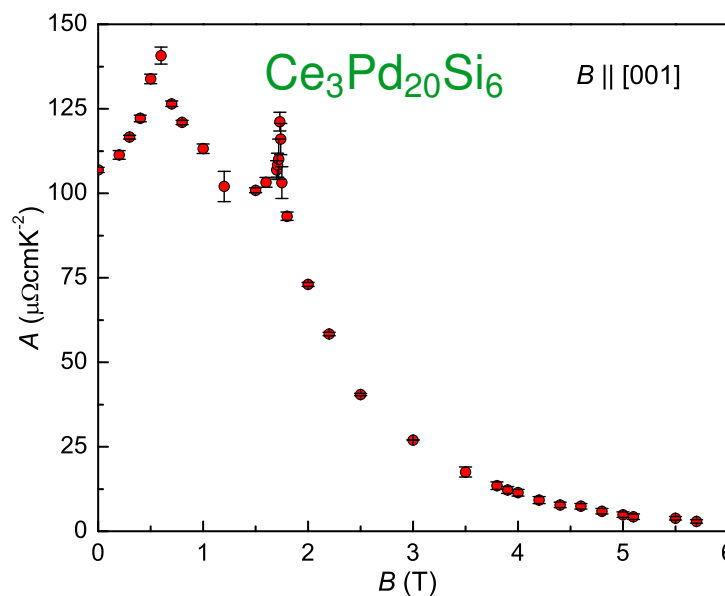
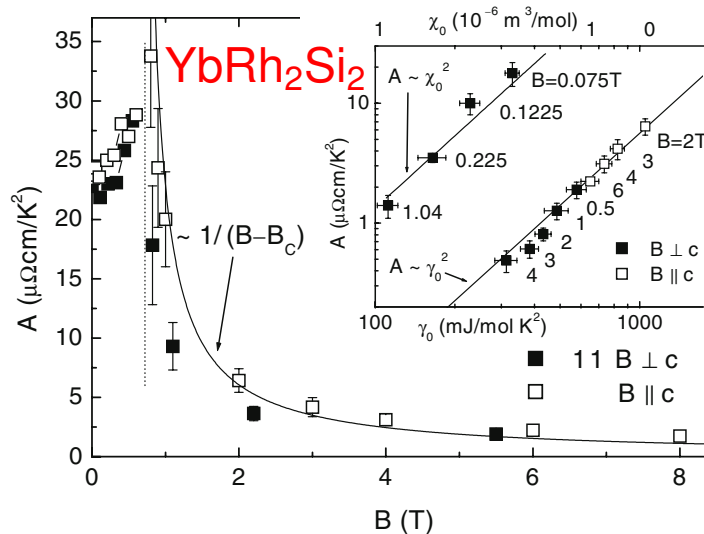
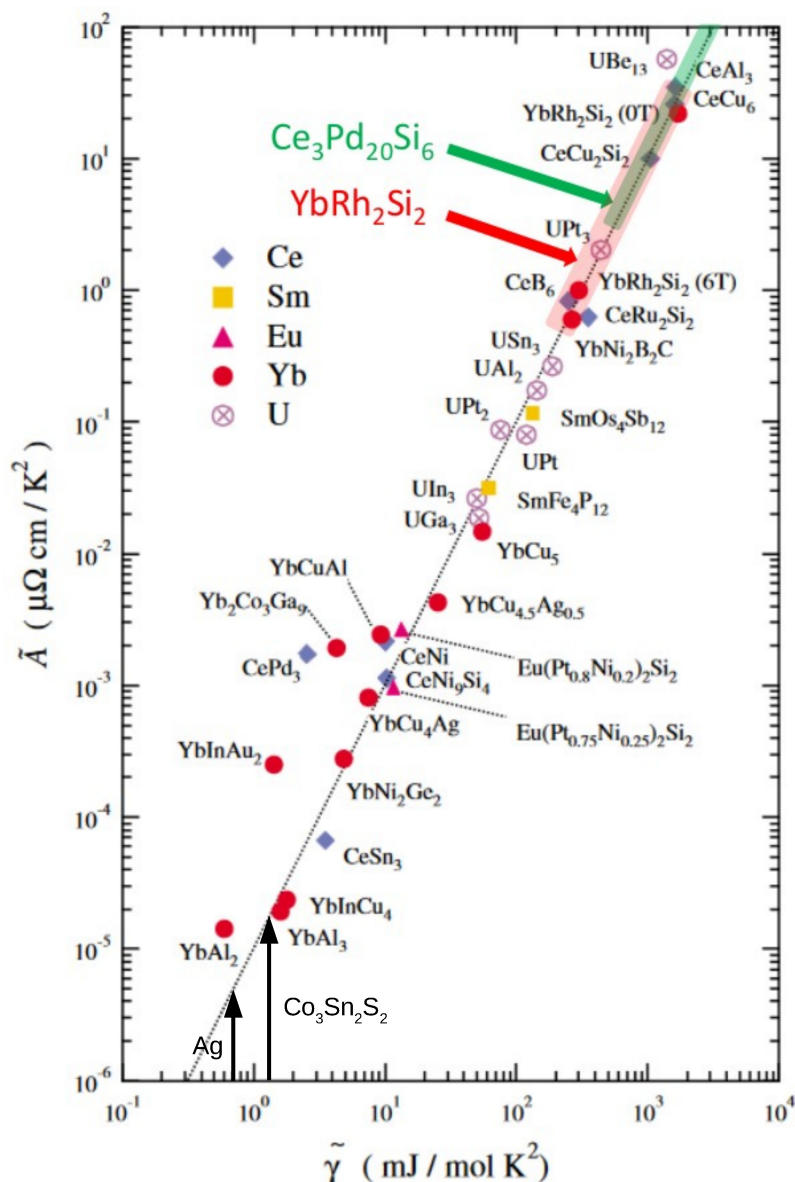


# Tunability due to competing interaction: Kondo vs RKKY



(Doniach, Physica B 91 (1977) 231)

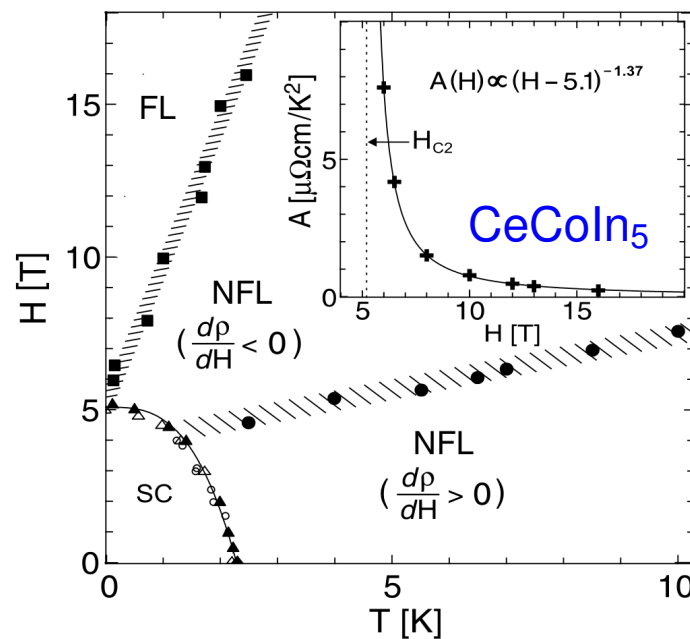
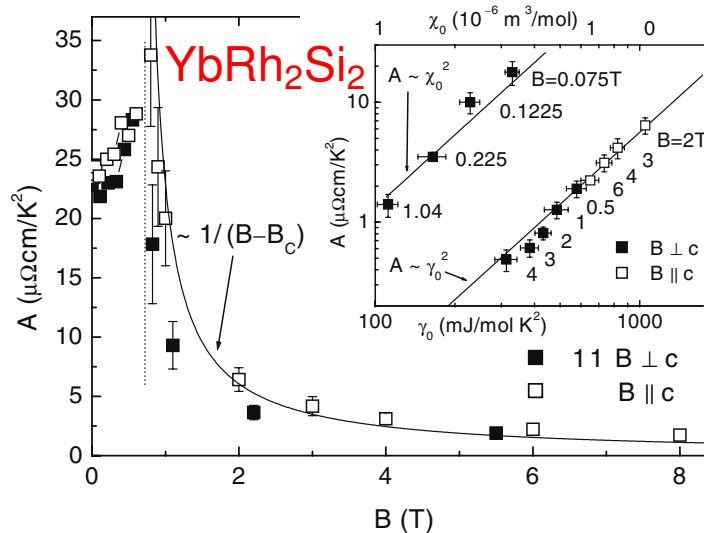
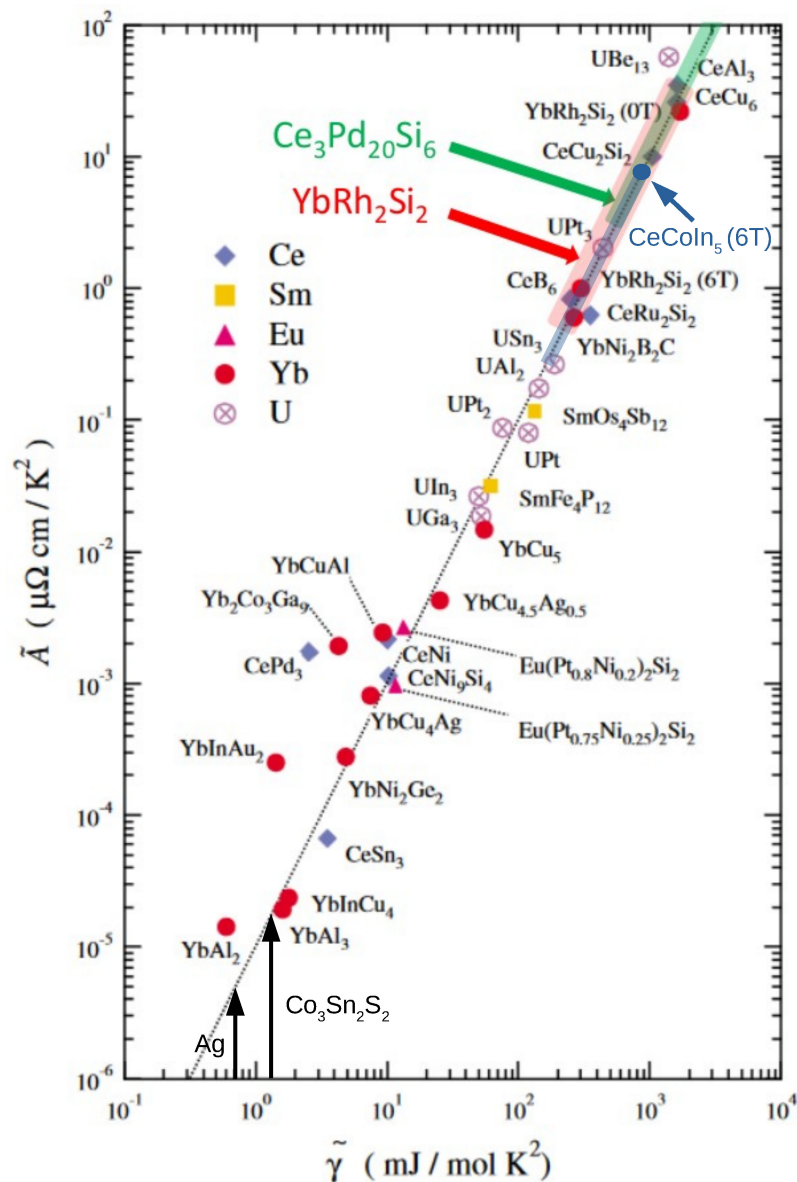
# Discontinuities appear upon tuning: FL parameters diverge



(SP & Si, Nat. Rev. Phys. 3 (2021) 9, and refs. therein)



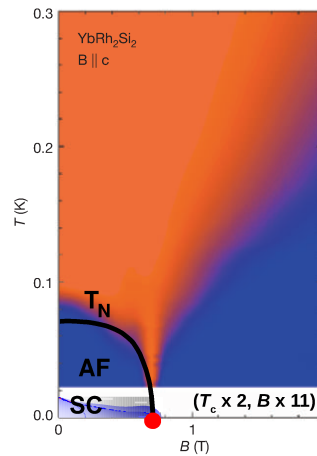
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(Nat. Rev. Phys. 3 (2021) 9; Paglione et al., Phys. Rev. Lett. 24 (2003) 246405)

# Heavy fermion systems

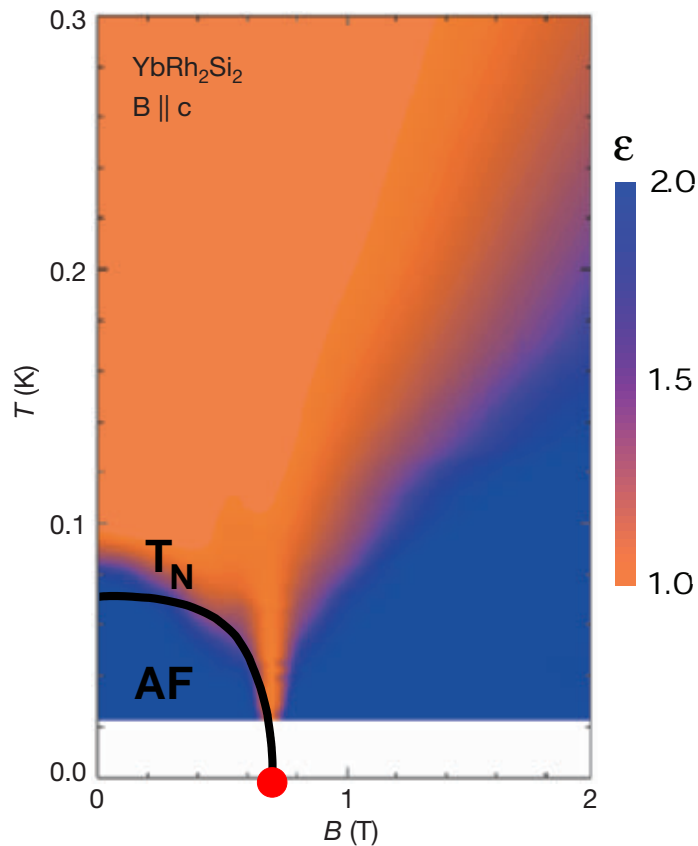
## From quantum criticality to electronic topology



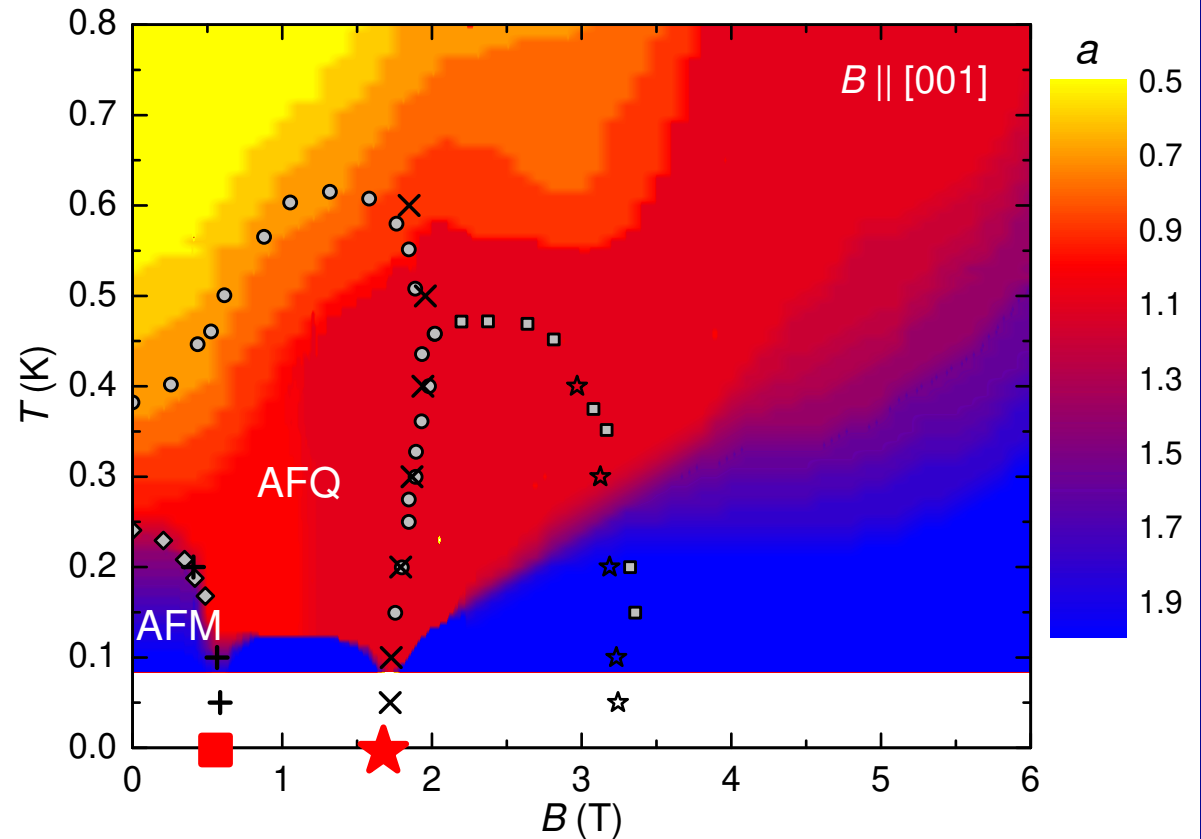
- Tunable correlation strength
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# Origin of divergences: Quantum critical points

**YbRh<sub>2</sub>Si<sub>2</sub>**



**Ce<sub>3</sub>Pd<sub>20</sub>Si<sub>6</sub>**



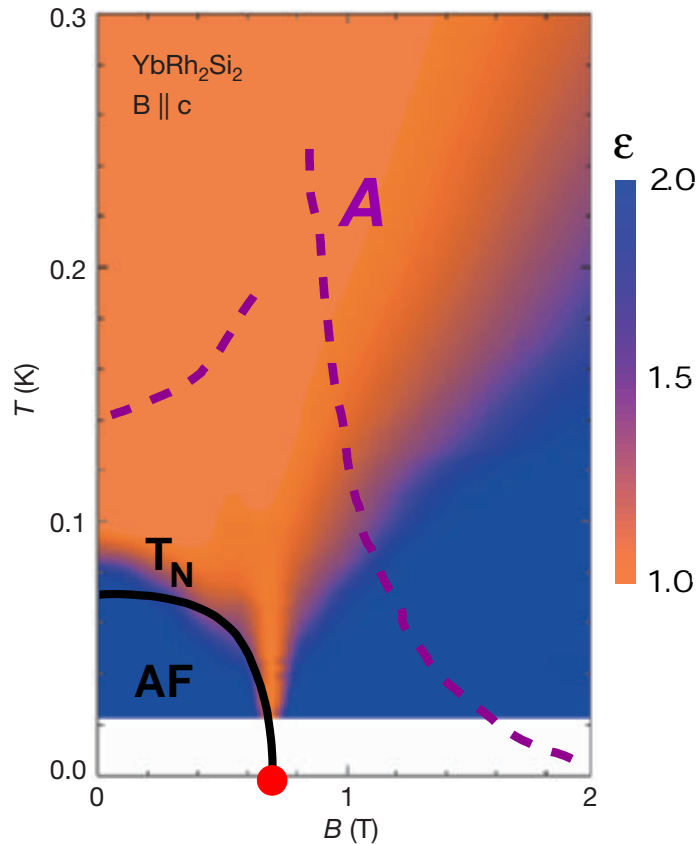
Fermi liquid:  $\rho = \rho_0 + AT^2$

Non-Fermi liquid:  $\rho = \rho'_0 + A'T^1$

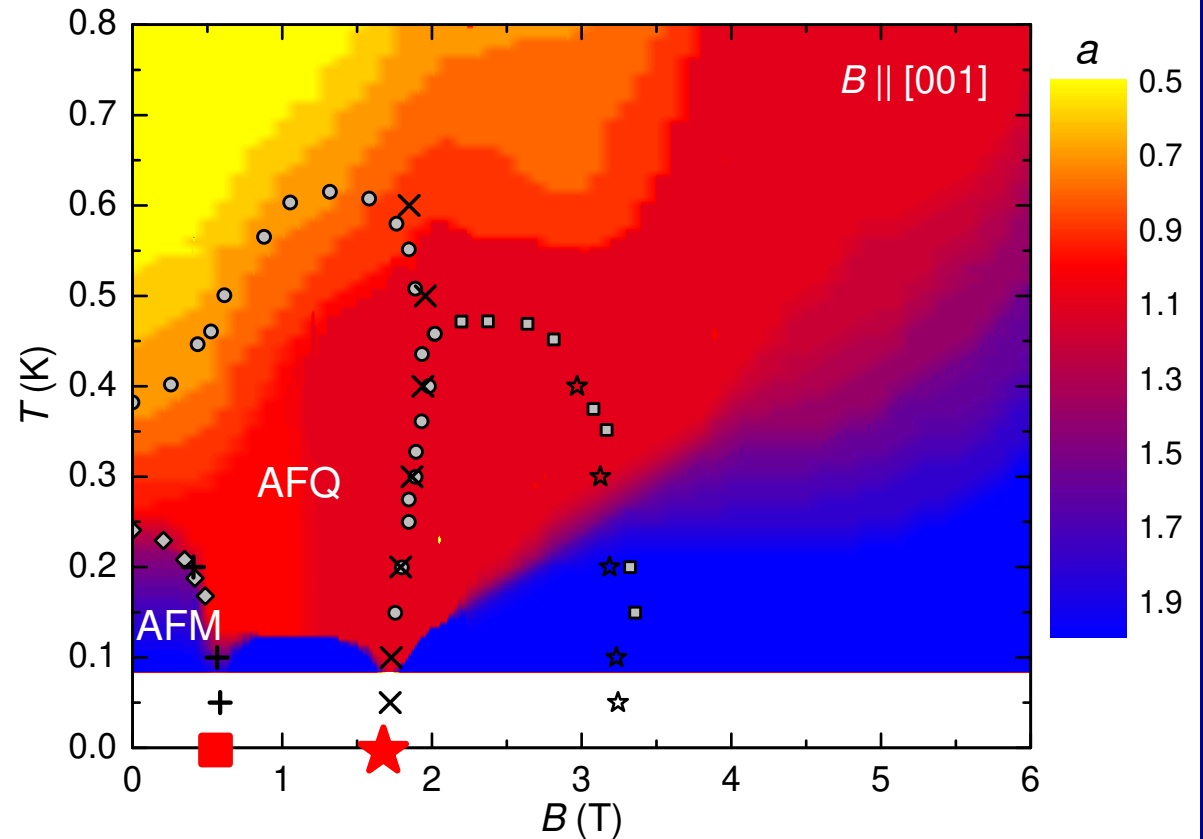
(Custers et al., Nature 424 (2003) 524; Martelli et al., PNAS 116 (2019) 08101)

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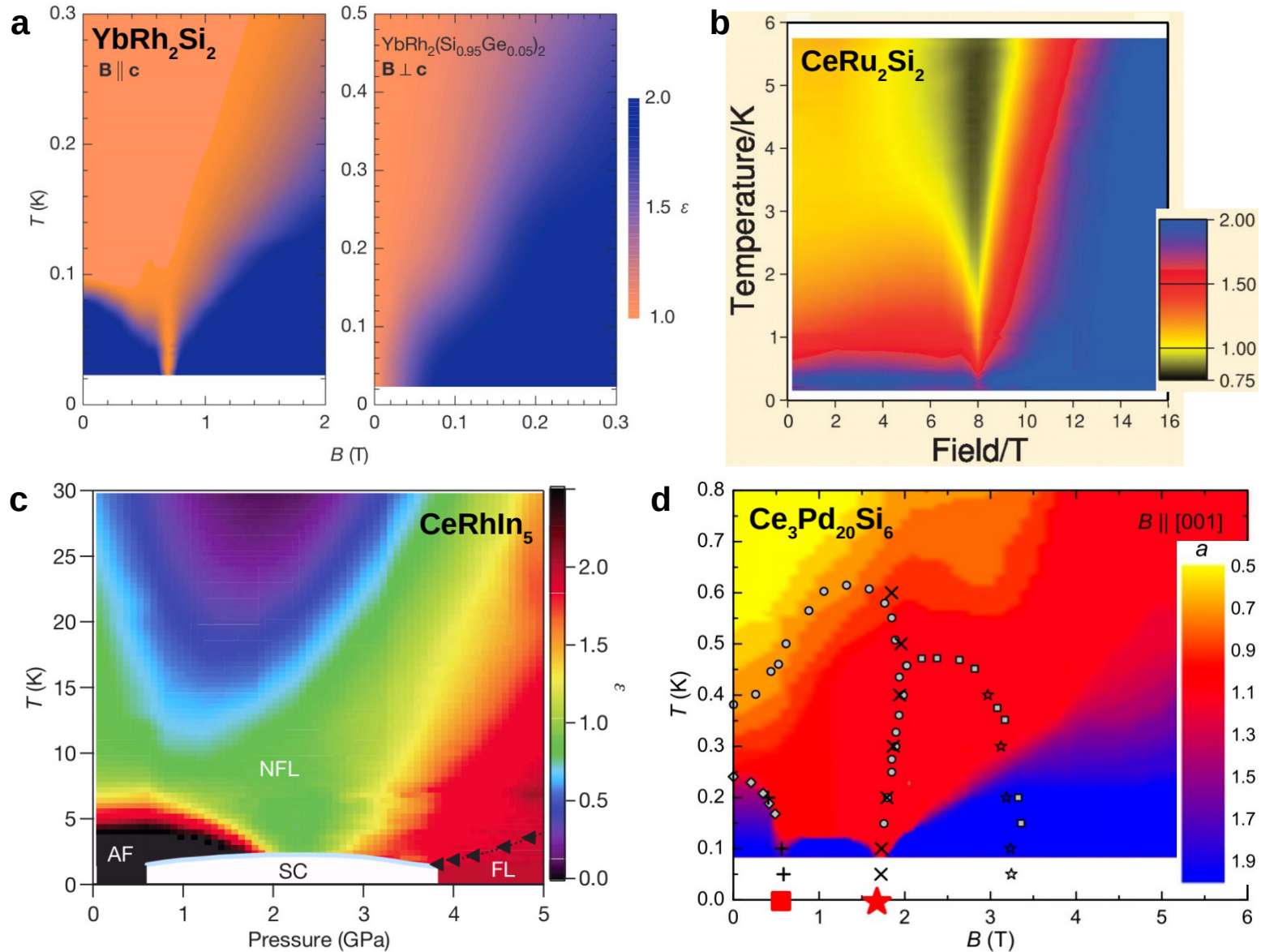


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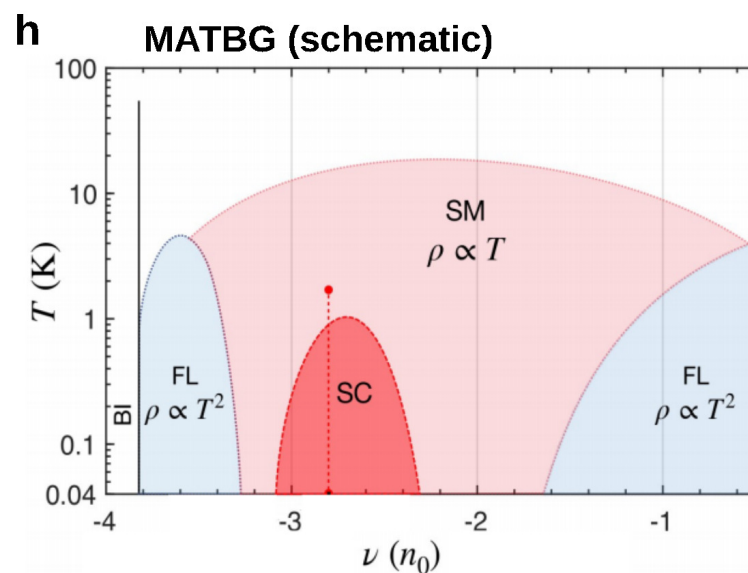
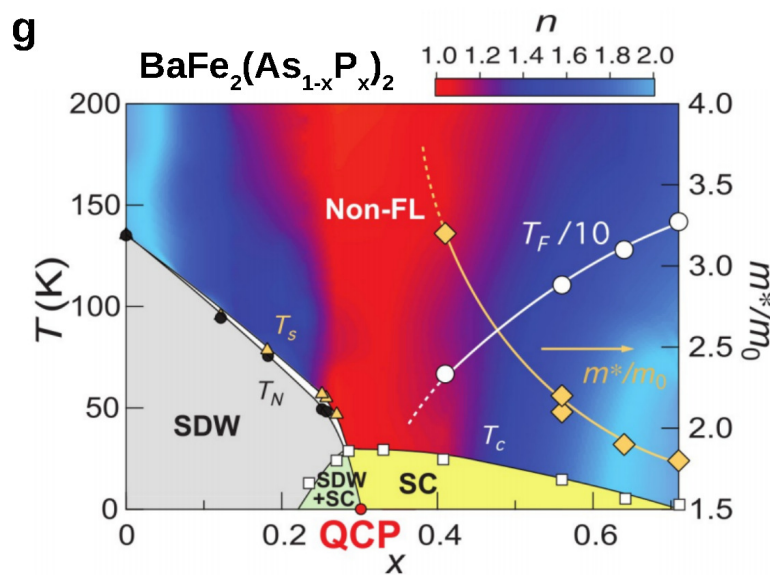
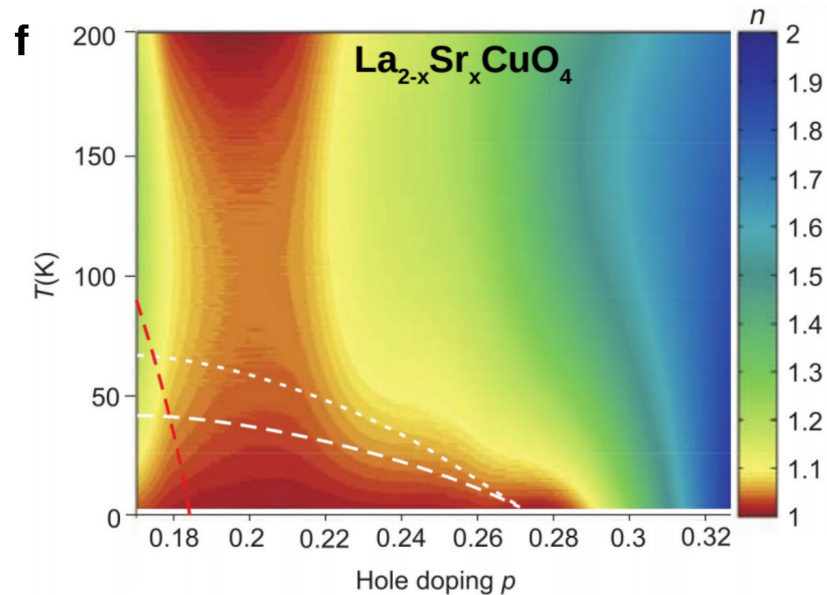
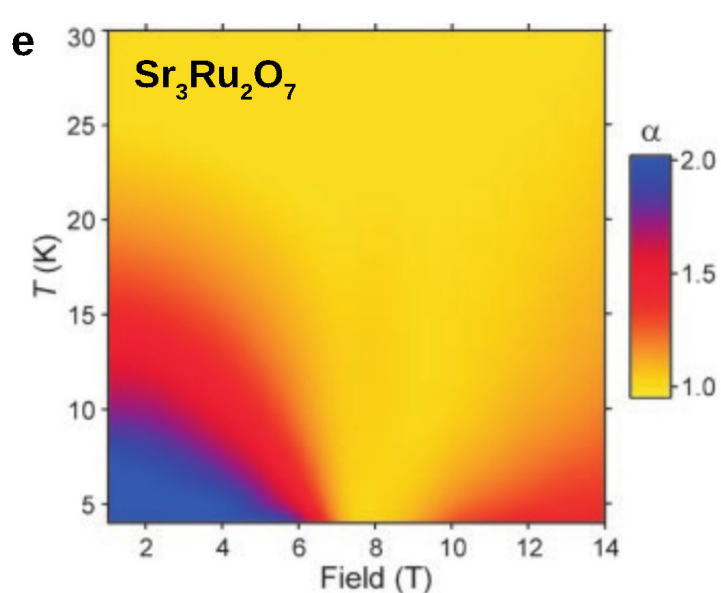
(Custers et al., Nature 424 (2003) 524; Martelli et al., PNAS 116 (2019) 08101)

# Phase diagrams governed by quantum fluctuations



(Taupin & SP, Crystals 12 (2022) 251 and refs. herein)

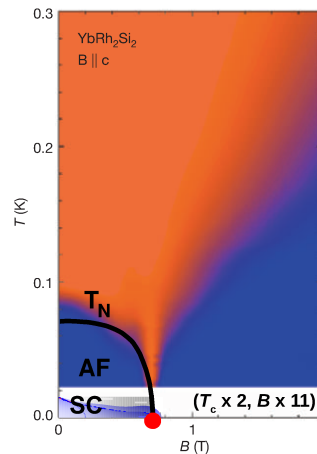
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(Taupin & SP, Crystals 12 (2022) 251 and refs. herein)

# Heavy fermion systems

## From quantum criticality to electronic topology



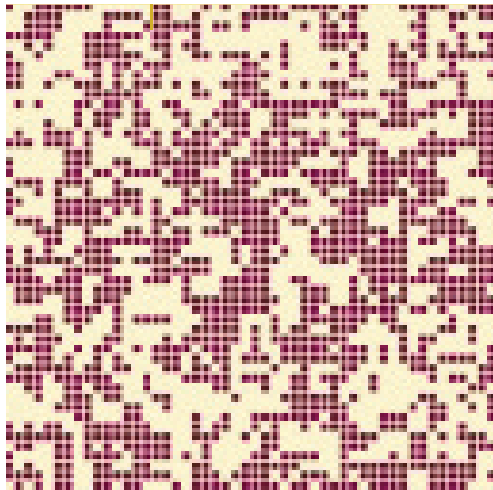
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# Classical continuous phase transitions

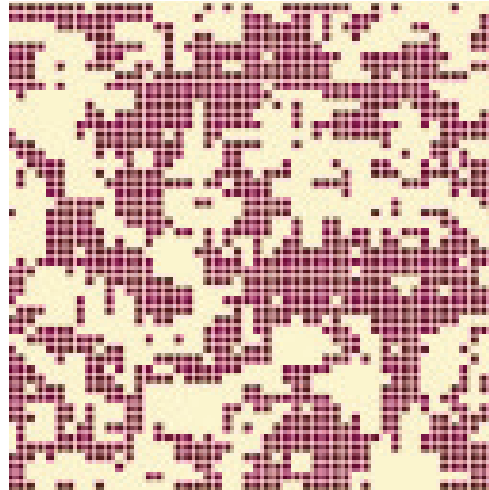
## Classical criticality from order parameter fluctuations

- Order parameter: condensate wave function (normal conductor  $\rightarrow$  superconductor), uniform magnetization (para-  $\rightarrow$  ferromagnet), ...
- Correlation length:  $\xi \sim |T - T_c|^{-\nu}$ , correlation time:  $\tau \sim |T - T_c|^{-\nu Z}$
- Scale invariance, universality

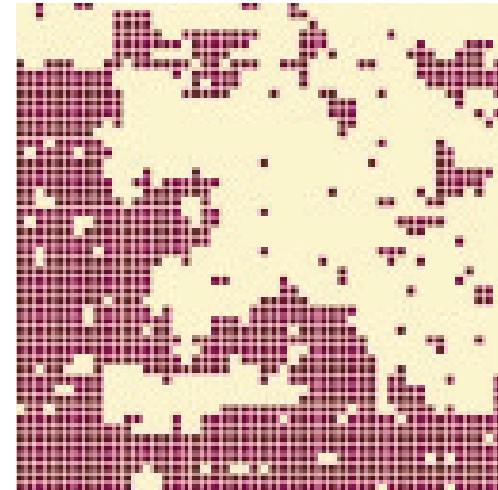
$$T = 2 \cdot T_c$$



$$T = 1.3 \cdot T_c$$



$$T = T_c$$



(T. Vojta, Physik in unserer Zeit 32 (2001) 38: 2D Ising model)

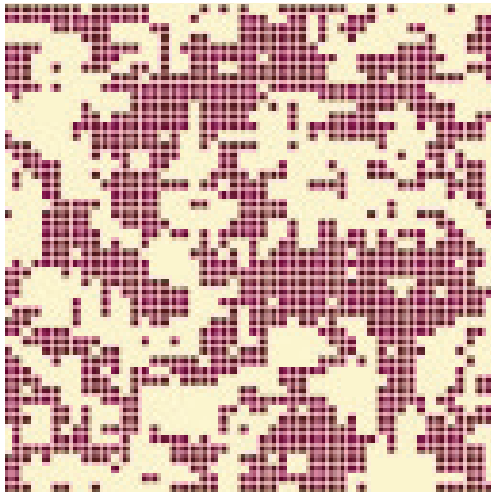


# Classical continuous phase transitions

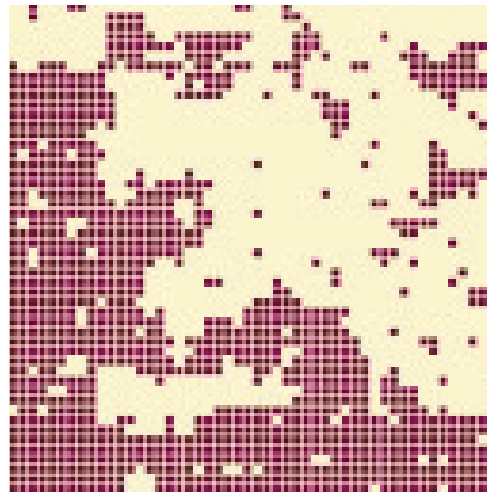
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- Correlation length:  $\xi \sim |T - T_c|^{-\nu}$ , correlation time:  $\tau \sim |T - T_c|^{-\nu Z}$
- Scale invariance, universality

$$T = 1.3 \cdot T_c$$



$$T = T_c$$



$$T = 0K$$



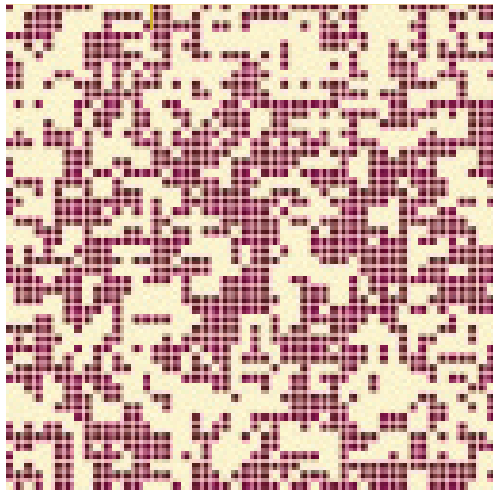
(T. Vojta, Physik in unserer Zeit 32 (2001) 38: 2D Ising model)

# Continuous **quantum** phase transitions

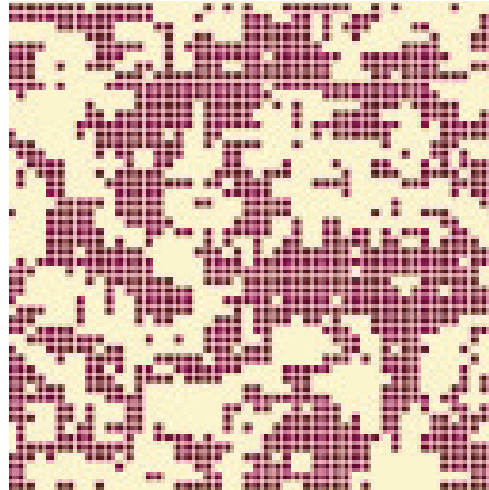
## **Quantum** criticality from order parameter fluctuations

- Order parameter: condensate wave function (normal conductor  $\rightarrow$  superconductor), uniform magnetization (para-  $\rightarrow$  ferromagnet), ...
- Correlation length:  $\xi \sim |B - B_c|^{-\nu}$ , correlation time:  $\tau \sim |B - B_c|^{-\nu Z}$
- Scale invariance, universality;  $\nu_{qc} \neq \nu_{cc}$

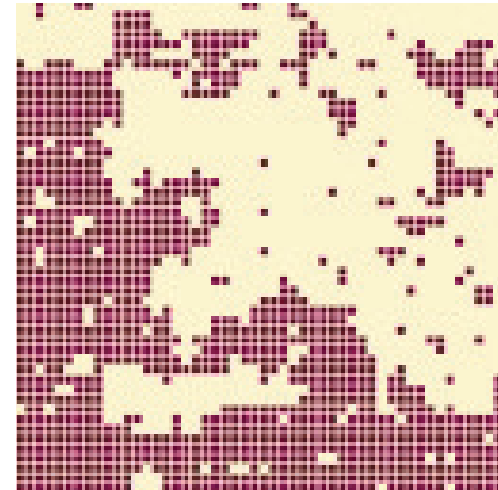
**At  $T = 0$ :**  $B = 2 \cdot B_c$



$B = 1.3 \cdot B_c$



$B = B_c$



(T. Vojta, Physik in unserer Zeit 32 (2001) 38: 2D Ising model)

# “Normal” QCPs follow Ginzburg, Landau, Wilson paradigm

## Predictions for some thermodynamic properties

$d = 2$	$d = 3$	$d = 2$	$d = 3$
$z = 2$	$z = 2$	$z = 3$	$z = 3$

---

$$\alpha_{\text{cr}} \sim \begin{array}{cccc} \ln \ln \frac{1}{T} & T^{1/2} & \ln \frac{1}{T} & T^{1/3} \end{array}$$

$$C_{\text{cr}} \sim \begin{array}{cccc} T \ln \frac{1}{T} & -T^{3/2} & T^{2/3} & T \ln \frac{1}{T} \end{array}$$

$$\Gamma_{r,\text{cr}} \sim \begin{array}{cccc} \frac{\ln \ln \frac{1}{T}}{T \ln \frac{1}{T}} & -T^{-1} & T^{-2/3} \ln \frac{1}{T} & \left( T^{2/3} \ln \frac{1}{T} \right)^{-1} \end{array}$$

$d$ : dimension,  $z = 2$ : AFM metal,  $z = 3$ : FM metal

$\alpha$ : thermal expansion,  $C$ : specific heat,  $\Gamma = \alpha/C$ : Grüneisen ratio

(v. Löhneysen et al., Rev. Mod. Phys. 79 (2007) 1015; Hertz & Millis)

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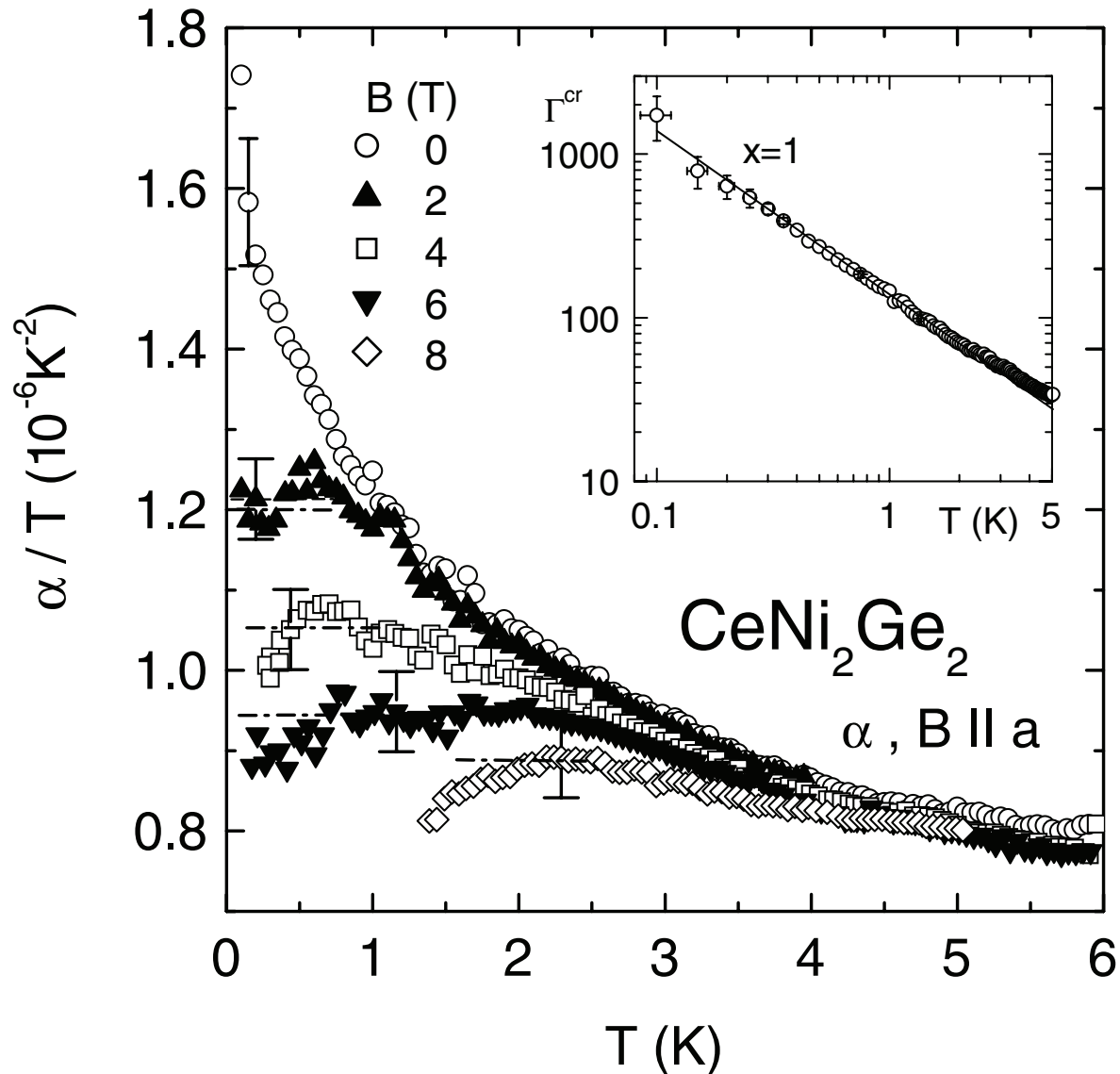
$$\Gamma_{r,\text{cr}} \sim \begin{array}{cccc} \frac{\ln \ln \frac{1}{T}}{T \ln \frac{1}{T}} & -T^{-1} & T^{-2/3} \ln \frac{1}{T} & \left( T^{2/3} \ln \frac{1}{T} \right)^{-1} \end{array}$$

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(v. Löhneysen et al., Rev. Mod. Phys. 79 (2007) 1015; Hertz & Millis)

# Thermal expansion and Grüneisen ratio: $\text{CeNi}_2\text{Ge}_2$

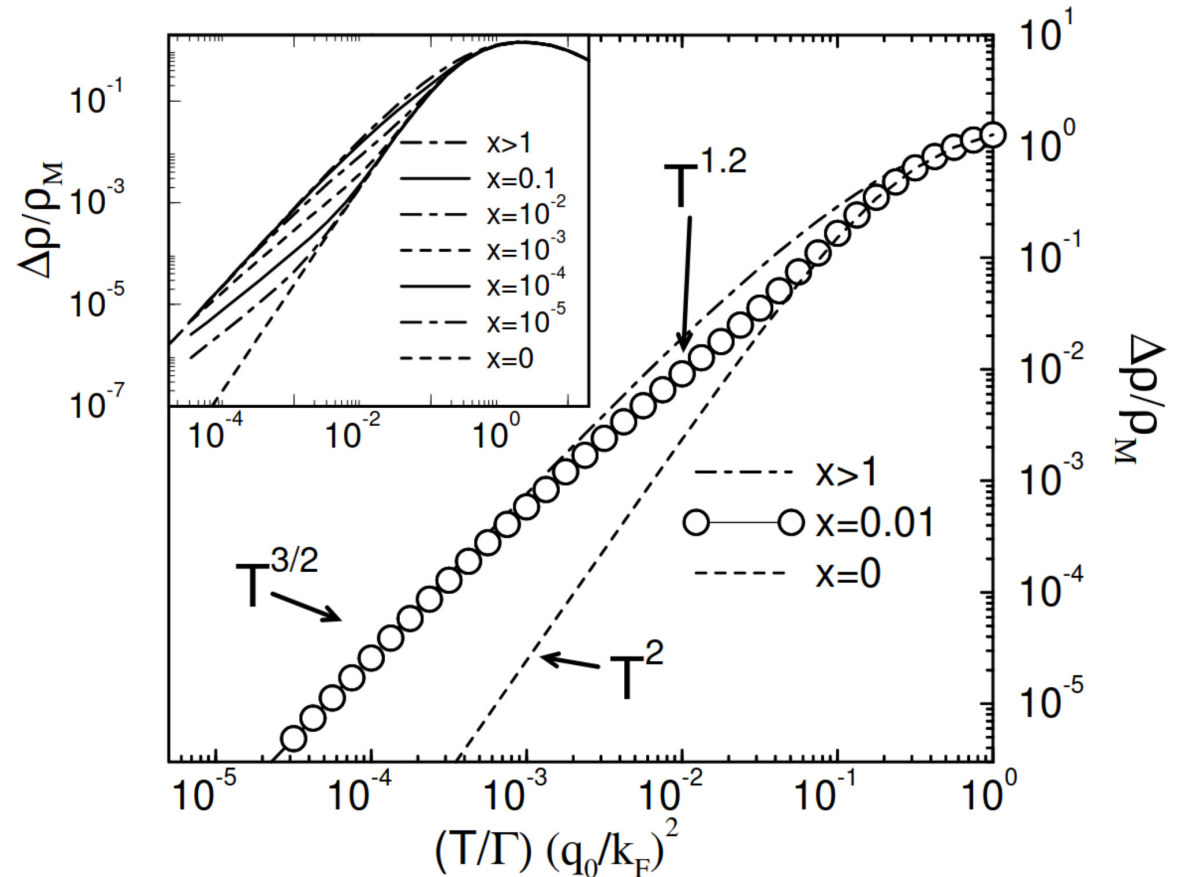
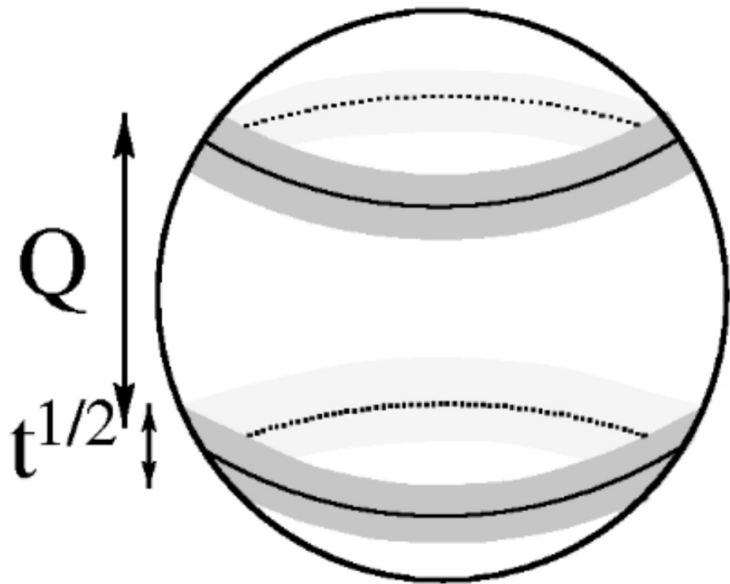


$\Gamma^{\text{cr}} \propto T^{-1}$ , 3D AFM Metall

(Küchler et al., PRL 91 (2003) 066405)

# “Normal” QCPs follow Ginzburg, Landau, Wilson paradigm

## Predictions for electrical resistivity



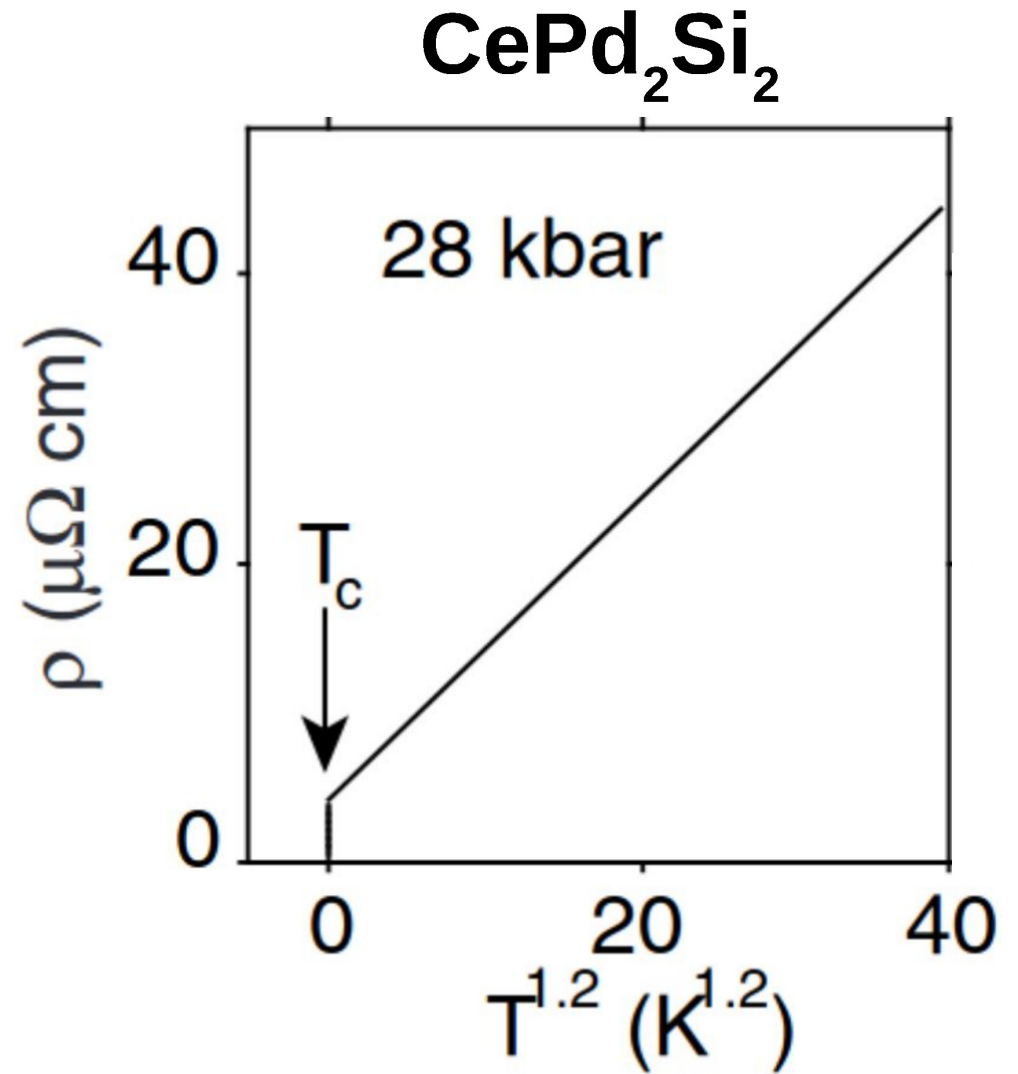
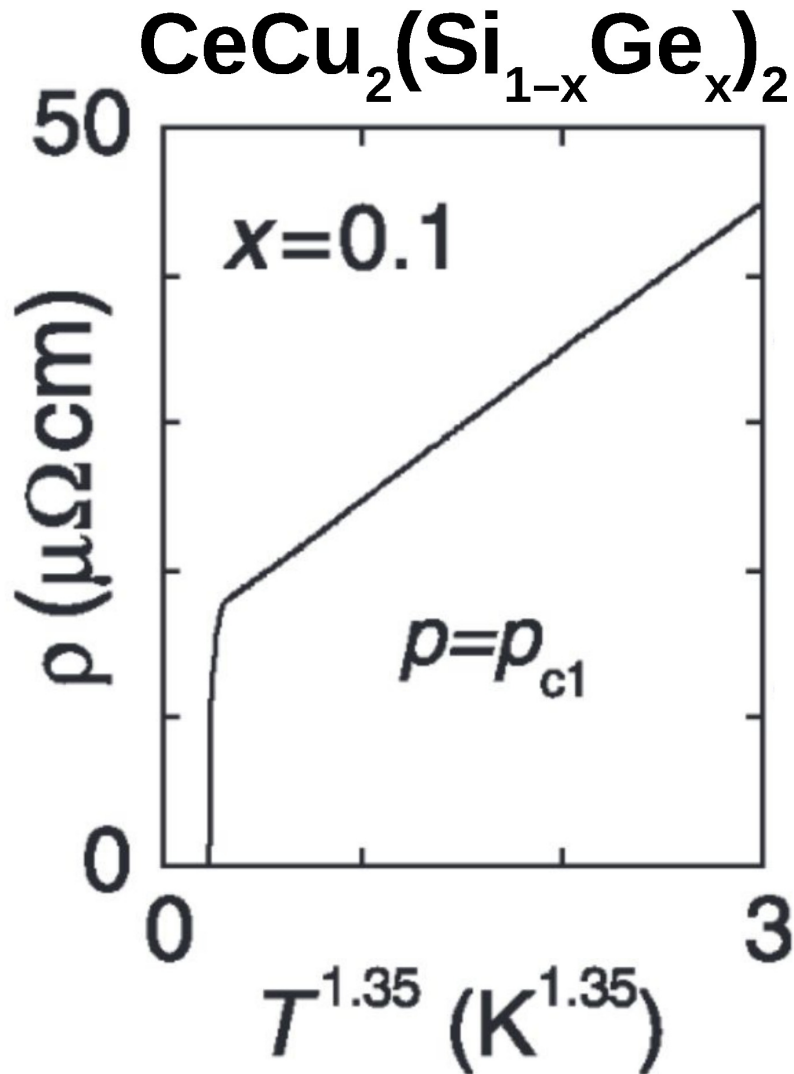
**No linear resistivity**

$x = 0$ : clean limit

$x > 1$ : dirty limit

(Rosch, Phys. Rev. Lett. 82 (1999) 4280)

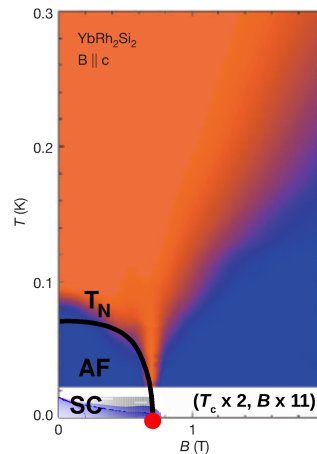
## Electrical resistivity at “normal” QCPs



(Mathur et al., Nature 394 (1998) 39; Yuan et al., Science 302 (2003) 2104)

# Heavy fermion systems

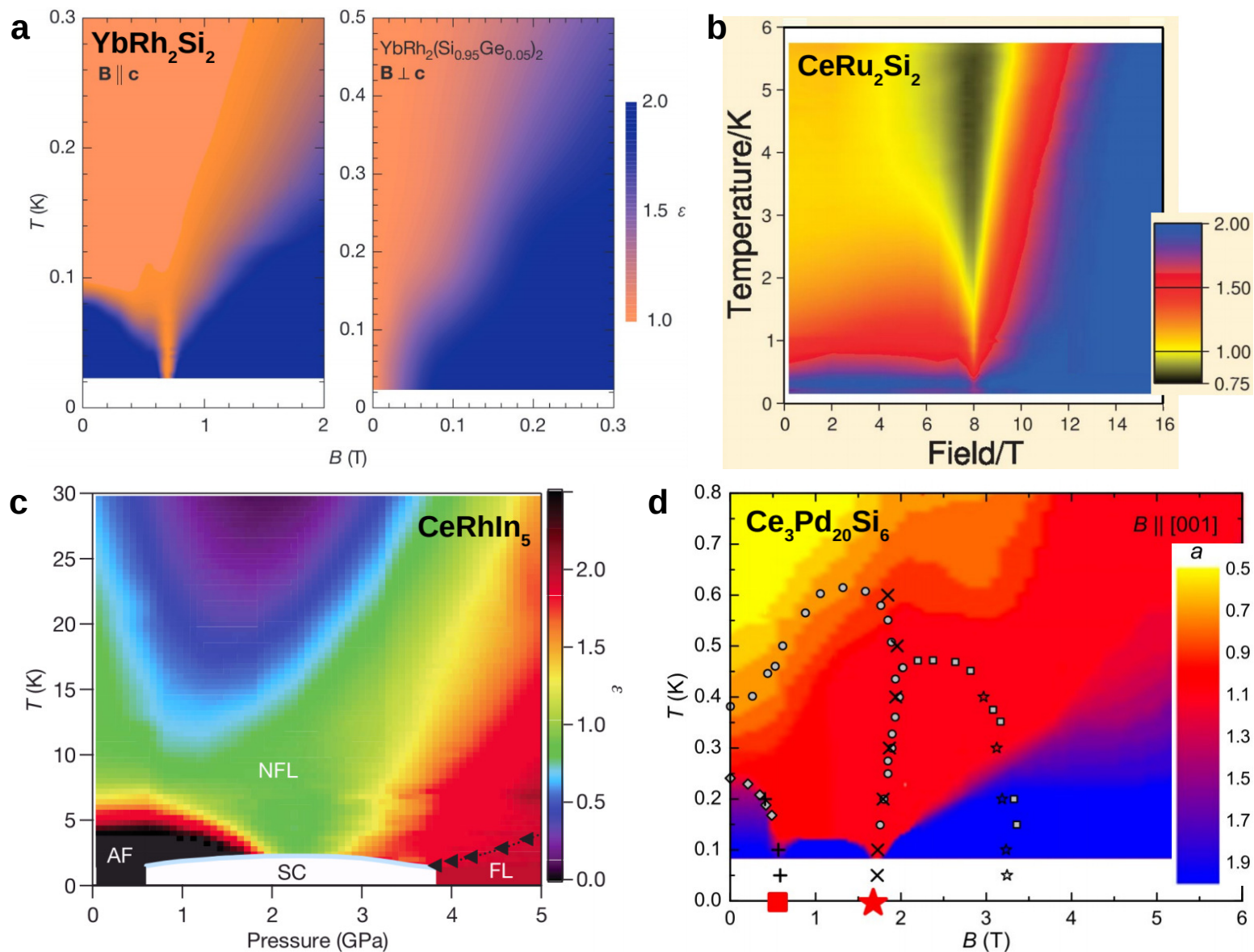
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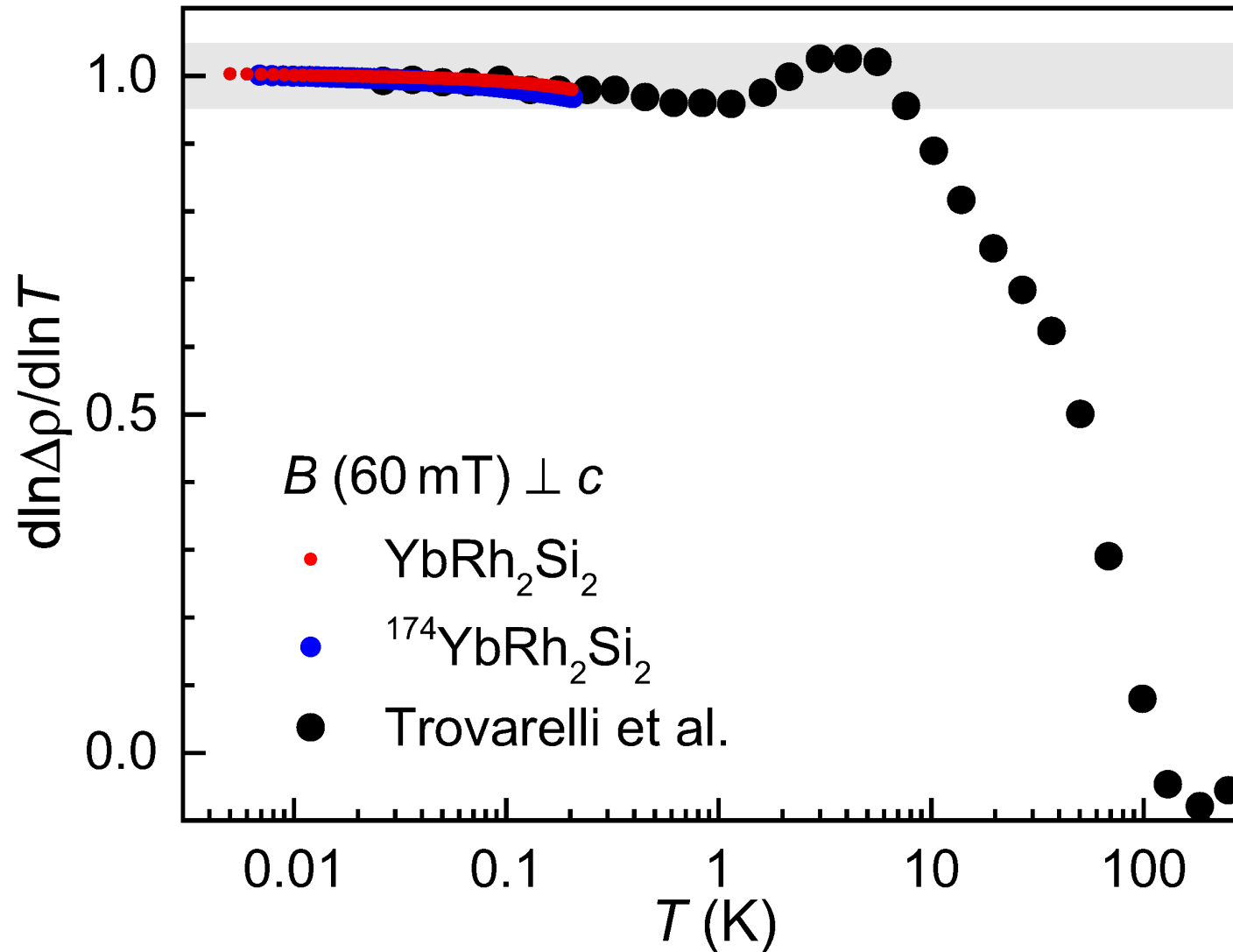


# Deviations from GLW: Linear-in- $T$ “strange metal” resistivity



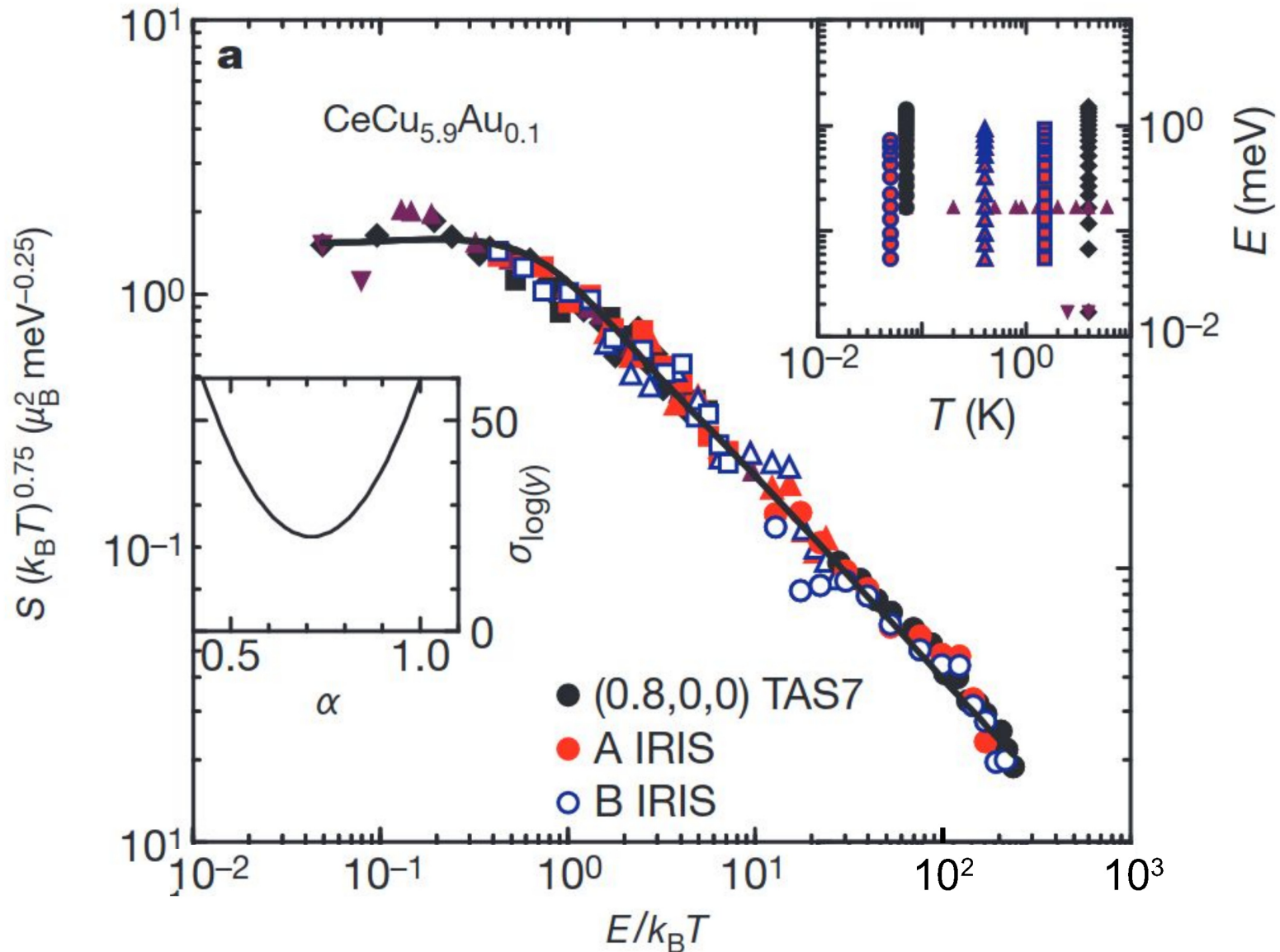
(Taupin & SP, Crystals 12 (2022) 251 and refs. herein)

# $\text{YbRh}_2\text{Si}_2$ : An extreme strange metal (3.5 order of magnitude)



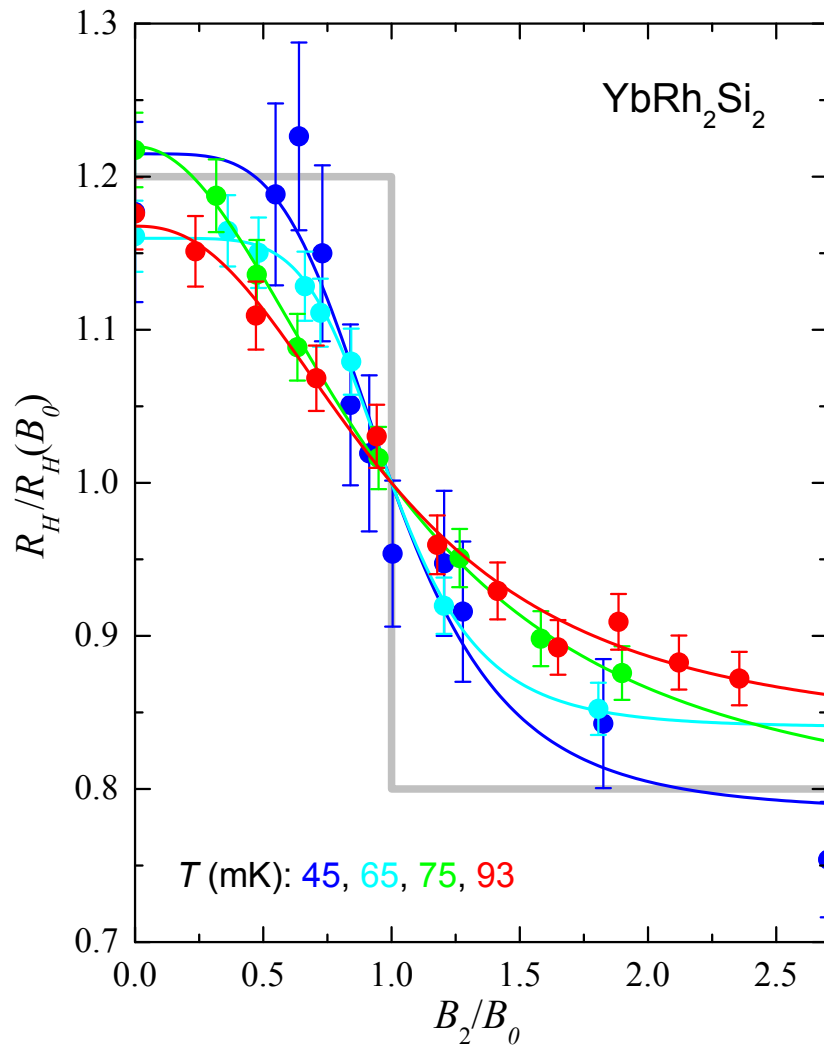
(Nguyen et al., Nat. Commun. 12 (2021) 4341)

# Deviations from GLW: $E/T$ scaling in INS in $\text{CeCu}_{5.9}\text{Au}_{0.1}$



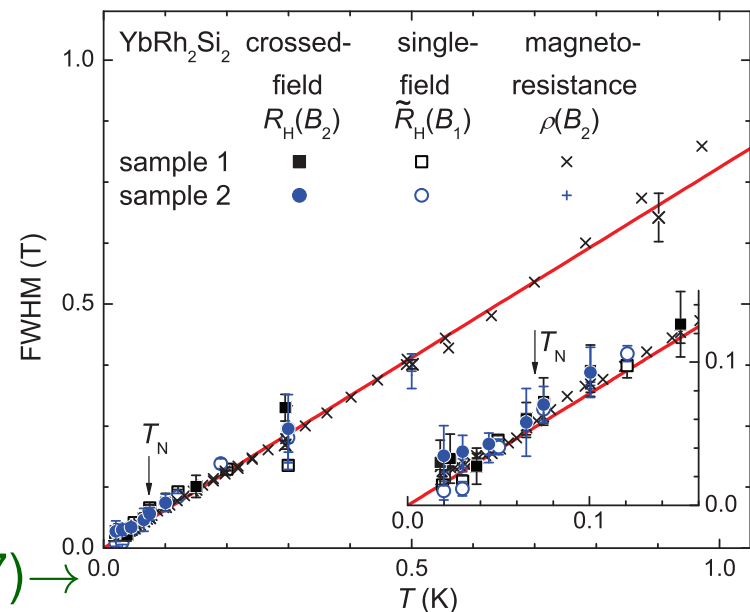
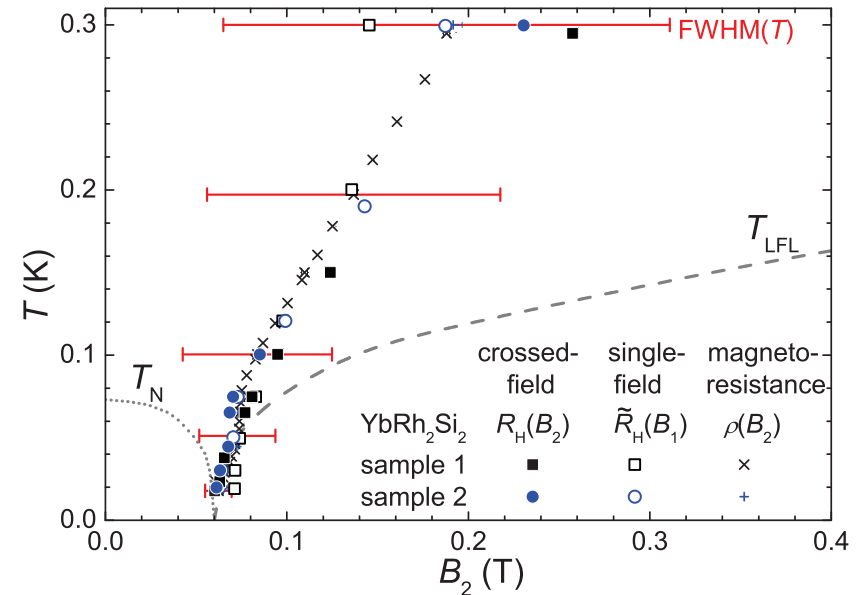
(Schröder et al., Nature 407 (2000) 351; theory: Si et al., ibid 413 (2001) 804)

# Deviations from GLW: Jump in Hall effect in $\text{YbRh}_2\text{Si}_2$

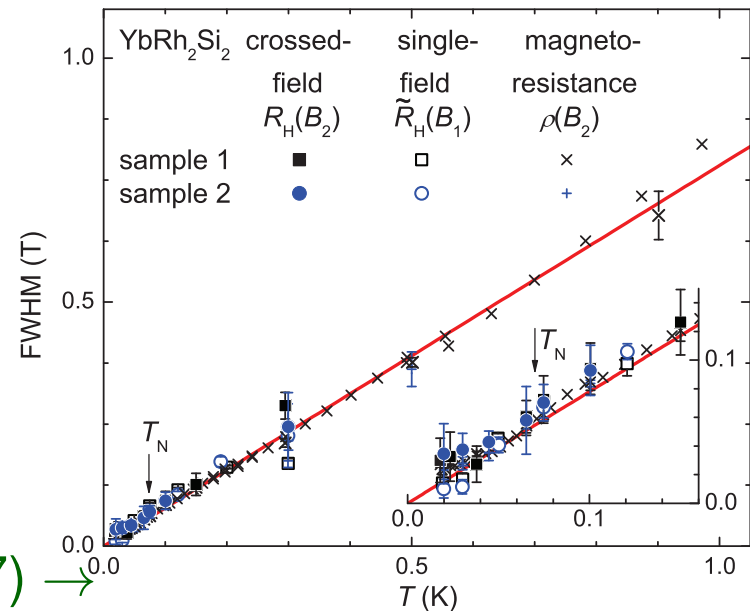
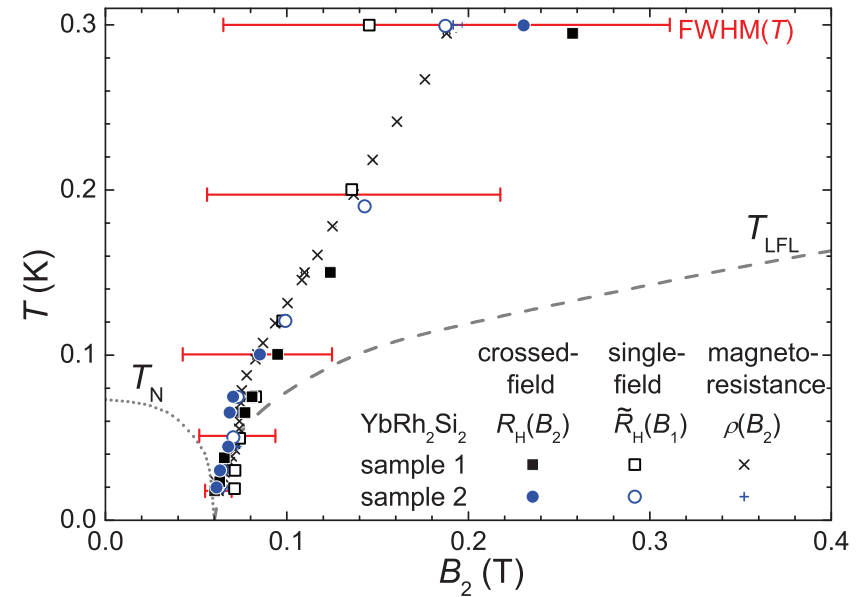
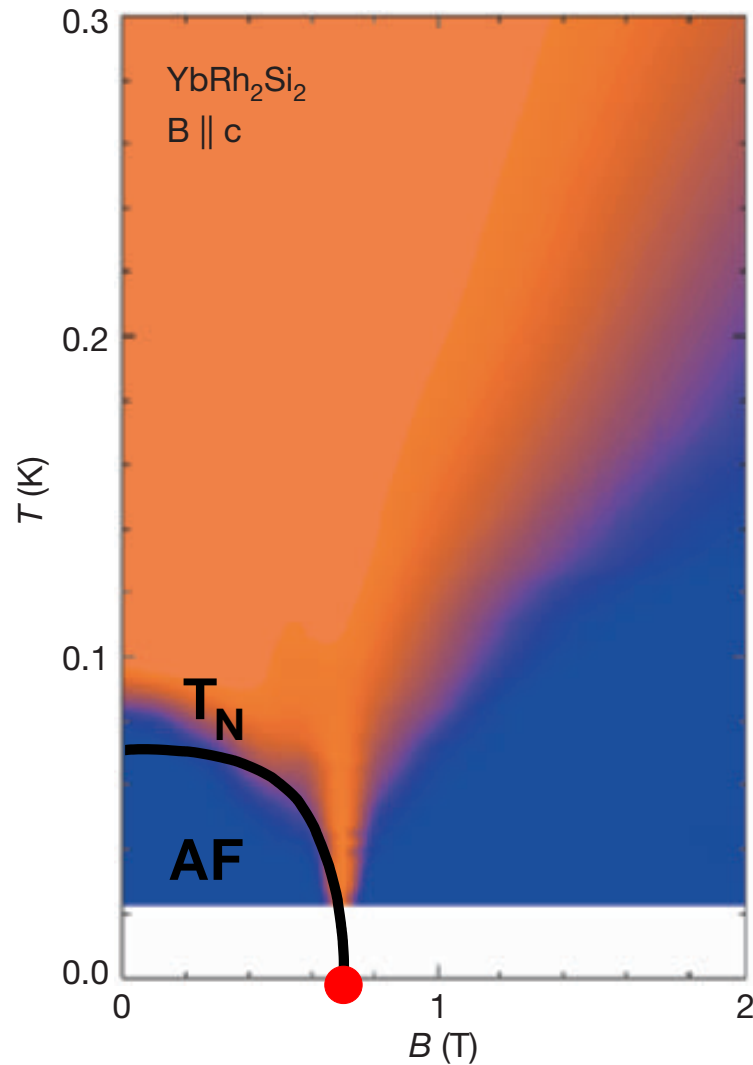


(SP et al., Nature 432 (2004) 881) ↑

(Friedemann et al., PNAS 107 (2010) 14547) →

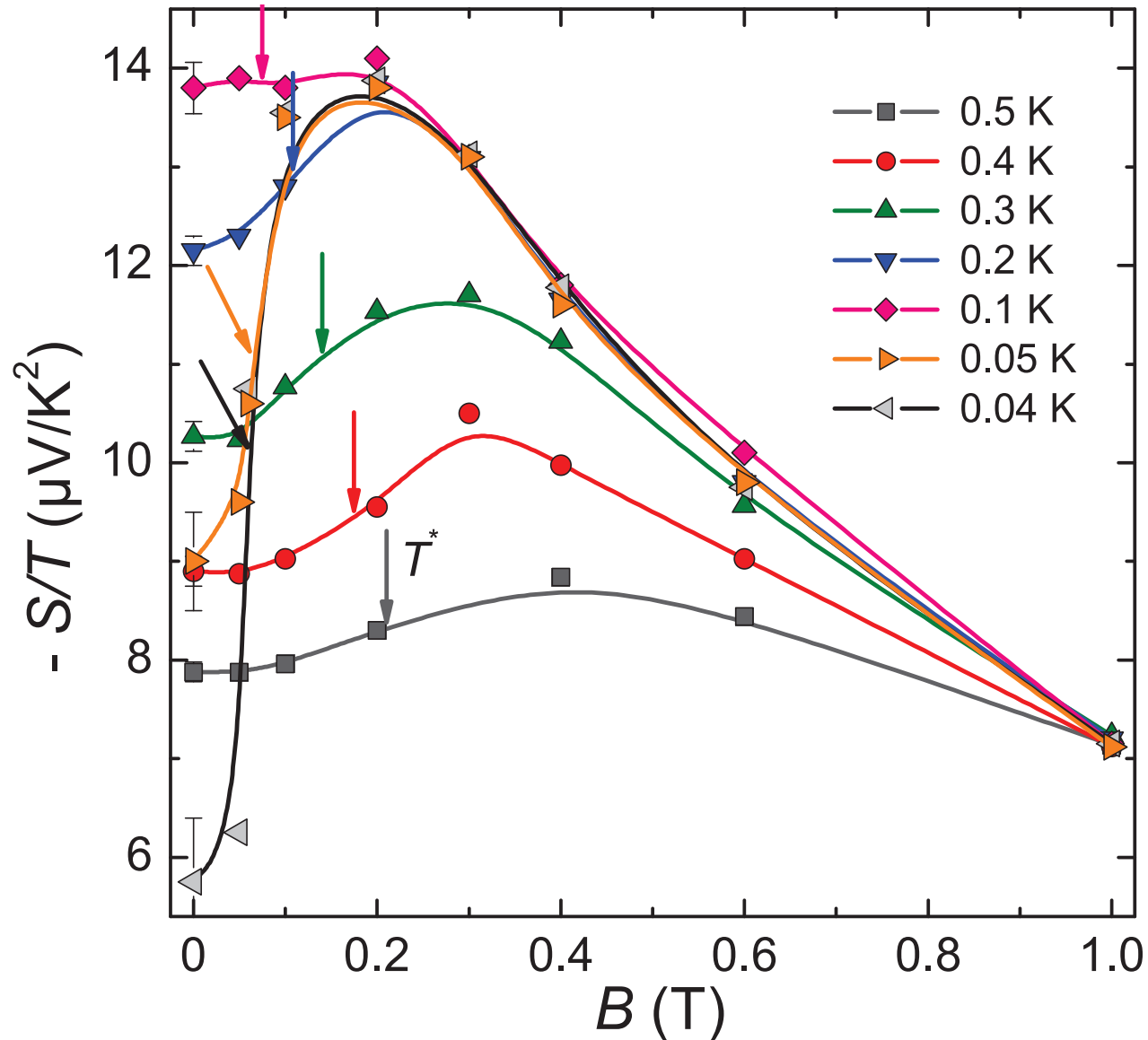


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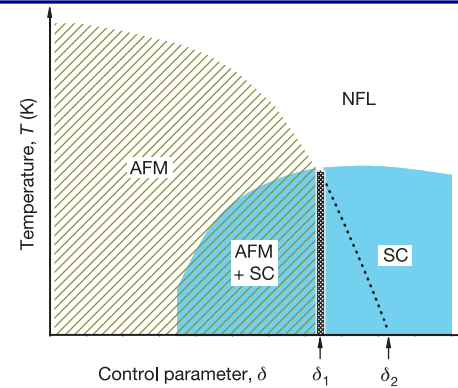
(Custers et al., Nature 424 (2003) 524) ↑  
(Friedemann et al., PNAS 107 (2010) 14547) →

# Deviations from GLW: Jump in thermopower in $\text{YbRh}_2\text{Si}_2$

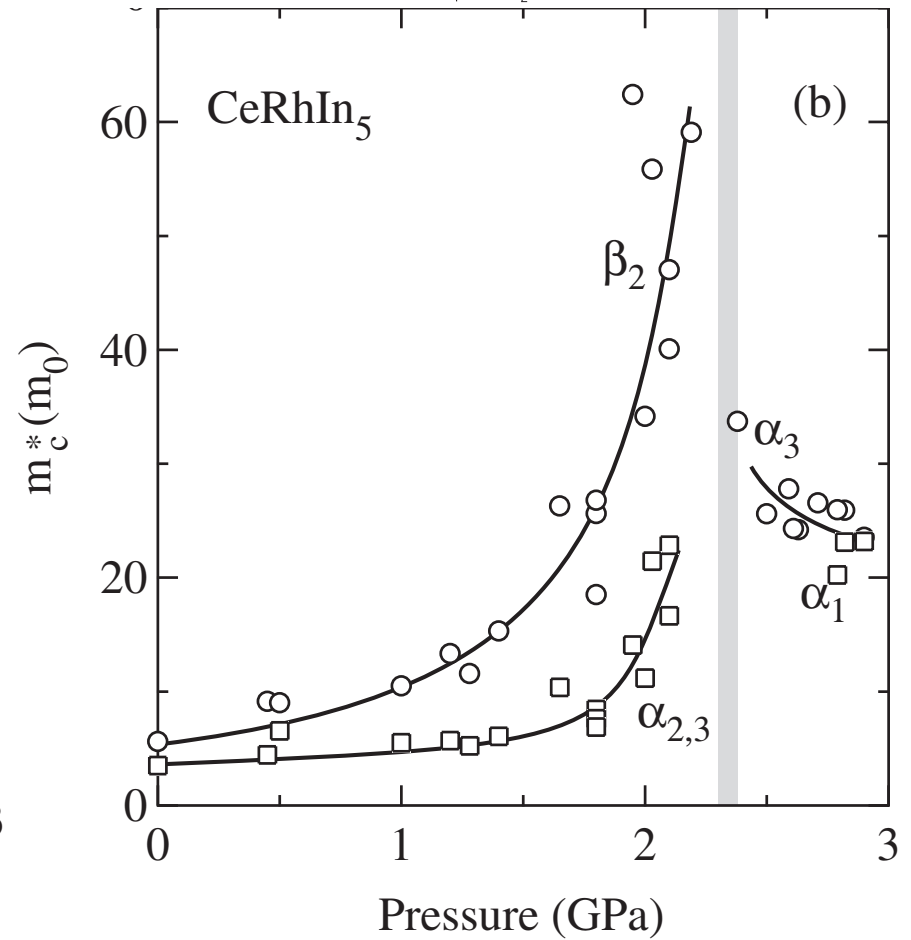
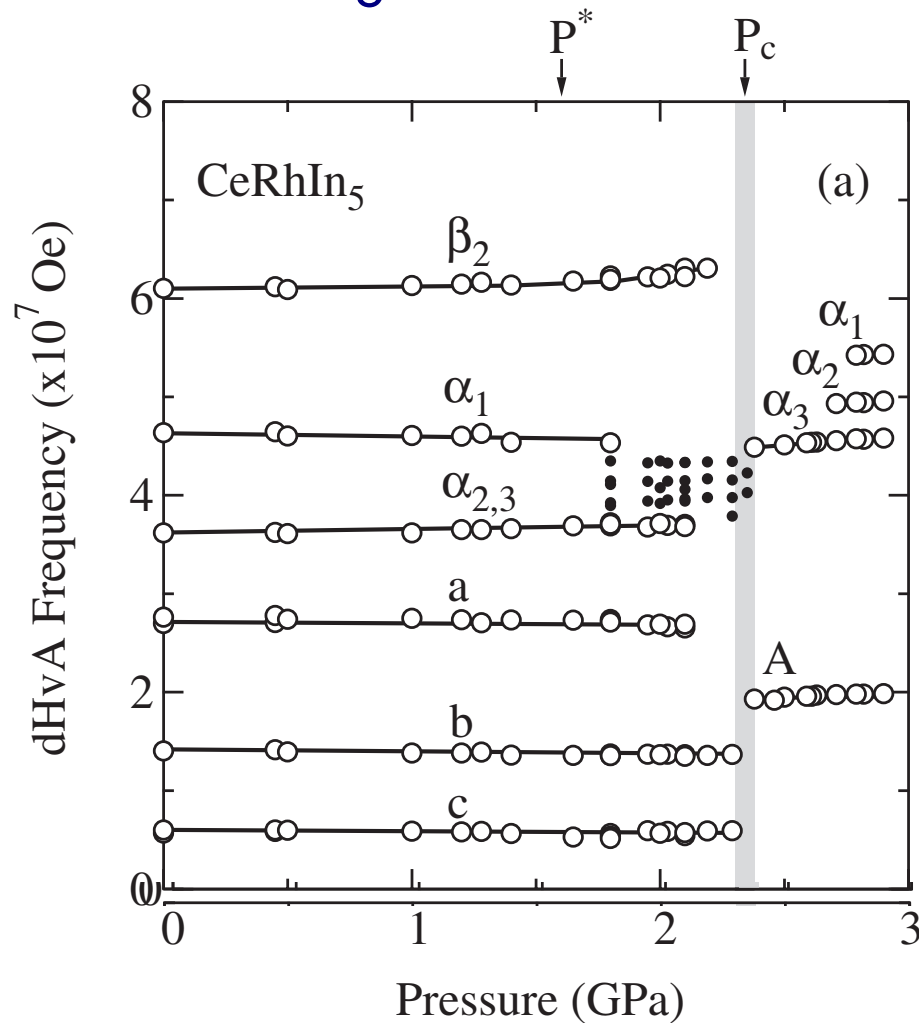


(Hartmann et al., Phys. Rev. Lett. 104 (2010) 096402)

# Deviations from GLW: De Haas-van Alphen study in $\text{CeRhIn}_5$

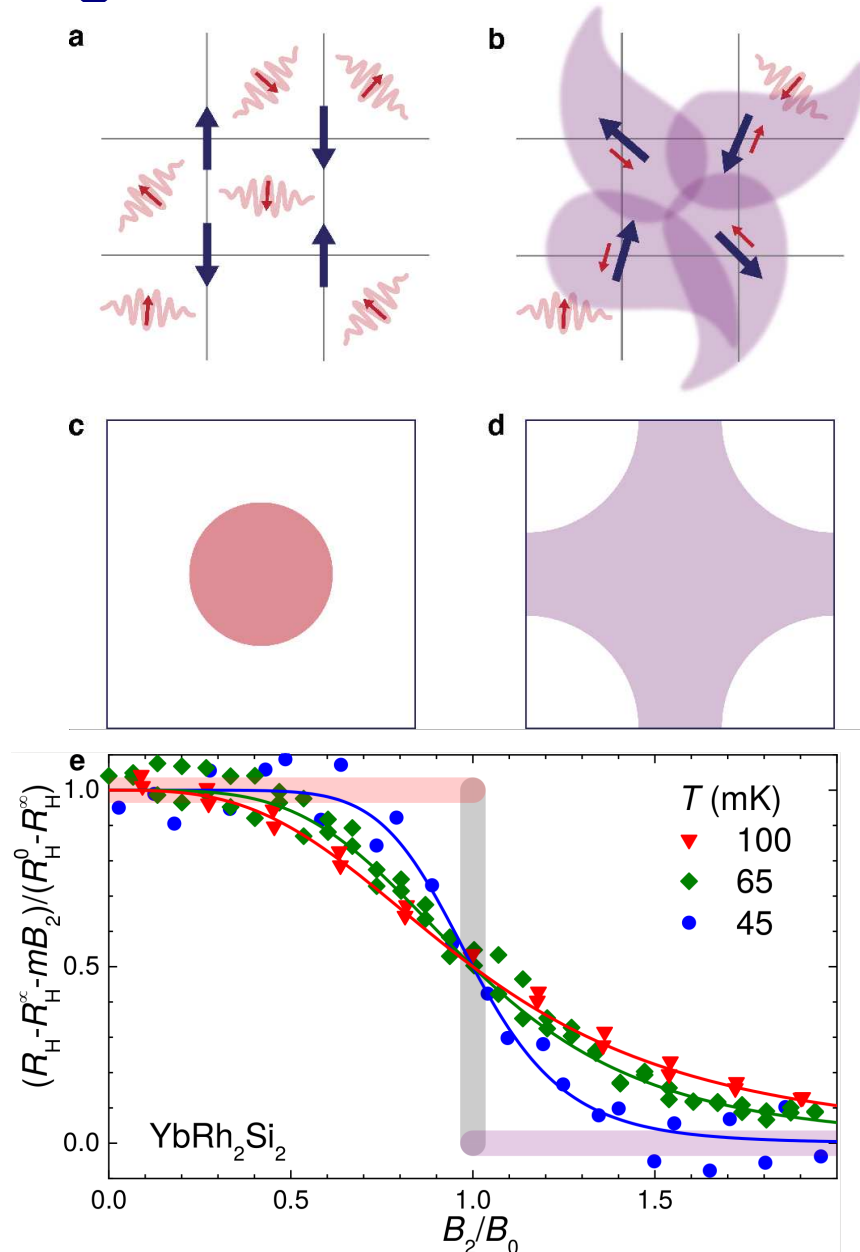


(Park et al.  
Nature 2006)



(Shishido et al., JPSJ 74 (2005) 1103)

# Understanding in terms of Kondo destruction

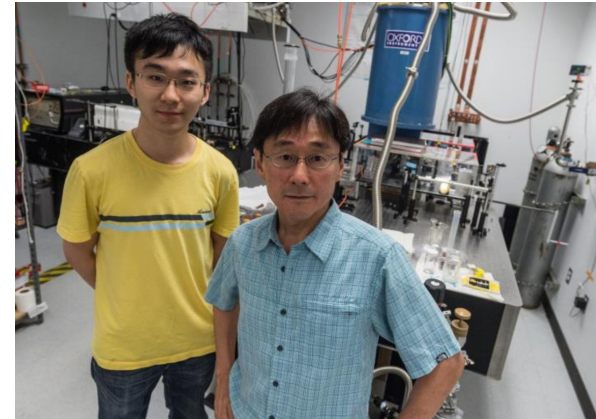


(SP & Si, Nat. Rev. Phys. 3 (2021) 9)

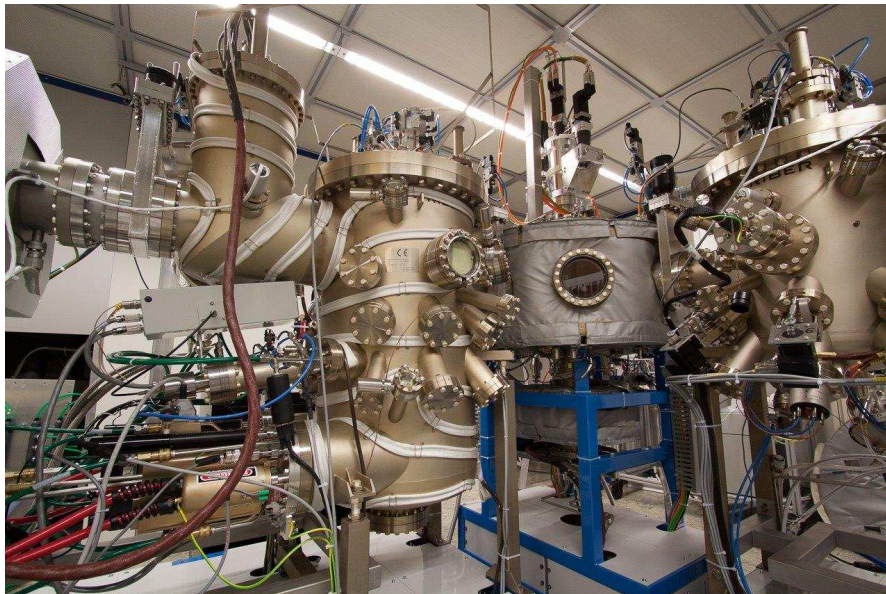


# Dynamical charge response: THz time-domain transmission spectroscopy

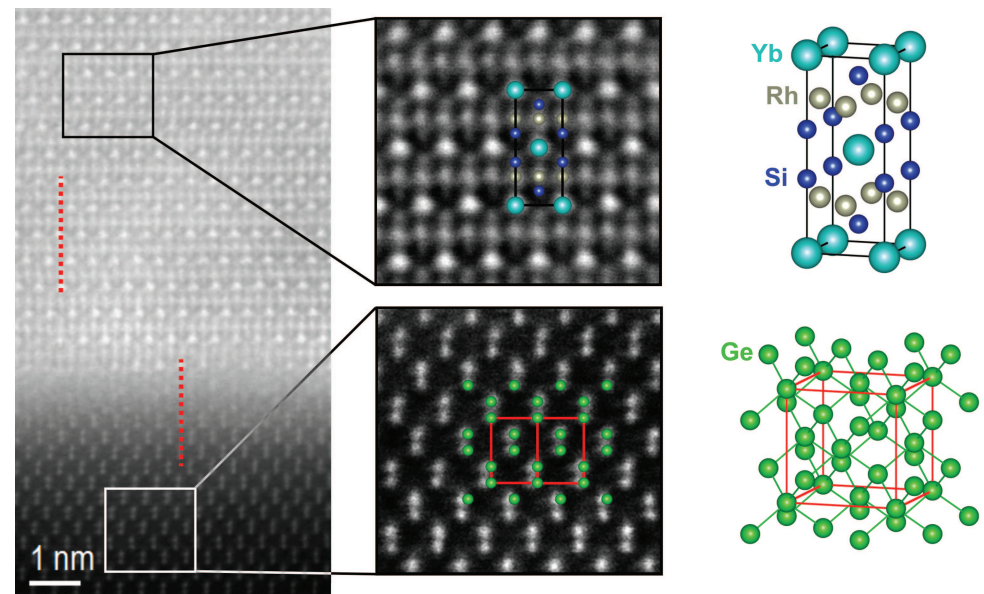
- Real and imag. part of  $\sigma(\omega)$
- No Kramers-Kronig transformation
- Thin films needed!



Molecular beam epitaxy system



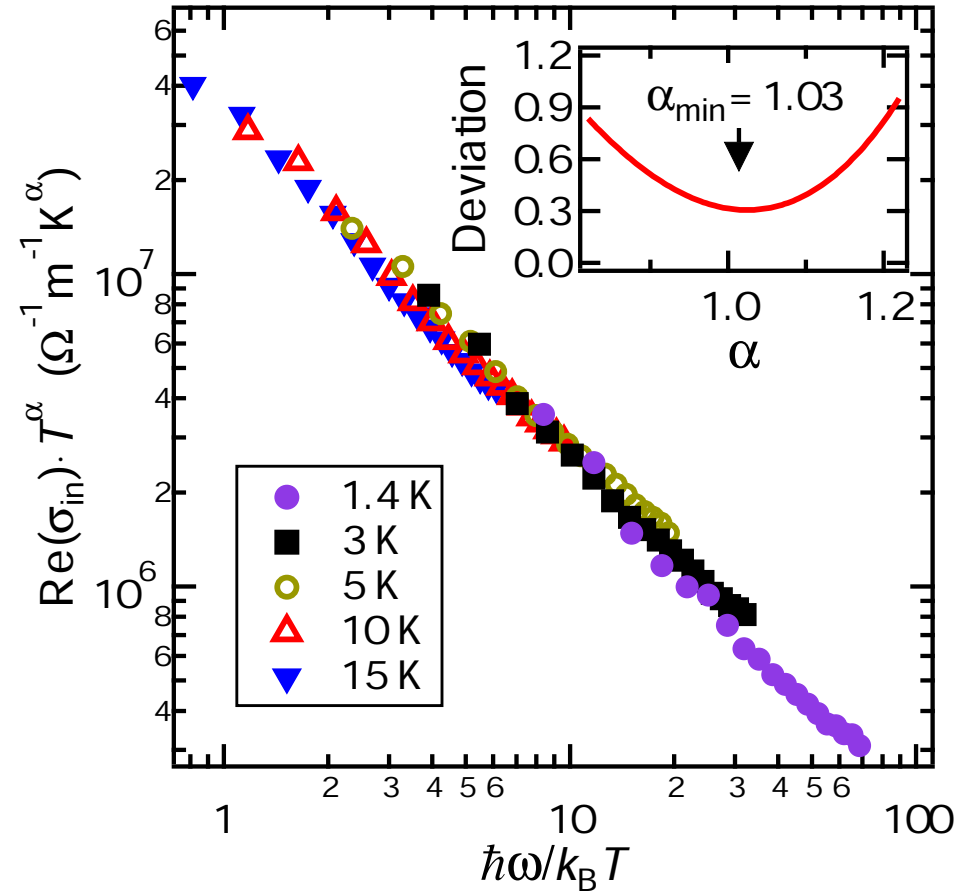
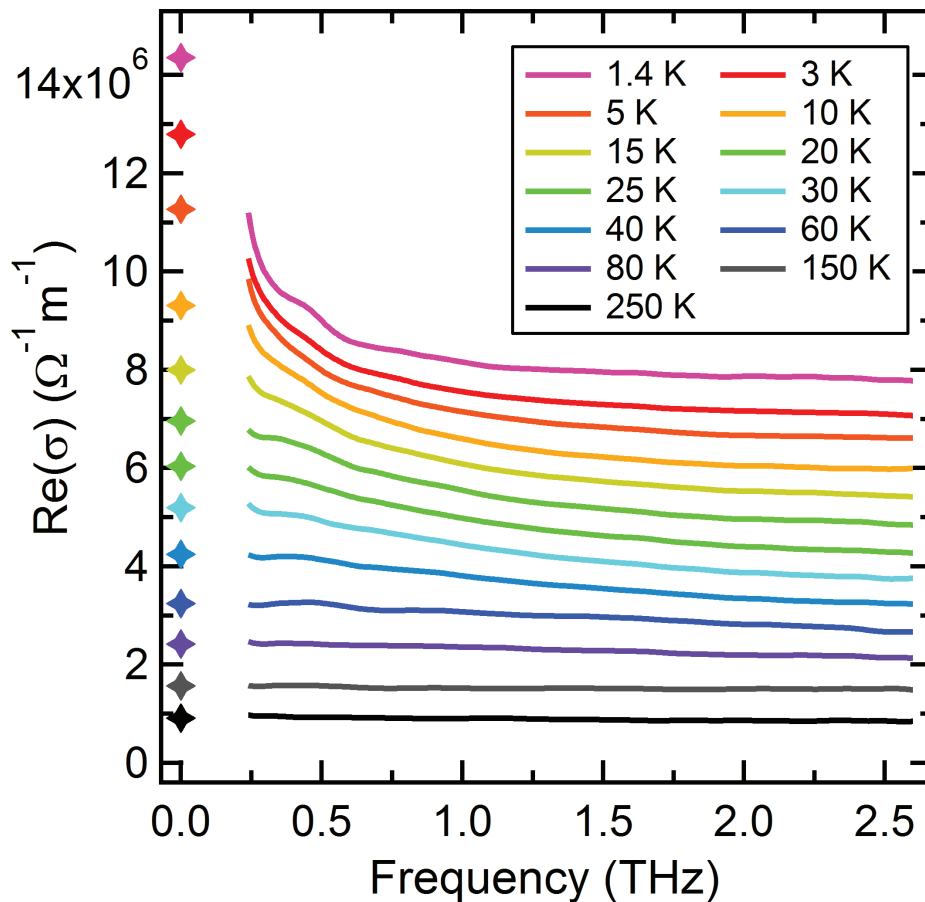
HAADF-STEM image



(Prochaska et al., Science 367 (2020) 285)

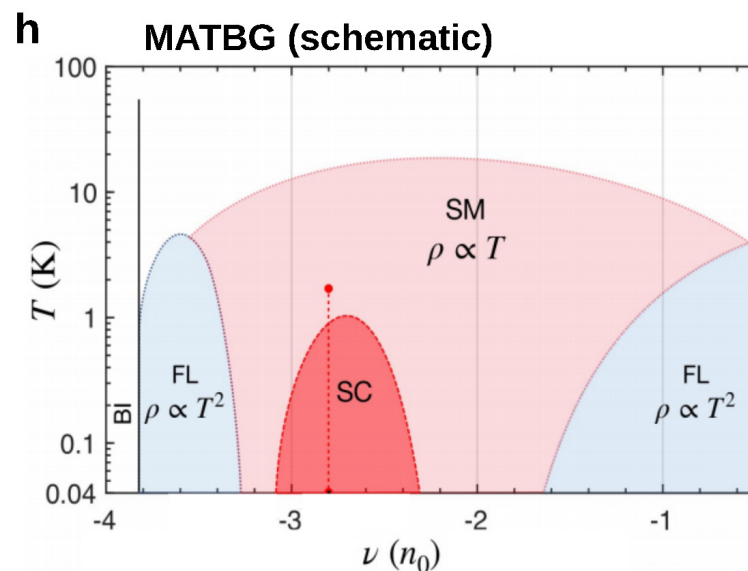
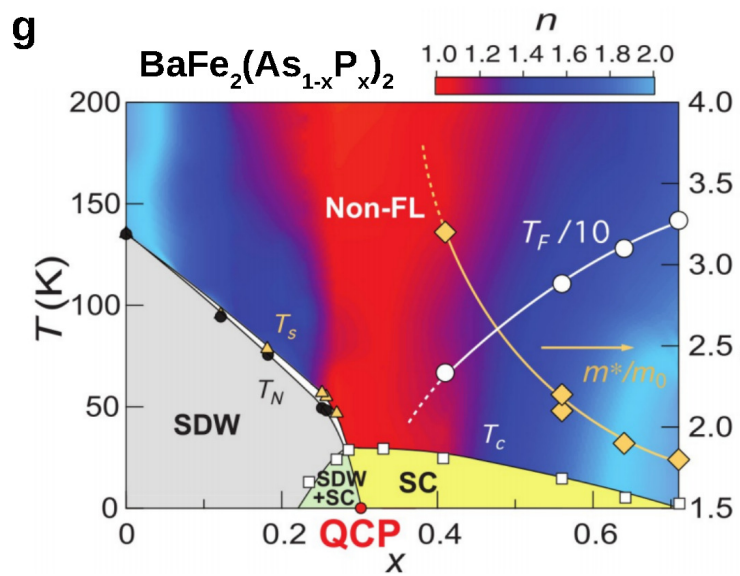
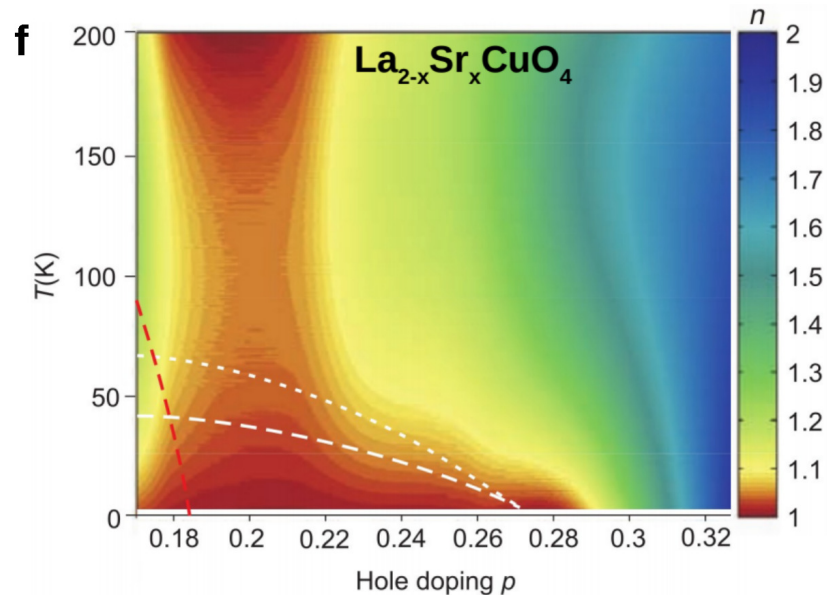
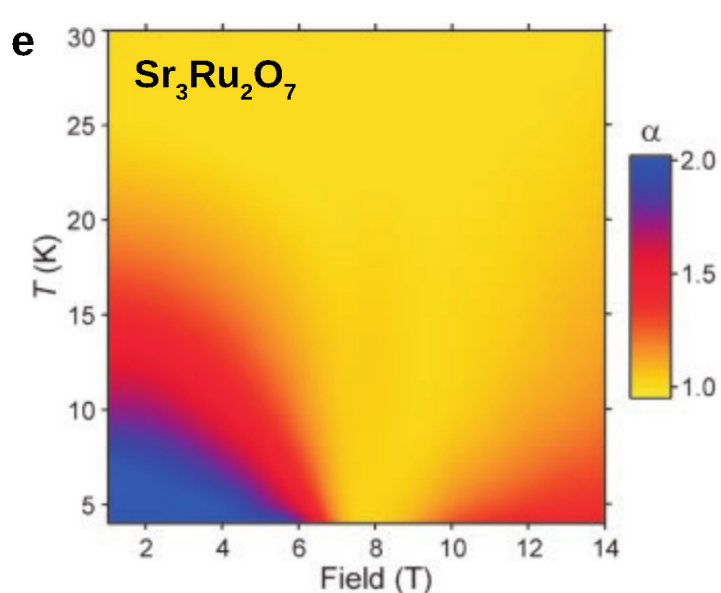
# Dynamical scaling of optical conductivity in $\text{YbRh}_2\text{Si}_2$

THz time-domain transmission spectroscopy on MBE films of  $\text{YbRh}_2\text{Si}_2$



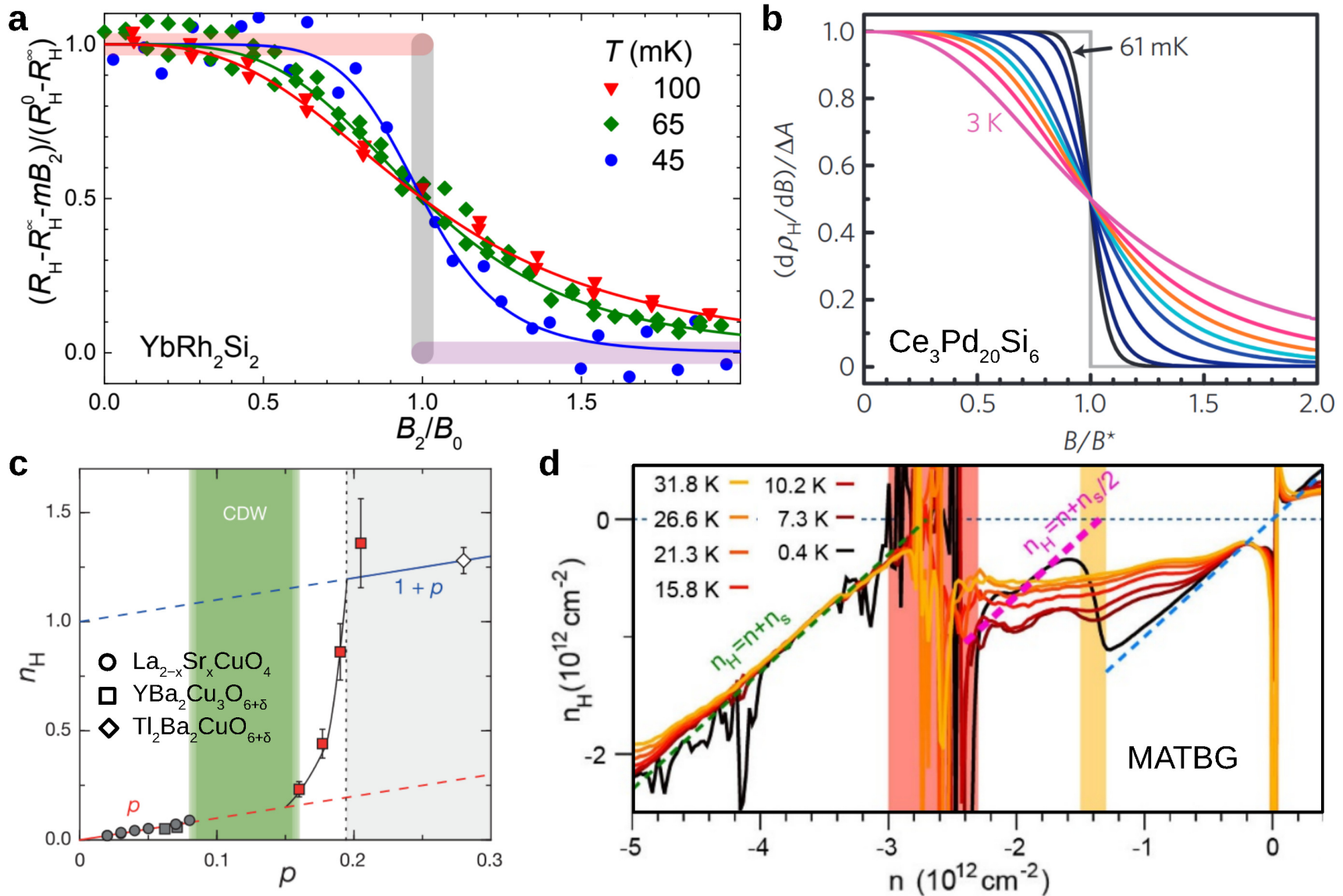
(Prochaska et al., Science 367 (2020) 285)

# Relation to other SCES: Strange metal behavior



(Taupin & SP, Crystals 12 (2022) 251 and refs. herein)

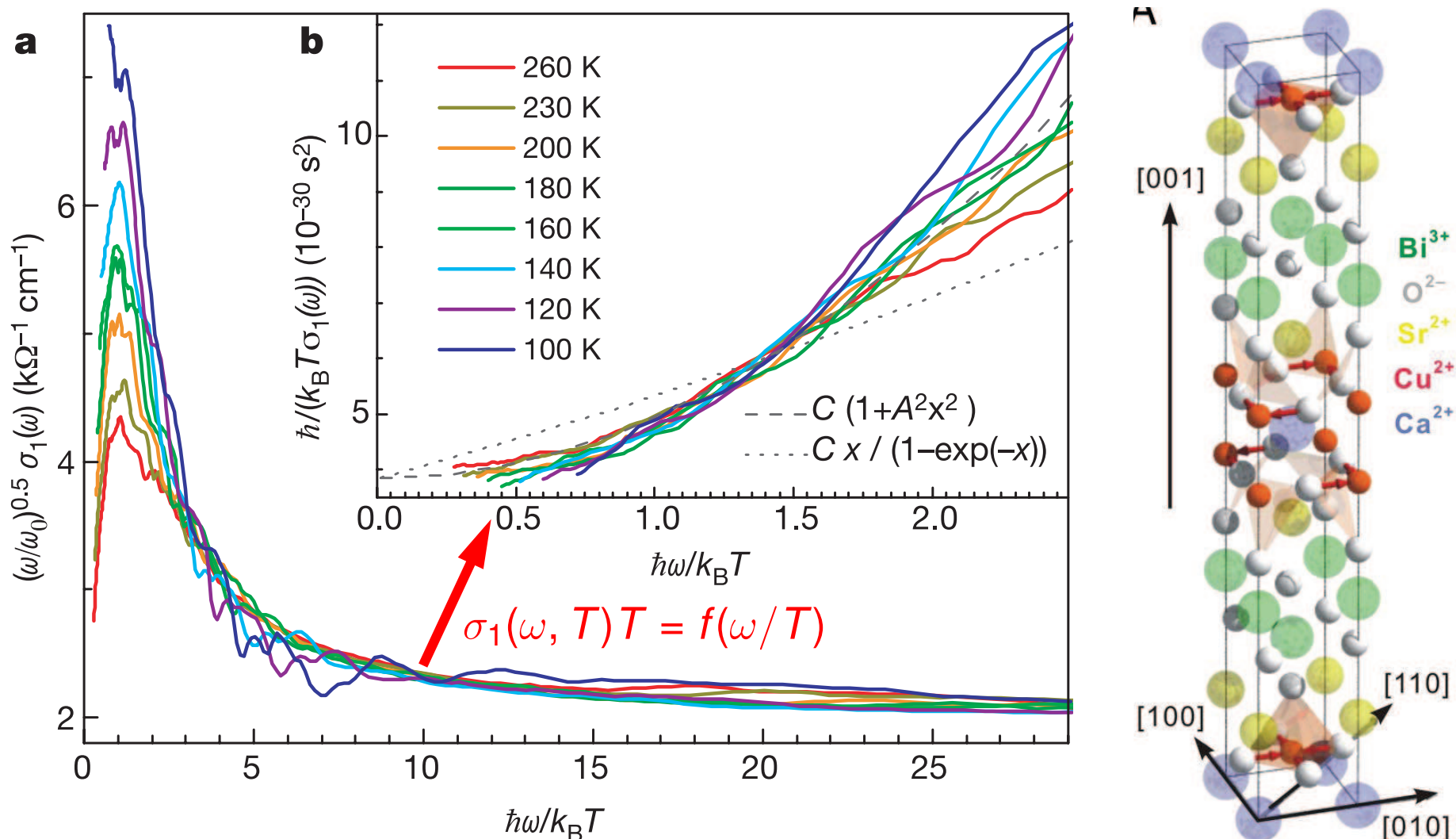
# Relation to other SCES: Carrier (de)localization



(M. Taupin & SP, Crystals 12 (2022) 251 and refs. herein)

# Relation to other SCES: Dynamical scaling in $\sigma(\omega)$ in cuprates

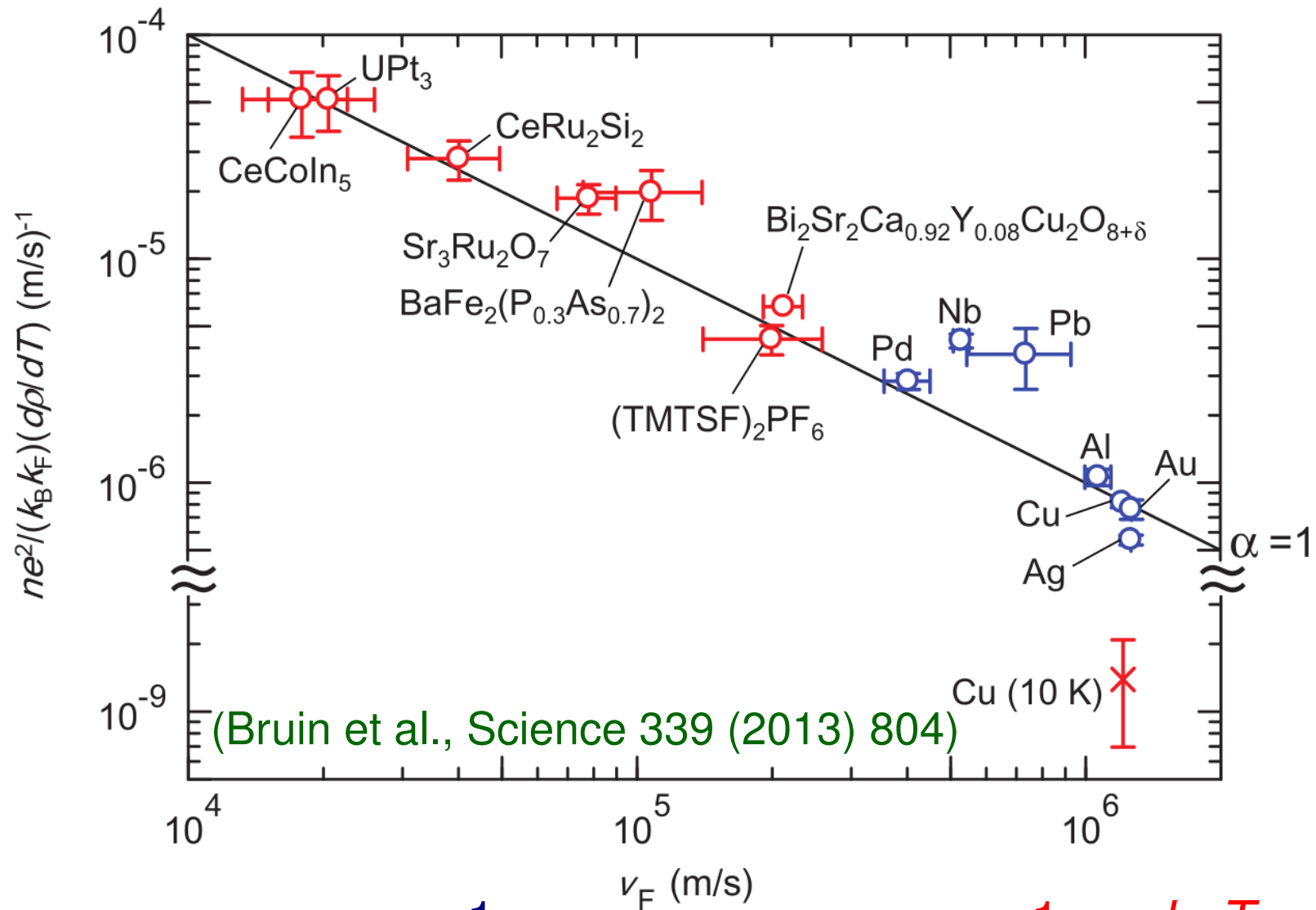
## Optical conductivity of optimally doped $\text{Bi}_{2.23}\text{Sr}_{1.9}\text{Ca}_{0.96}\text{Cu}_2\text{O}_{8+\delta}$



(van der Marel et al., Nature 425 (2003) 271)

(Carbone et al., PNAS 2008)↑

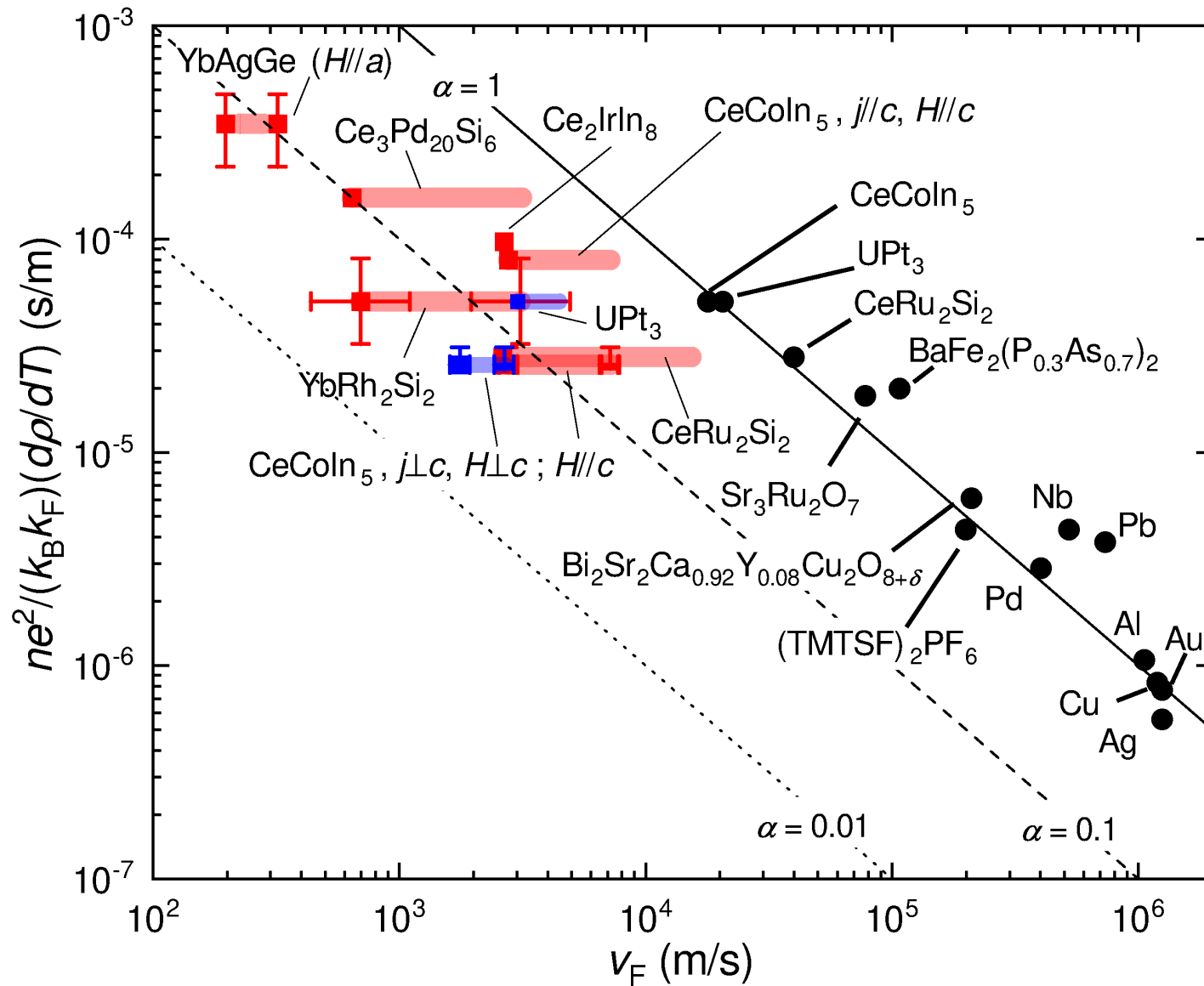
# Planckian dissipation? – Quantum oscillation data



$$\rho - \rho_0 = A' T = \frac{m}{ne^2} \frac{1}{\tau}$$

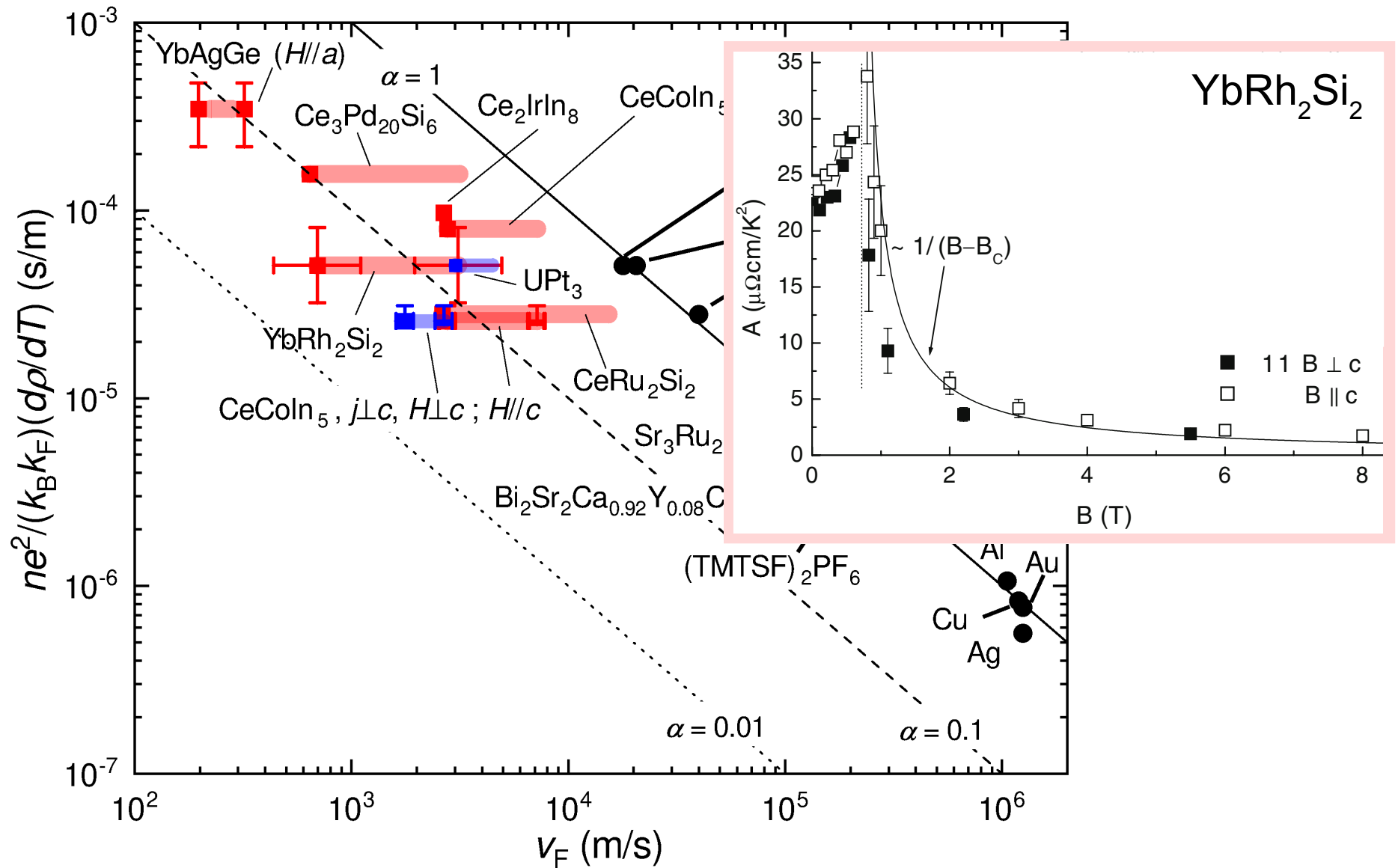
Drude model with  $\frac{1}{\tau} = \alpha \frac{k_B T}{\hbar}$

# Planckian dissipation? – Low- $T$ electrical transport



(M. Taupin & SP, Crystals 12 (2022) 251 and refs. herein)

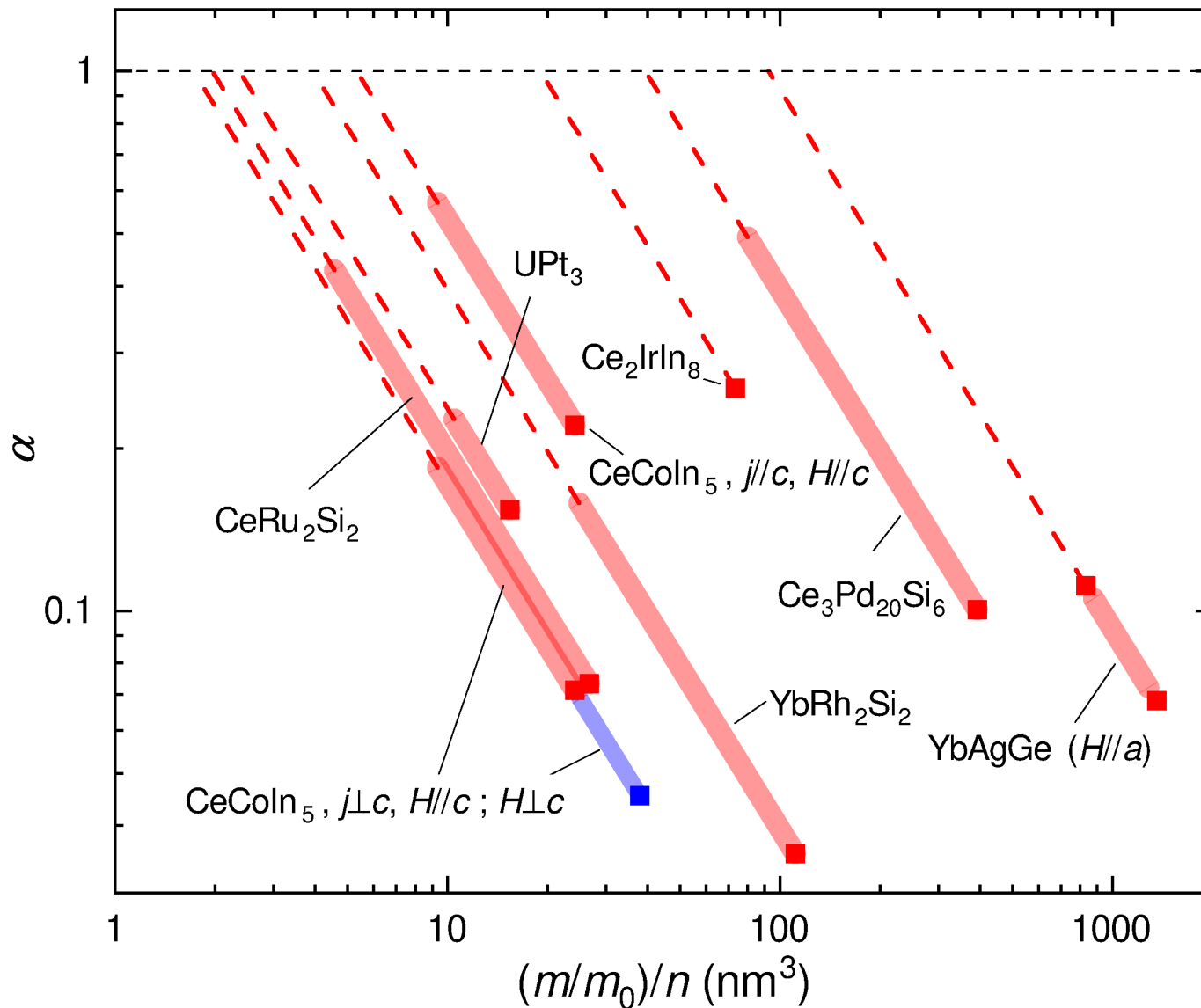
# Planckian dissipation? – Low- $T$ electrical transport



(M. Taupin & SP, Crystals 12 (2022) 251 and refs. herein)



# Planckian dissipation? – Low- $T$ electrical transport



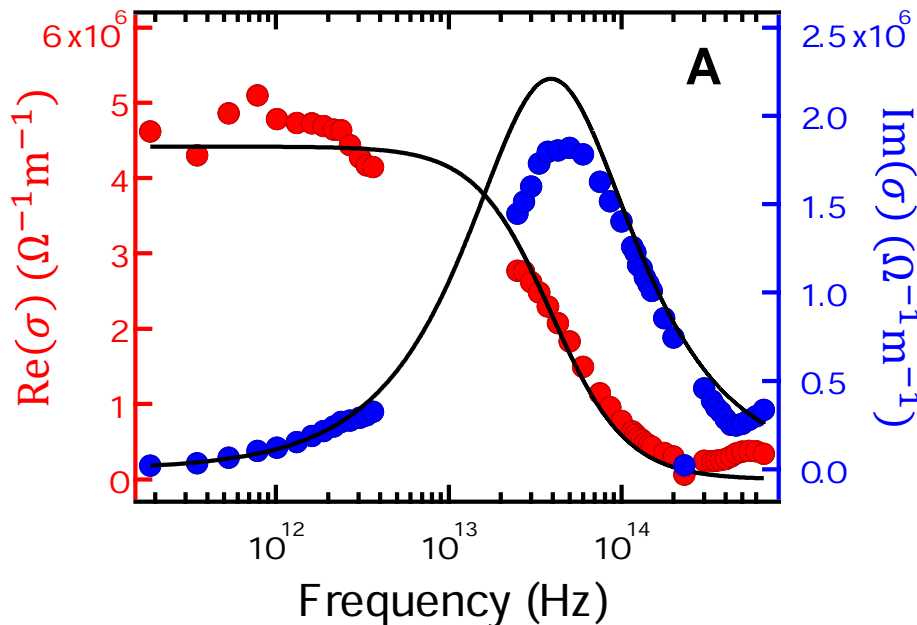
(M. Taupin & SP, Crystals 12 (2022) 251 and refs. herein)

# Planckian dissipation in optical conductivity?

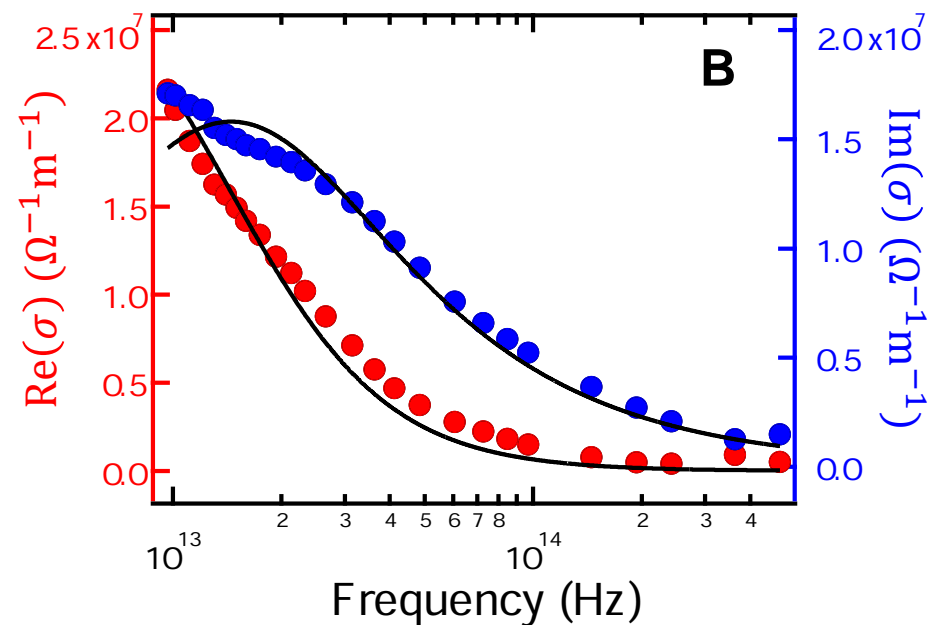
$$\text{Re}[\sigma(\omega)] = \sigma_1 = \frac{ne^2\tau}{m} \frac{1}{1 + \omega^2\tau^2}$$

$$\text{Im}[\sigma(\omega)] = \sigma_2 = \frac{ne^2\tau}{m} \frac{\omega\tau}{1 + \omega^2\tau^2}$$

Pb at room temperature

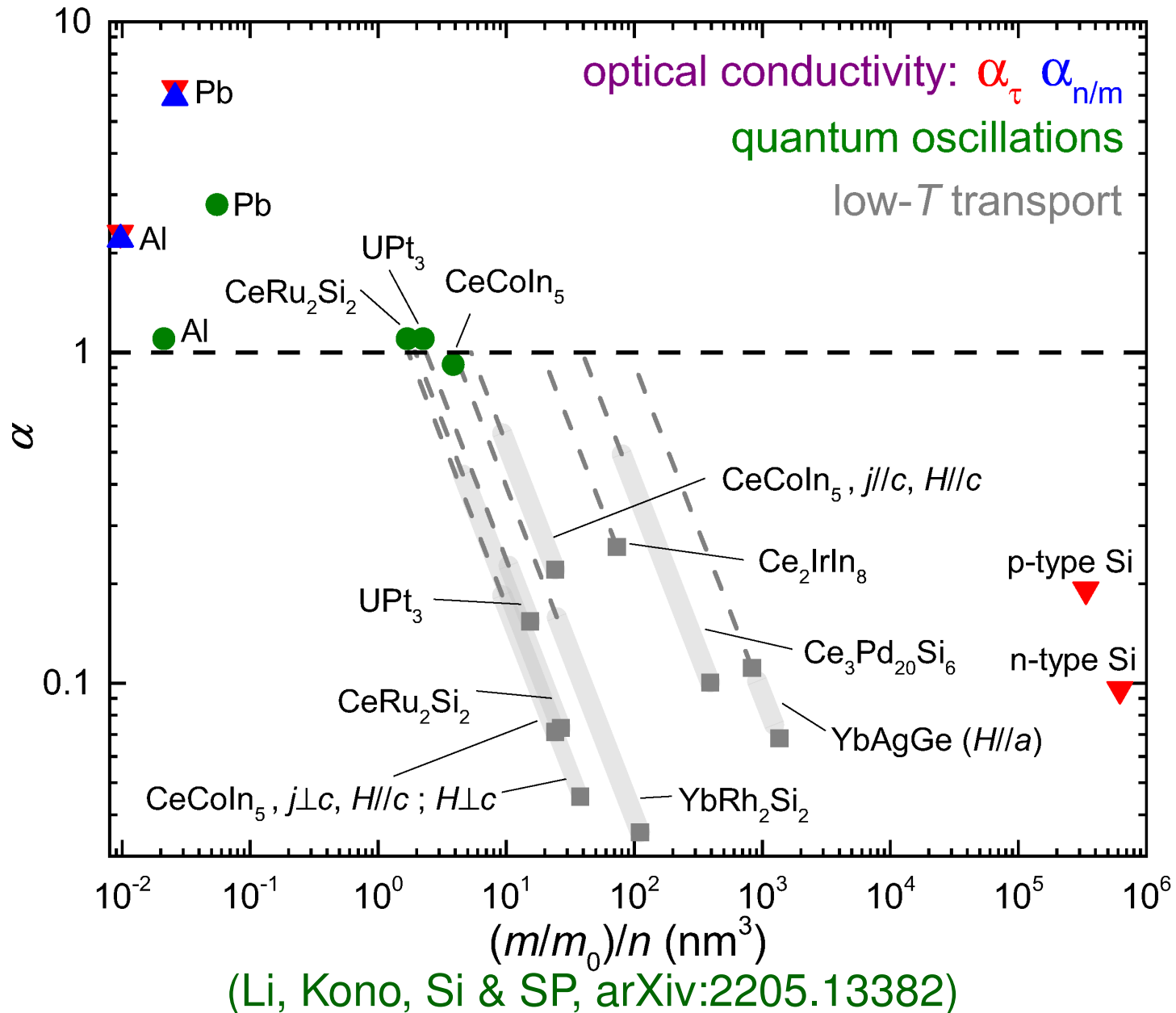


Al at room temperature



(Li, Kono, Si & SP, arXiv:2205.13382)

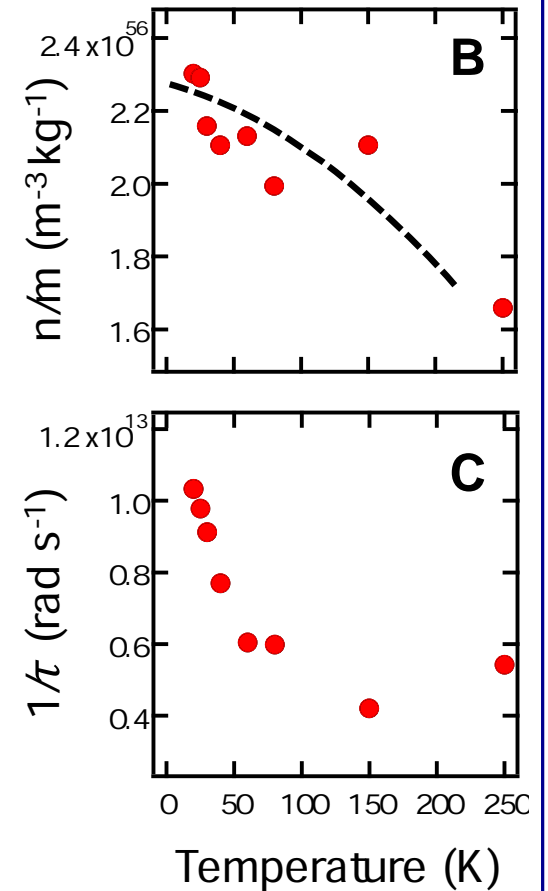
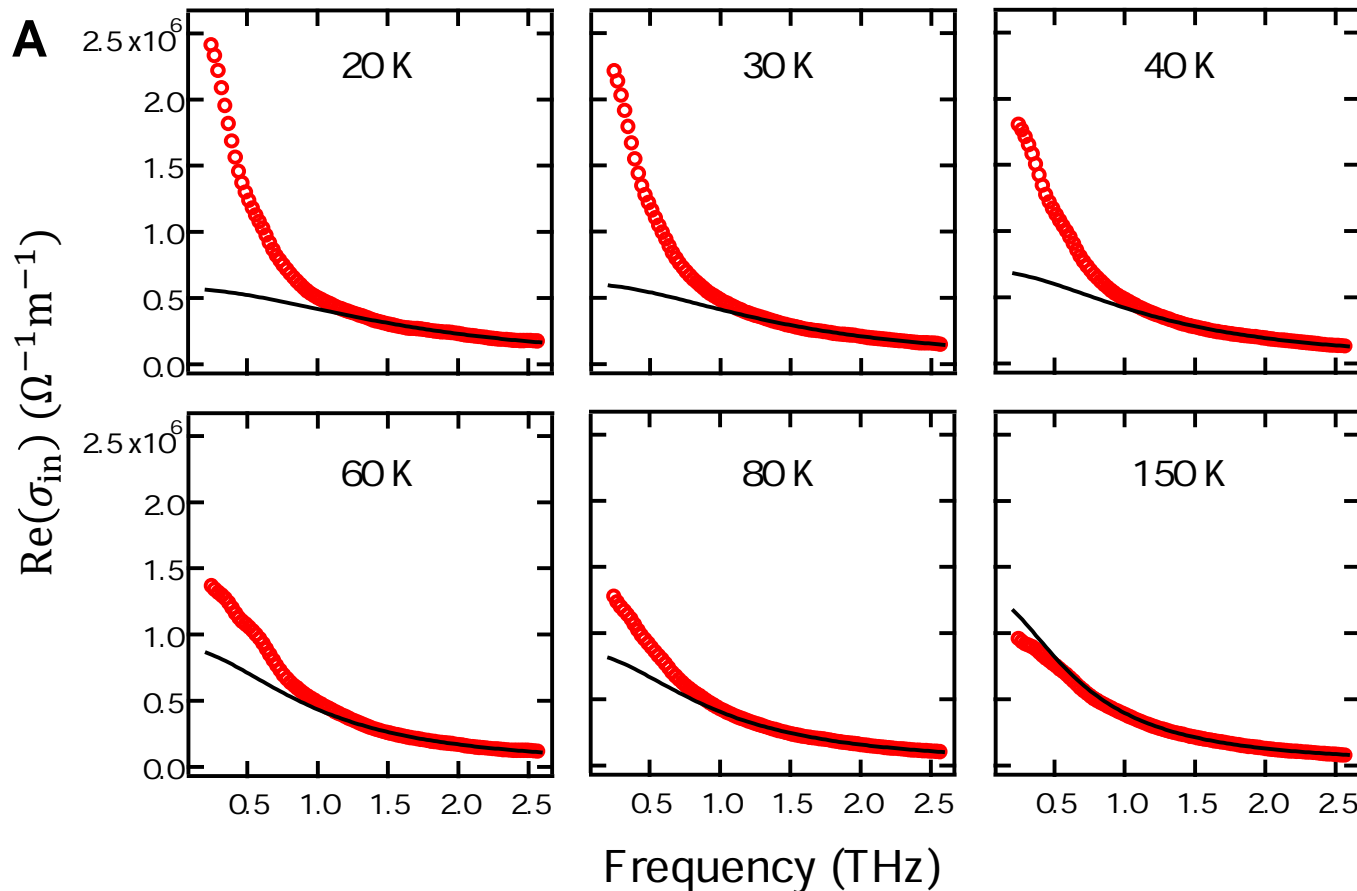
# Planckian dissipation? – Optical conductivity data



# Planckian dissipation in optical conductivity?

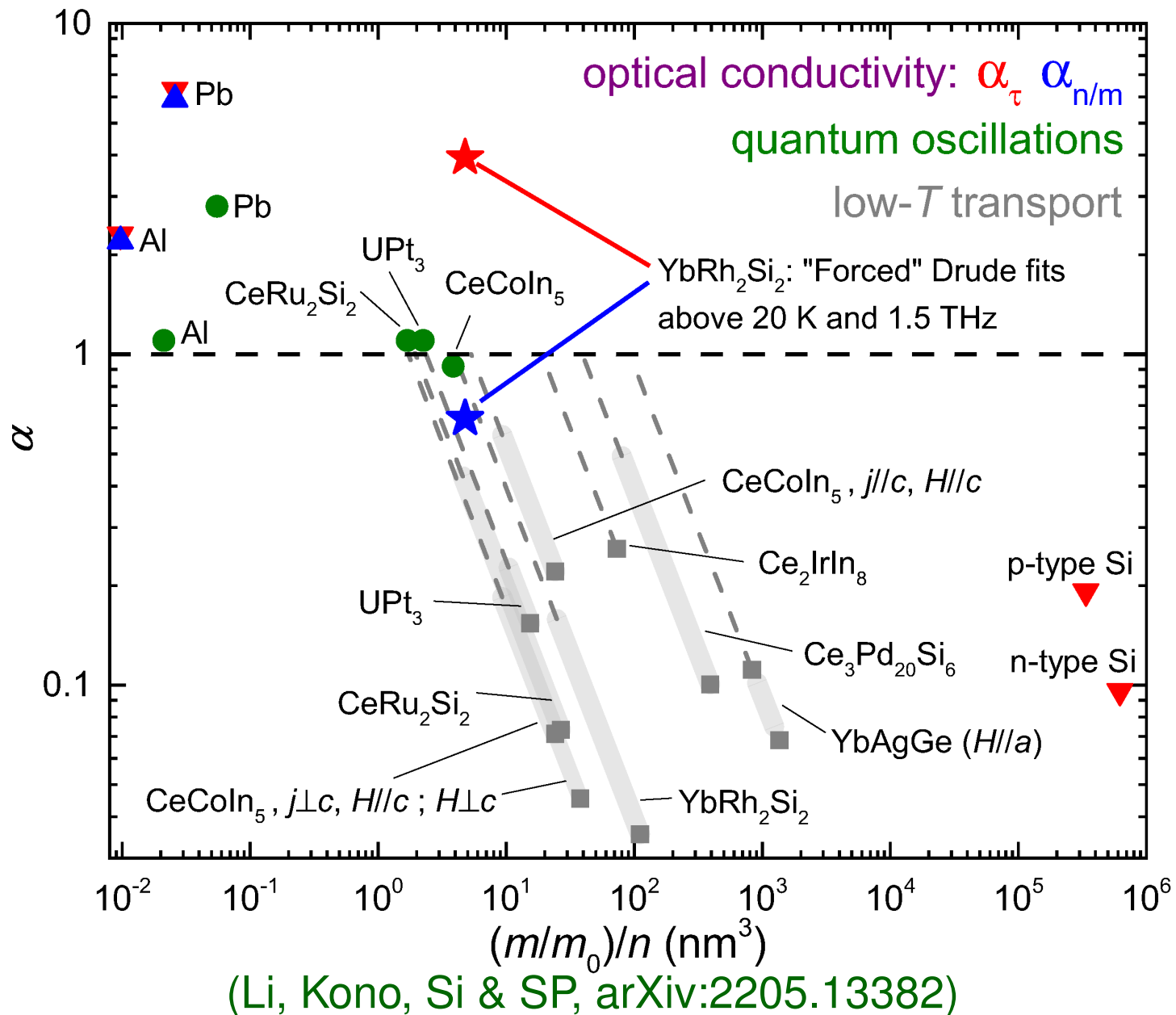
YbRh<sub>2</sub>Si<sub>2</sub>: Non-Drude behavior in strange metal regime

→ Force Drude fit to high  $T$  and  $\omega$  data



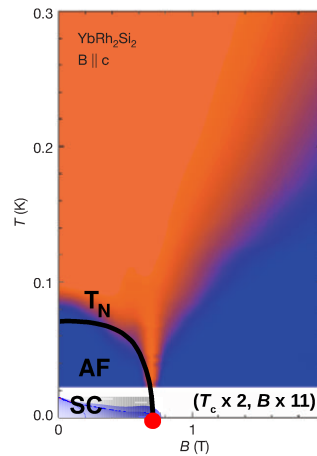
(Li, Kono, Si & SP, arXiv:2205.13382)

# Planckian dissipation? – Optical conductivity data



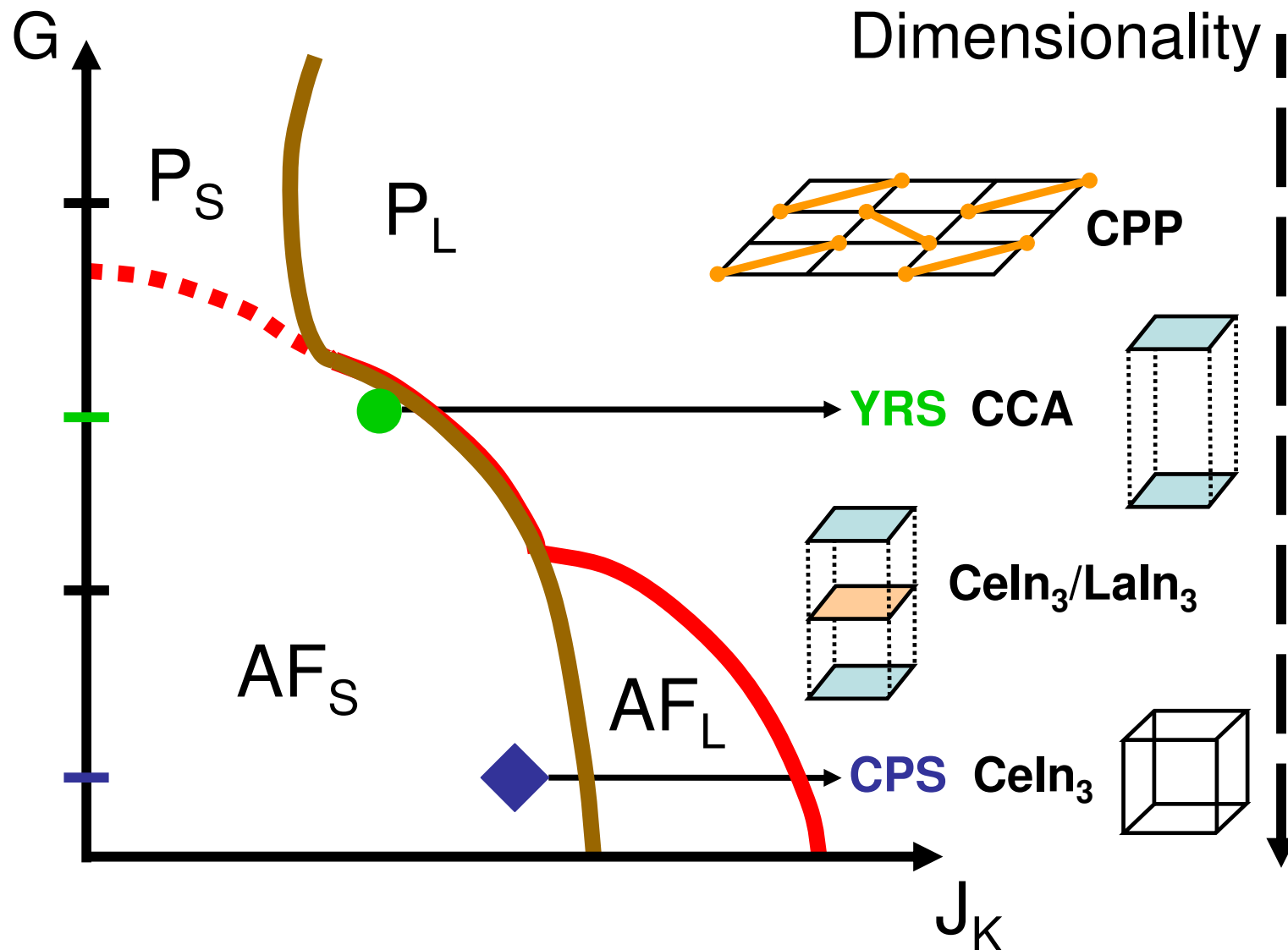
# Heavy fermion systems

## From quantum criticality to electronic topology



- Tunable correlation strength
- Phase diagrams governed by quantum fluctuations
- Quantum criticality from vanishing order parameter
- Beyond order-parameter quantum criticality
- **Global phase diagram of heavy fermion compounds**
- Emergent phases: Unconventional superconductivity

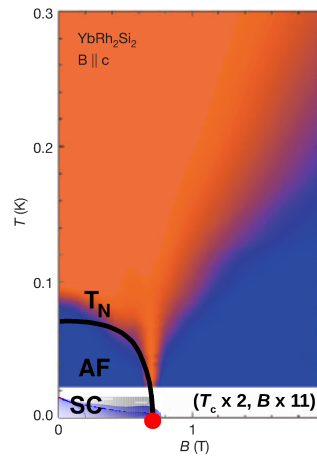
# Global phase diagram of heavy fermion metals



(Custers et al., Nature Mater. 11 (2012) 189; Si, Physica B 378-380 (2006) 23)

# Heavy fermion systems

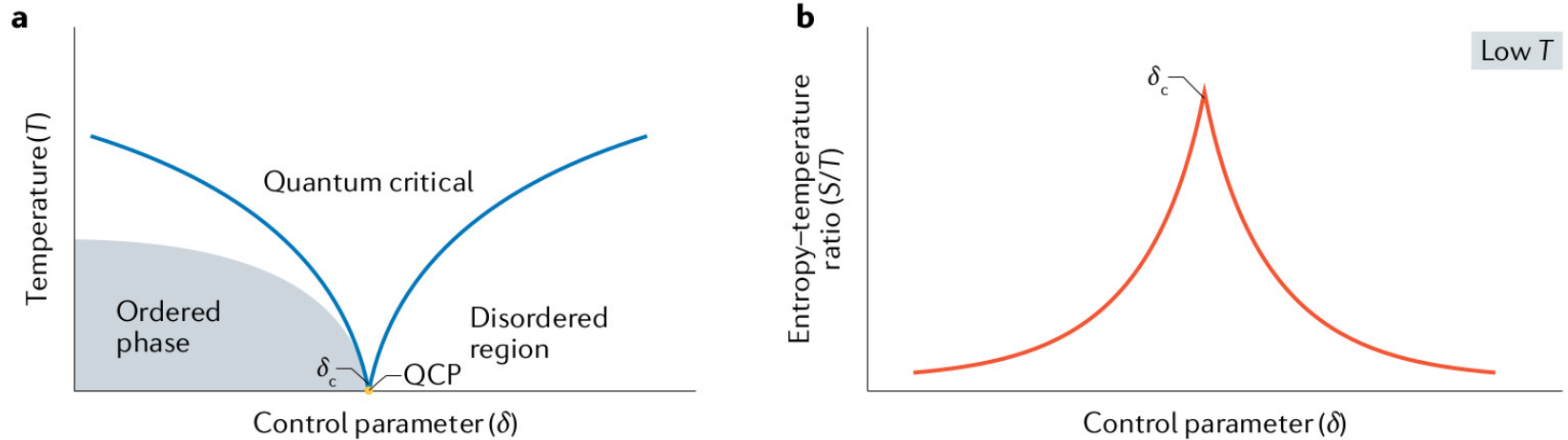
## From quantum criticality to electronic topology



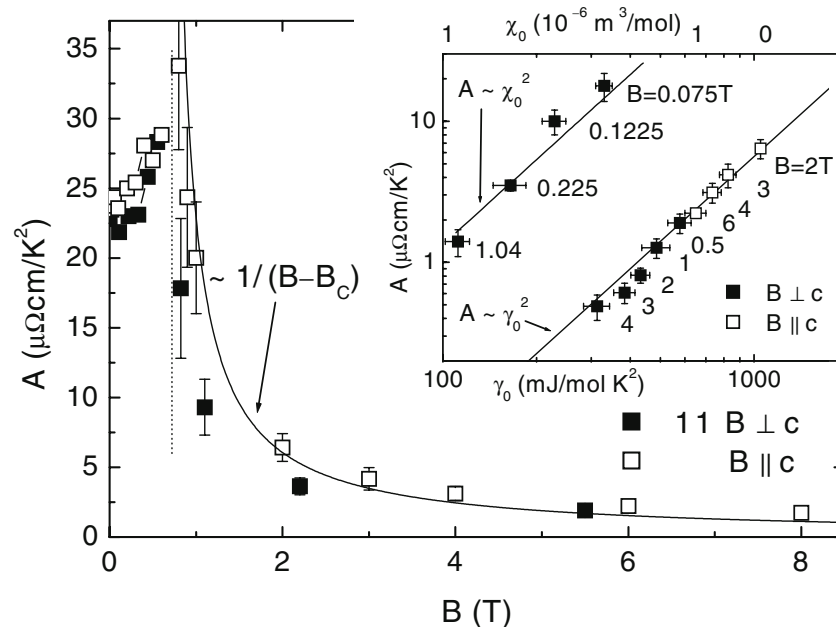
- Tunable correlation strength
- Phase diagrams governed by quantum fluctuations
- Quantum criticality from vanishing order parameter
- Beyond order-parameter quantum criticality
- Global phase diagram of heavy fermion compounds
- **Emergent phases: Unconventional superconductivity**



# Entropy accumulation at quantum critical point



YbRh<sub>2</sub>Si<sub>2</sub>:

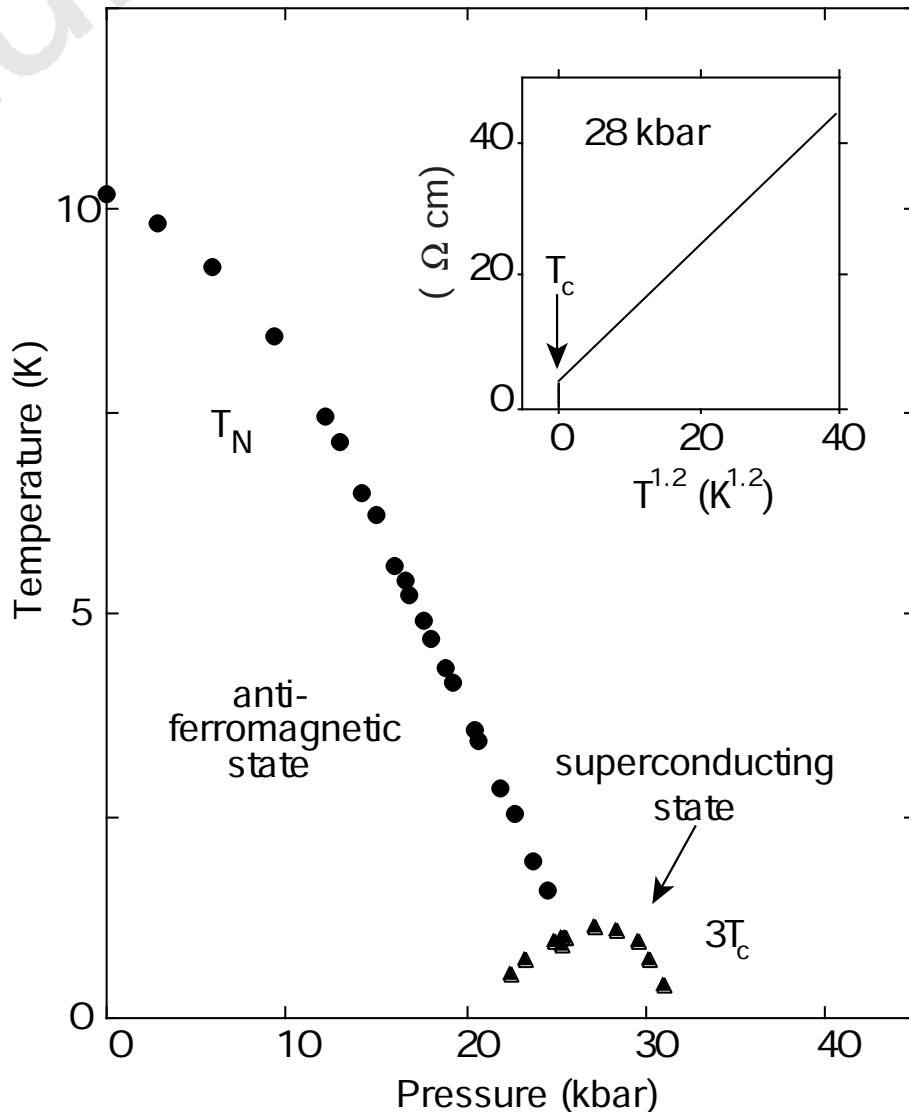


(SP & Si, Nat. Rev. Phys. 3 (2021) 9 (top); Gegenwart et al., Phys. Rev. Lett. 89 (2002) 056402 (bottom))

# LGW QCPs lead to spin fluctuation mediated sc pairing

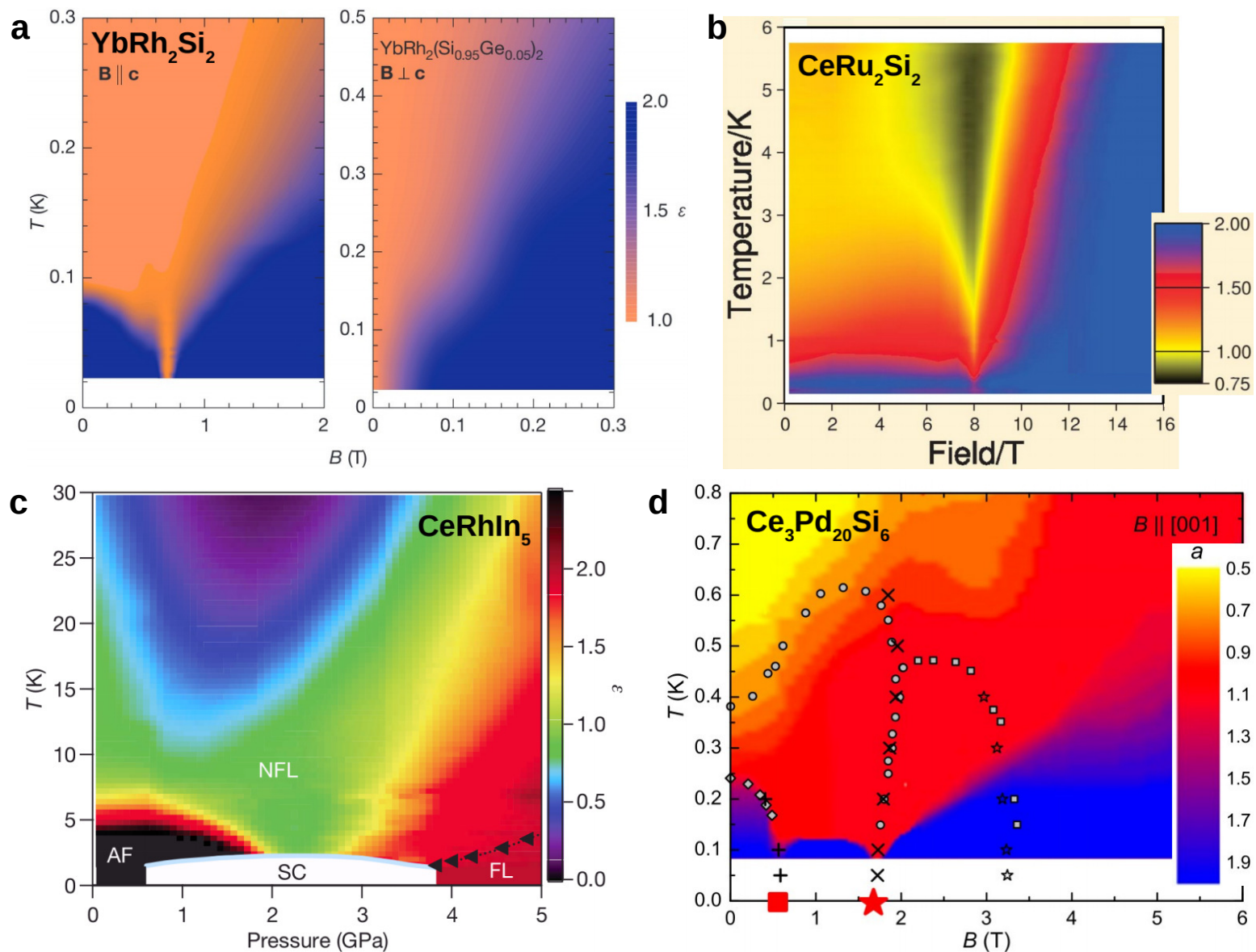
... it is still unconventional sc

- non- $f$  reference compound does not superconduct
- $\Delta C/(\gamma_0 T_c) \sim 1$  and huge  $dB_{c2}/dT(T_c)$ : quasiparticles are heavy fermions
- BCS pairing unlikely:  $v_F \sim v_{ph}$ , no retardation, Coulomb interaction important  $\rightarrow$  magnetic pairing
- strong pairbreaking by *nonmagnetic* impurities



(Mathur et al., Nature 394 (1998) 39)

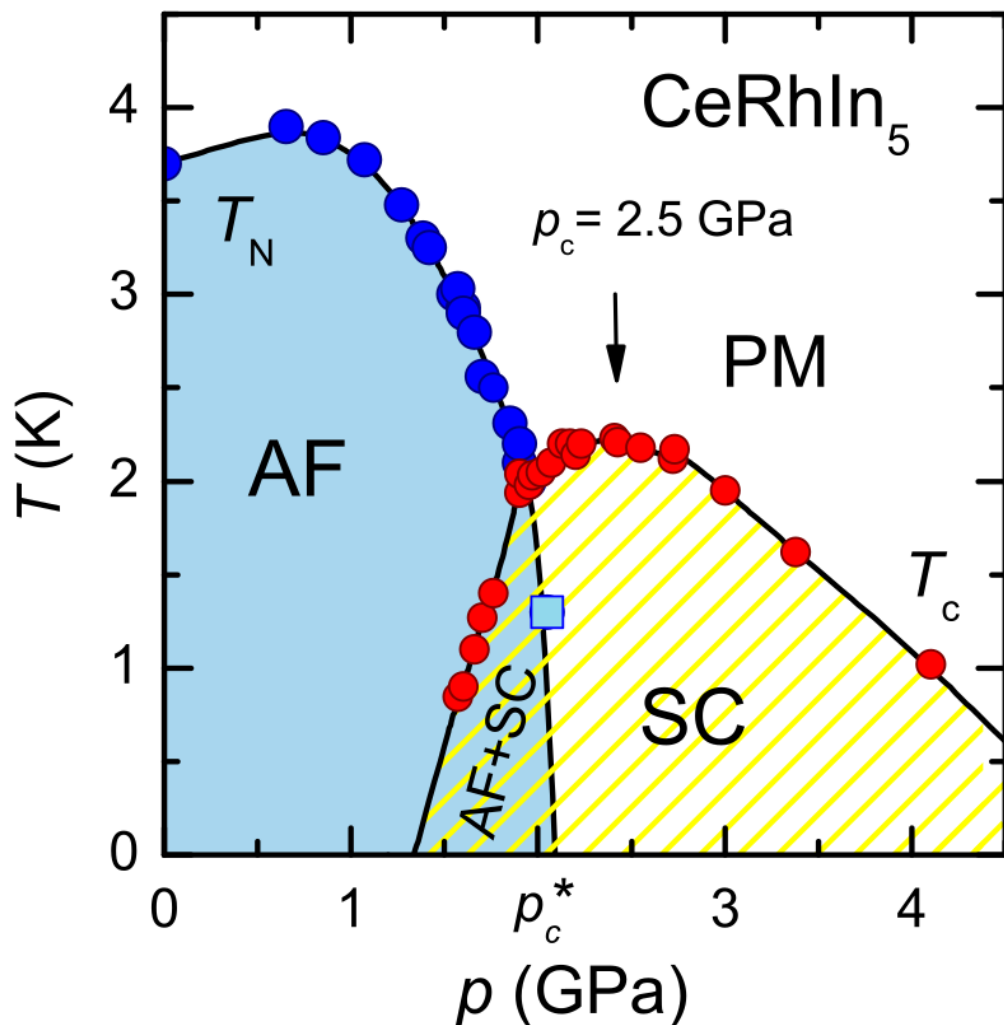
# Superconductivity at a beyond-GLW QCP?



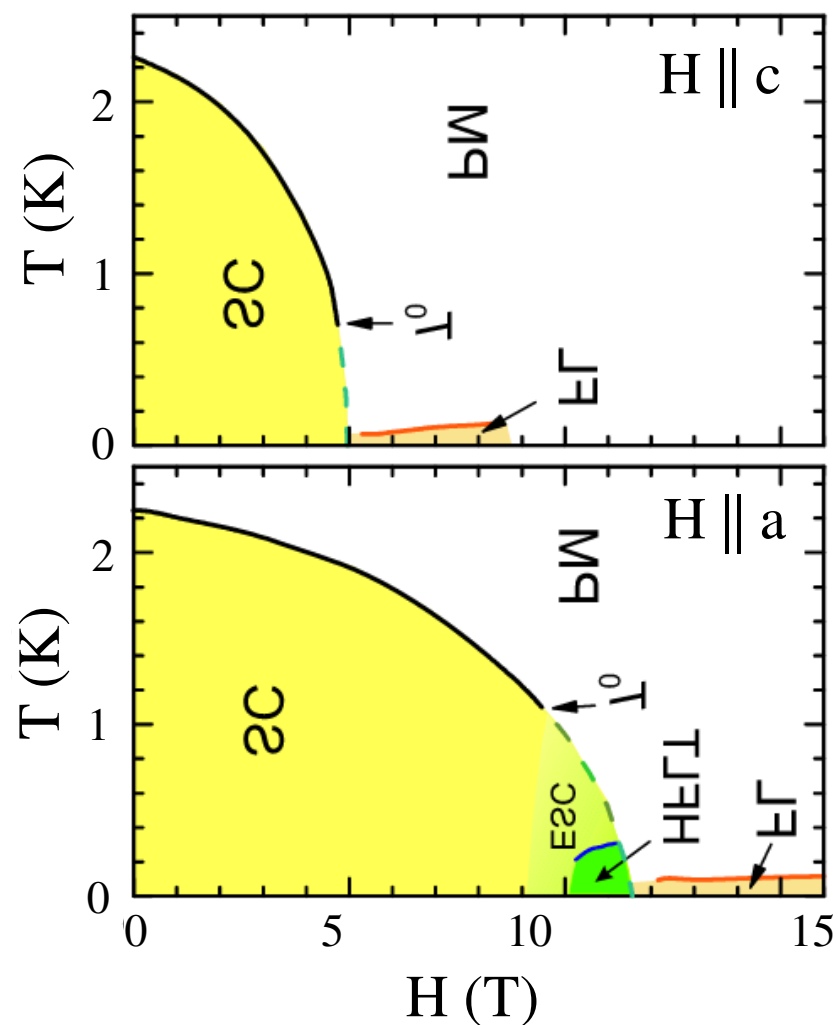
(Taupin & SP, Crystals 12 (2022) 251 and refs. herein)

# Superconductivity at a beyond-GLW QCP

## CeRhIn<sub>5</sub>



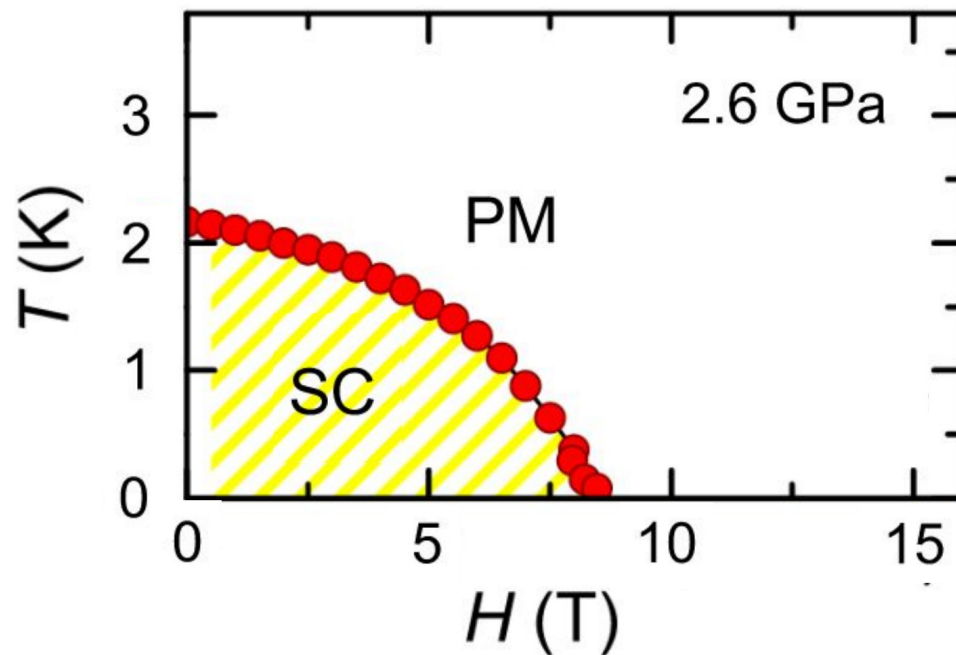
## CeCoIn<sub>5</sub>



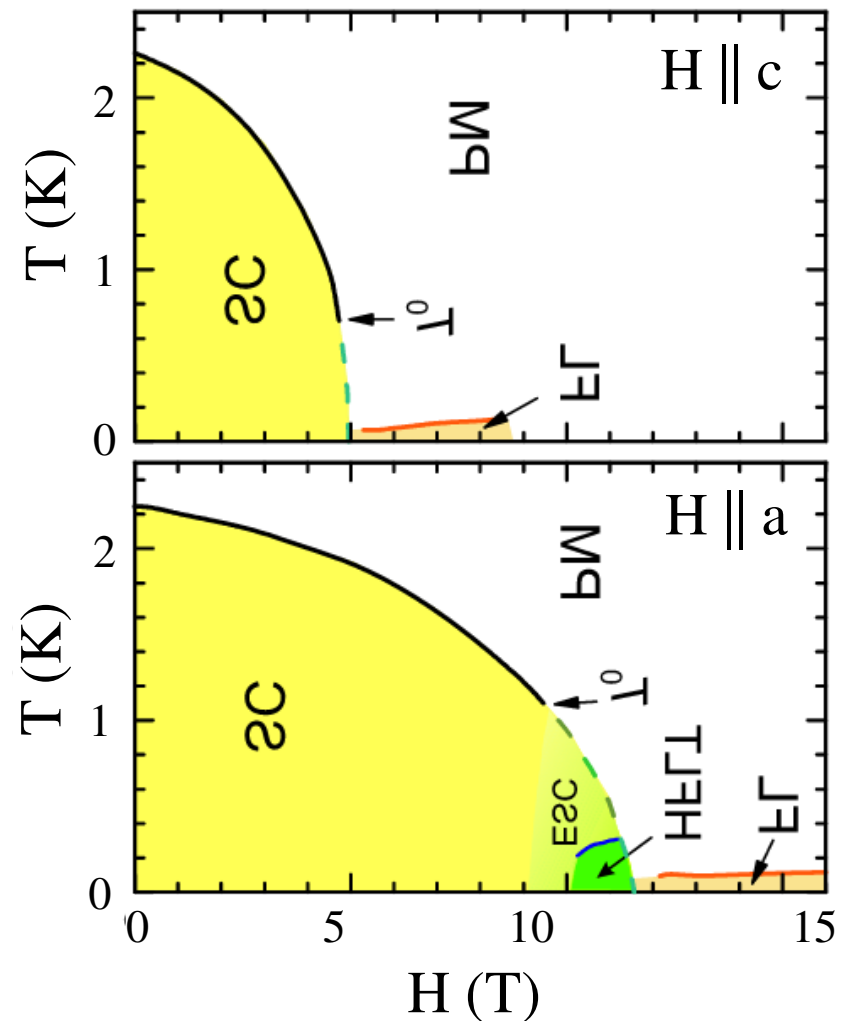
(Knebel et al., C. R. Physique 12 (2011) 542 and refs. herein)

# Superconductivity at a beyond-GLW QCP

CeRhIn<sub>5</sub>

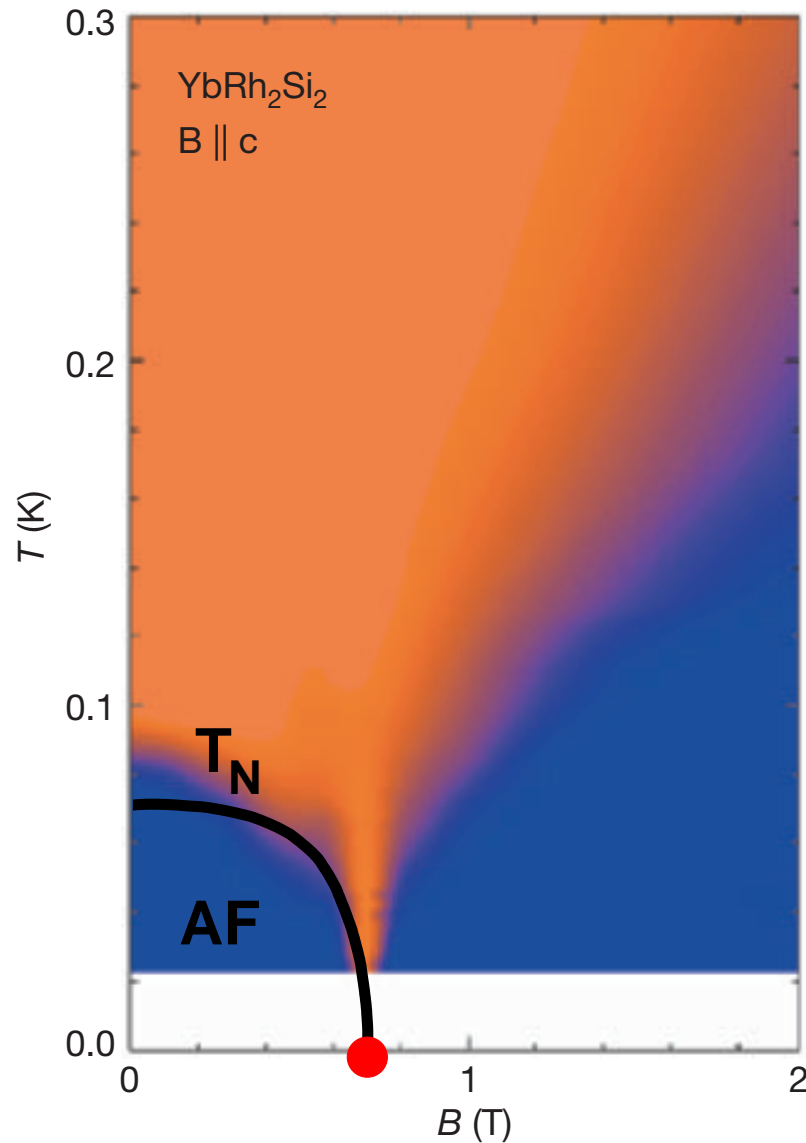


CeCoIn<sub>5</sub>



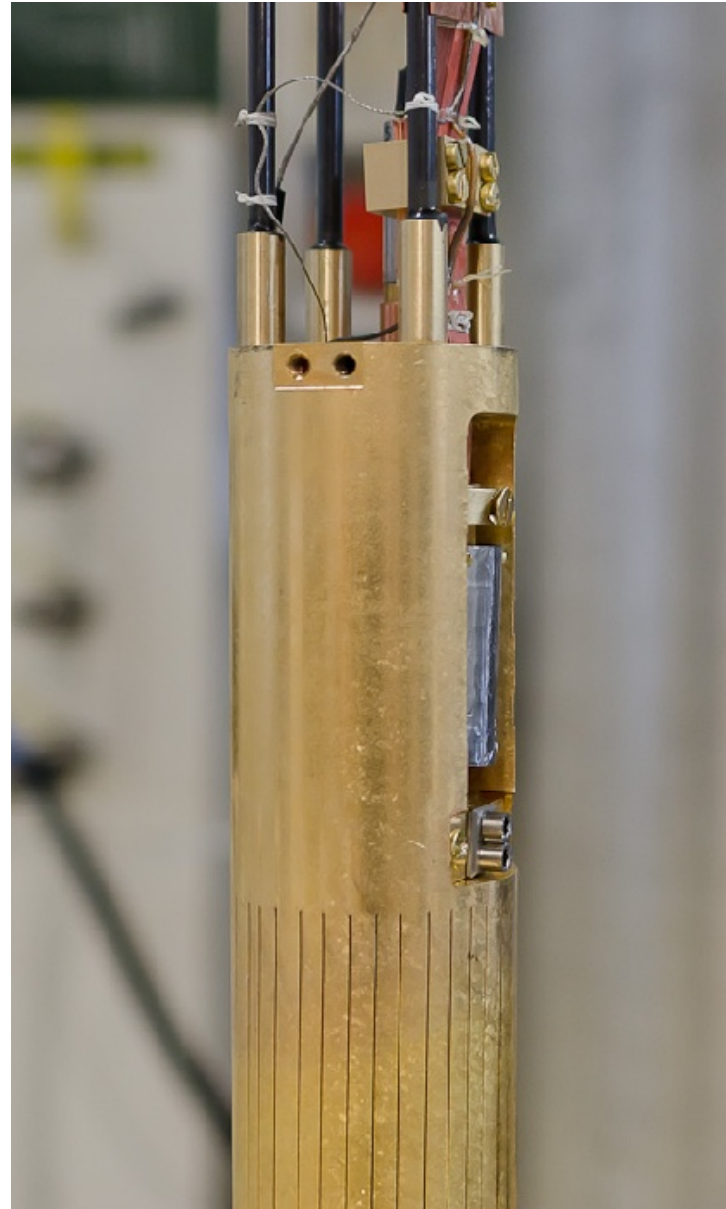
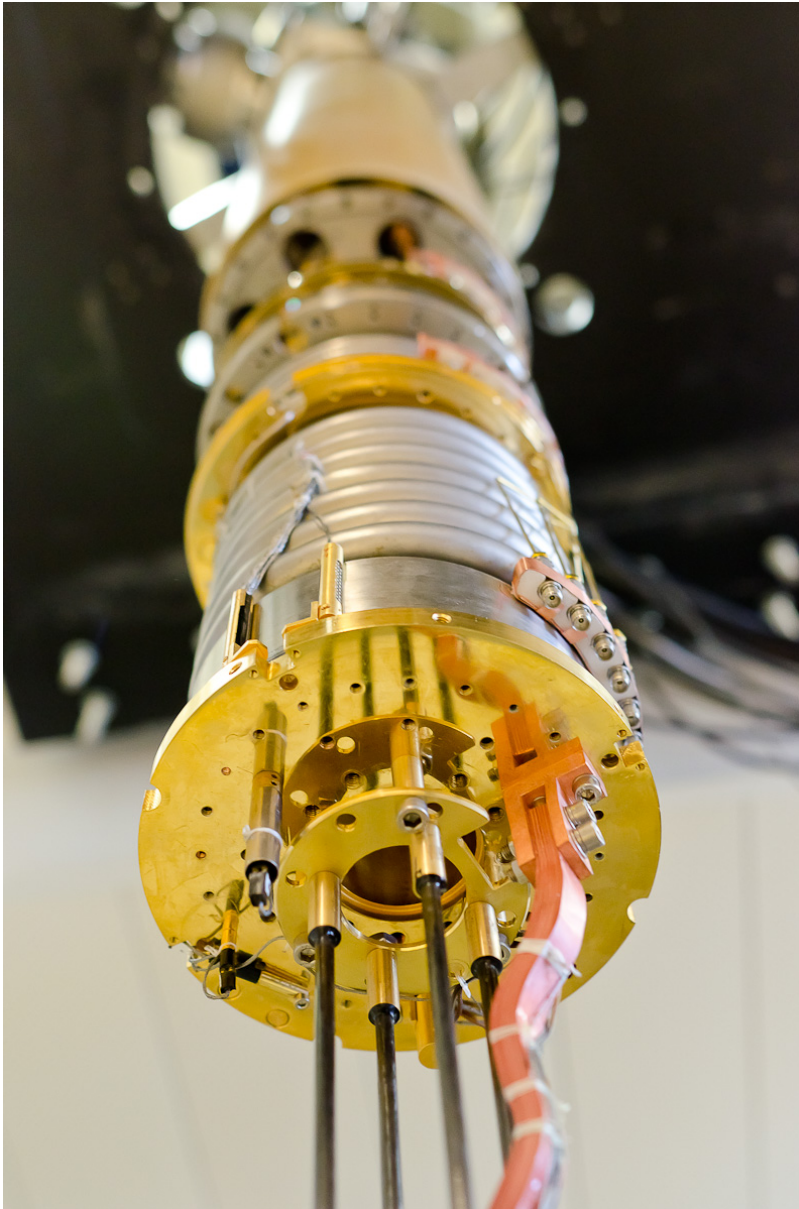
(Knebel et al., C. R. Physique 12 (2011) 542 and refs. herein)

# Is there superconductivity at the QCP of $\text{YbRh}_2\text{Si}_2$ ?

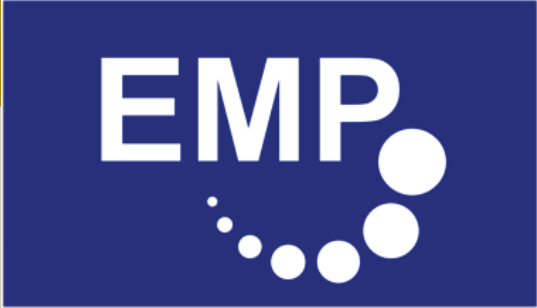
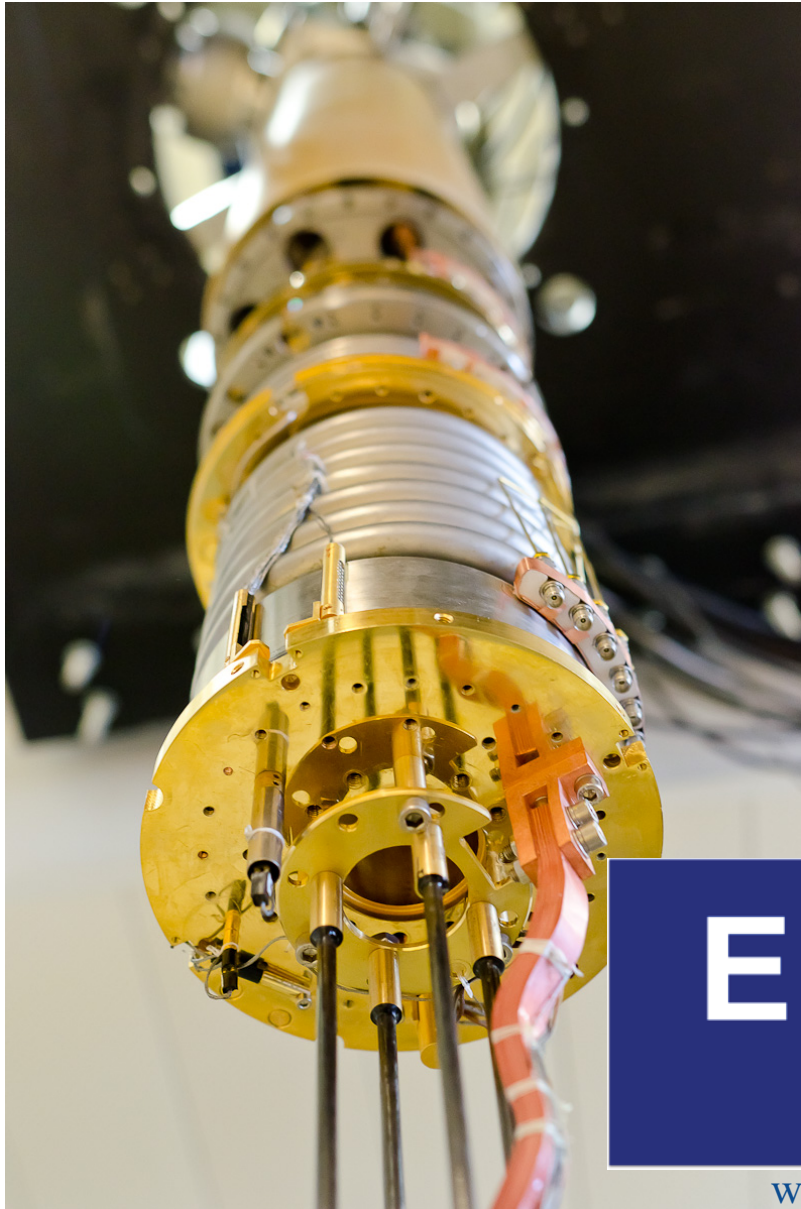


(Custers et al., Nature 424 (2003) 524)

# The Vienna Microkelvin Laboratory



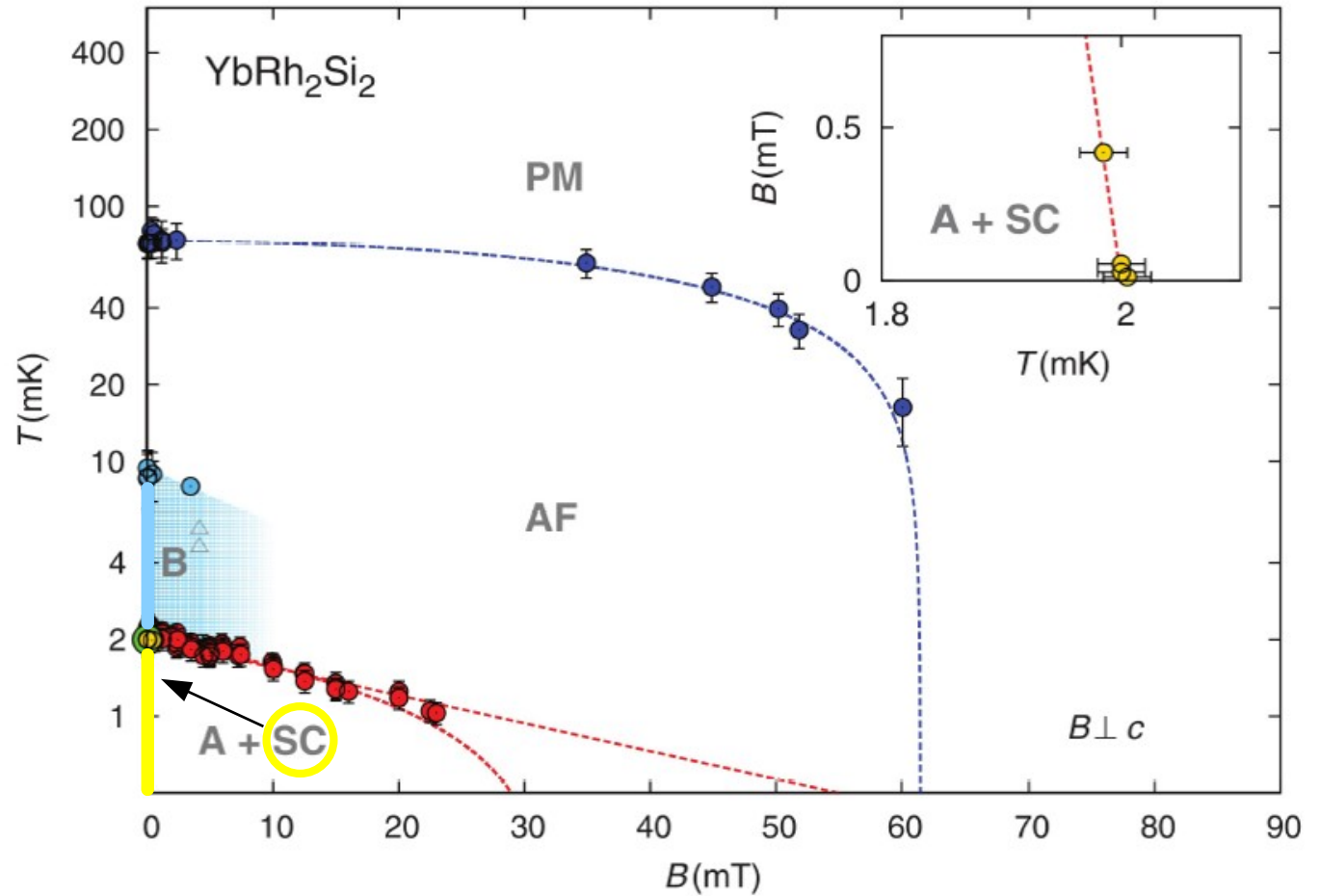
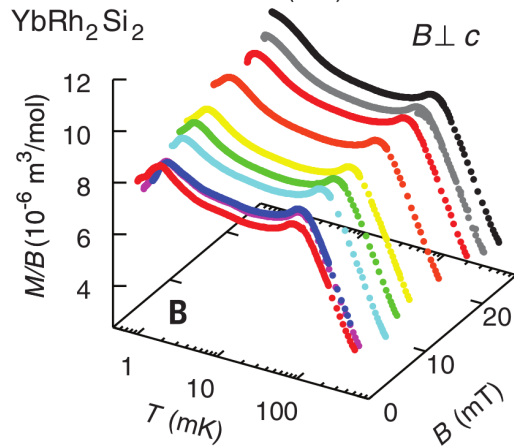
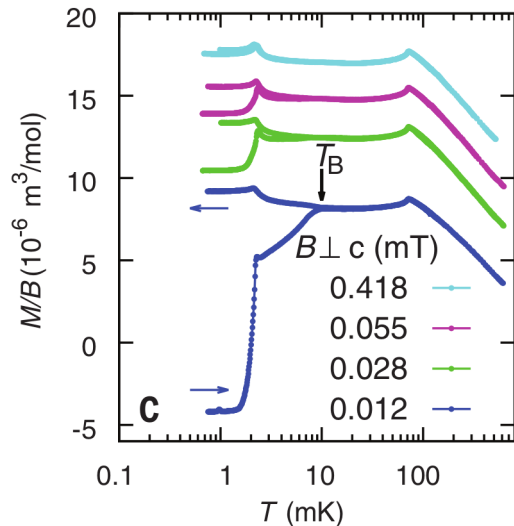
# The Vienna Microkelvin Laboratory



[www.EMPlatform.eu](http://www.EMPlatform.eu)



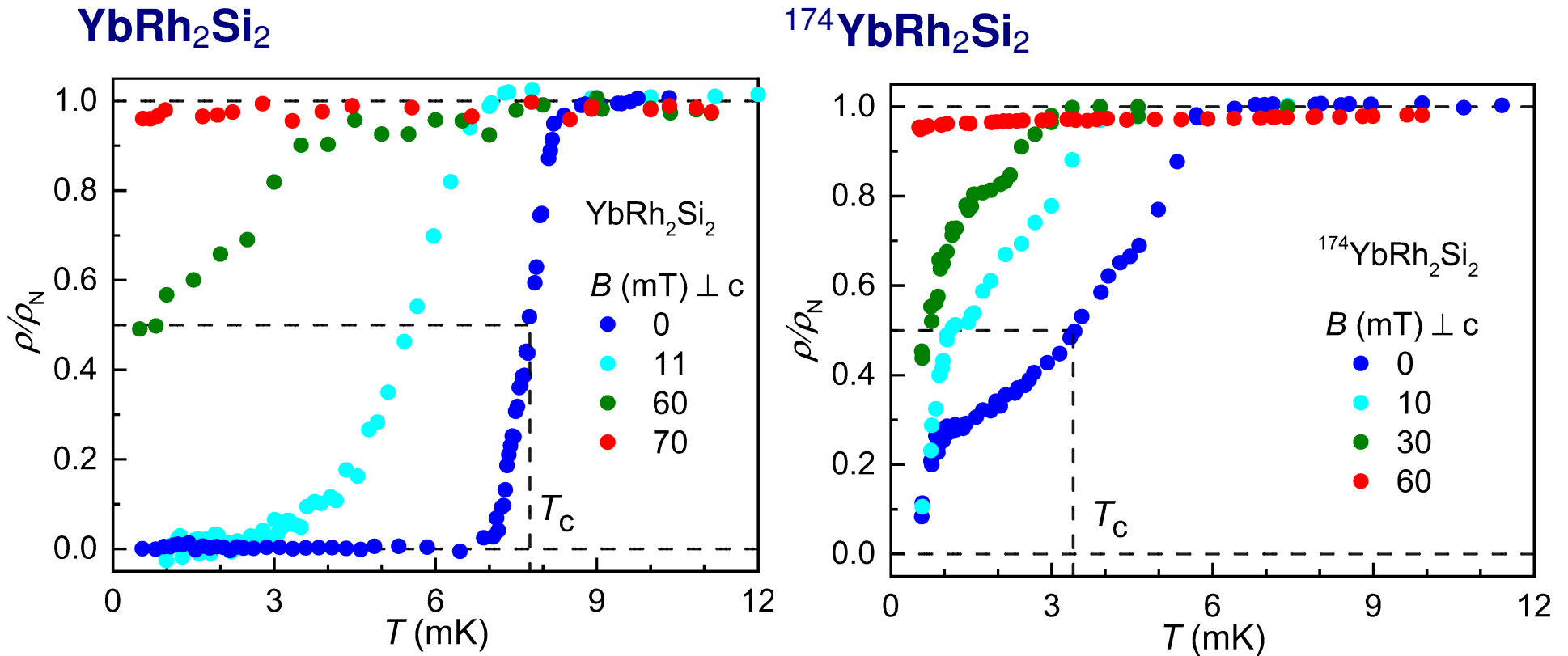
# Magnetization measurements at ultralow temperatures



Shielding below 2 mK at  $H = 0.012$  mT; field-cooled magnetization curves show kinks up to  $\sim 25$  mT; **hybrid electronic-nuclear spin order?**

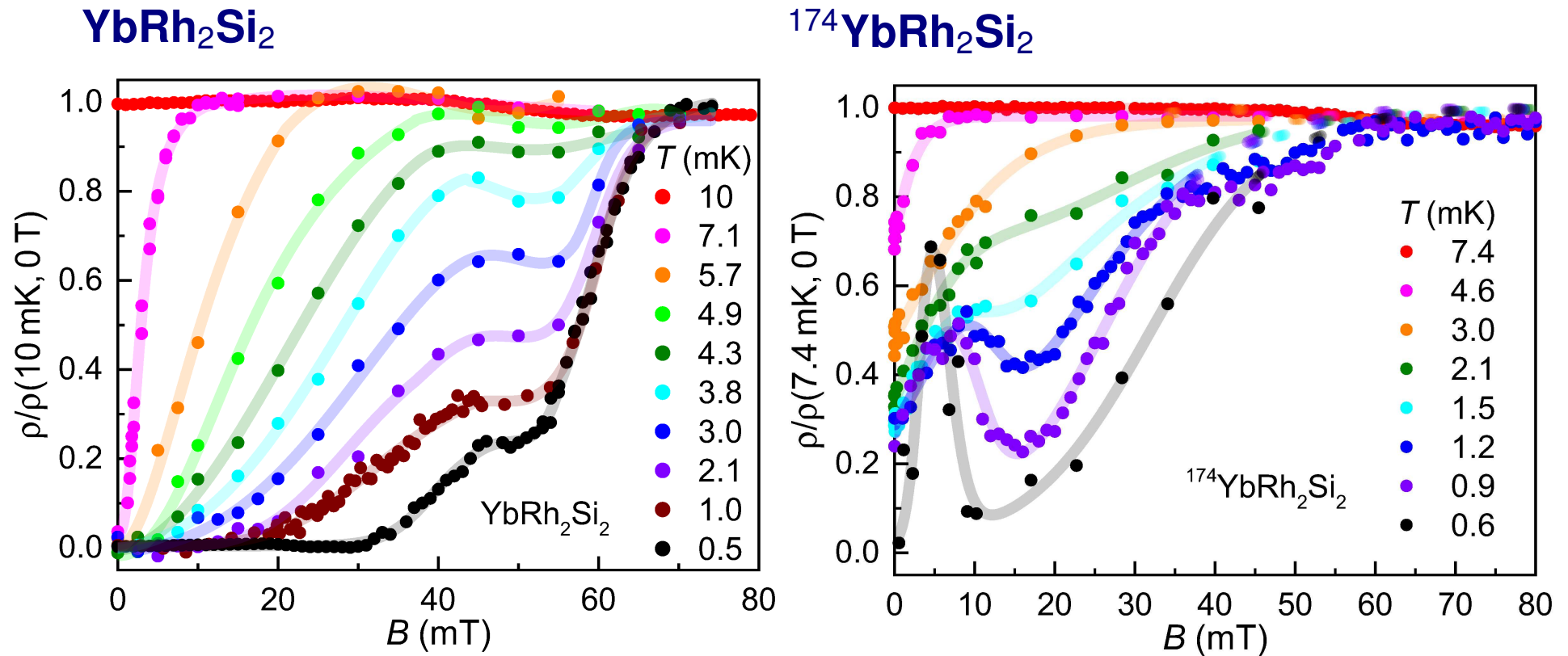
(Schuberth et al., Science 351 (2016) 485)

# Electrical resistivity at ultralow temperatures: Iso- $B$ curves



(Nguyen et al., Nat. Commun. 12 (2021) 4341)

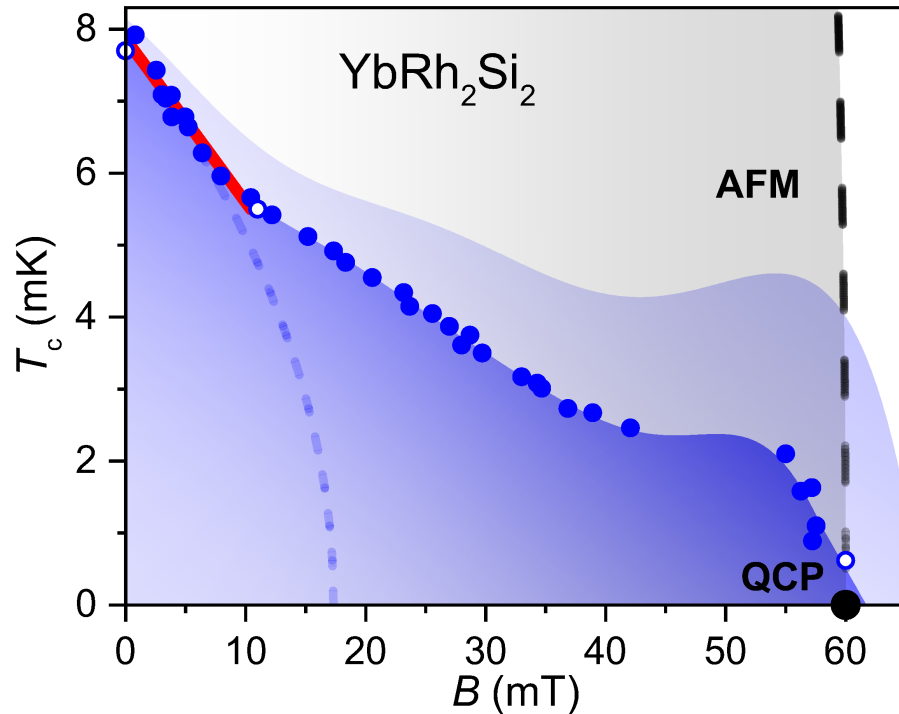
# Electrical resistivity at ultralow temperatures: Iso- $T$ curves



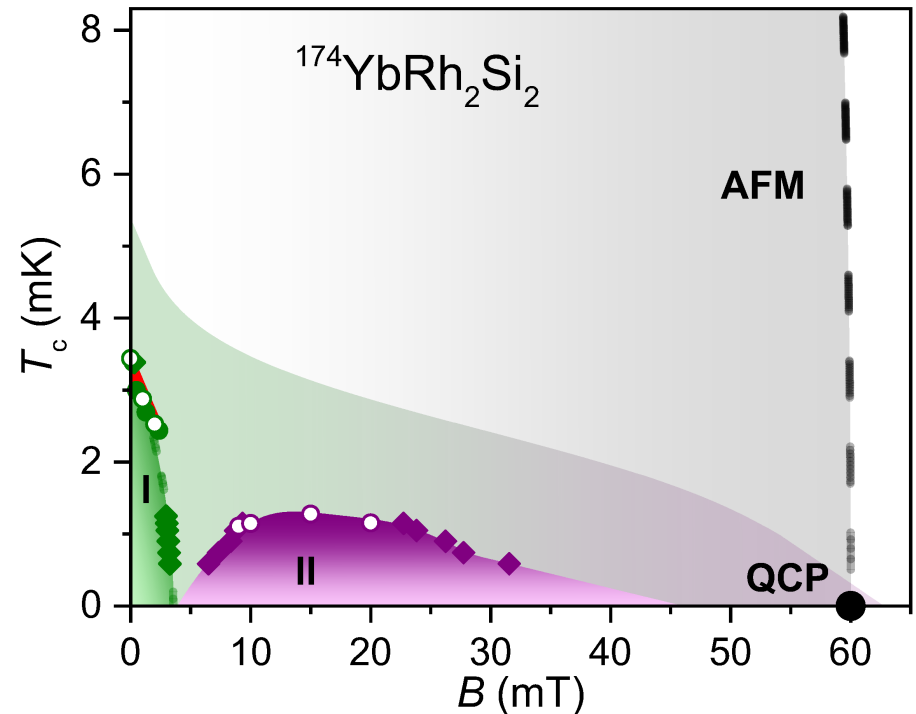
(Nguyen et al., Nat. Commun. 12 (2021) 4341)

# Temperature–magnetic field phase diagrams

## $\text{YbRh}_2\text{Si}_2$



## $^{174}\text{YbRh}_2\text{Si}_2$

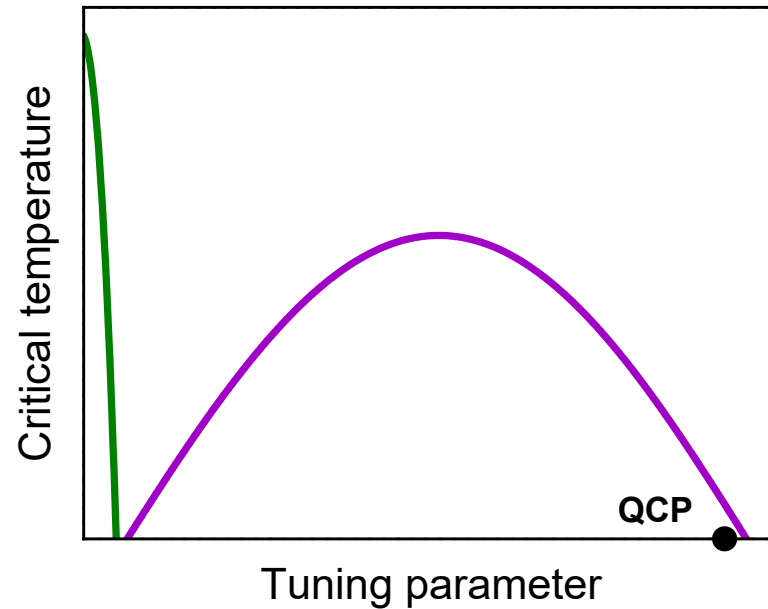
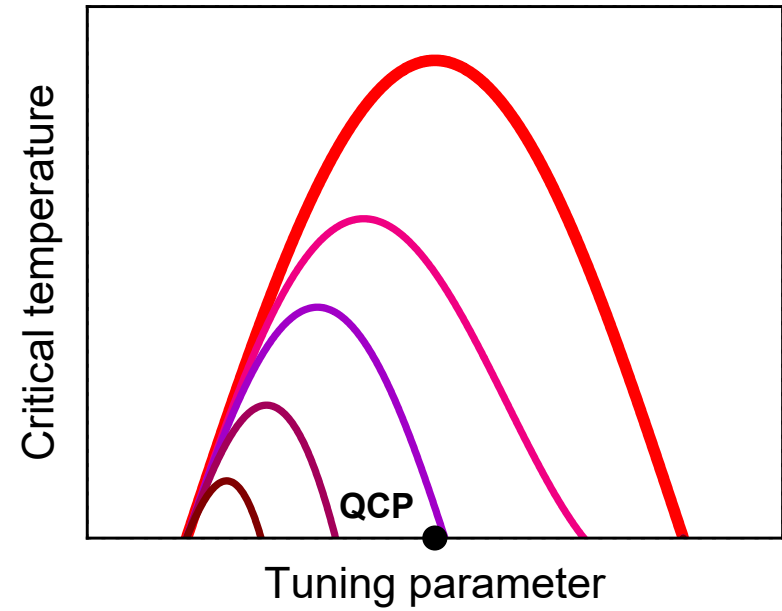
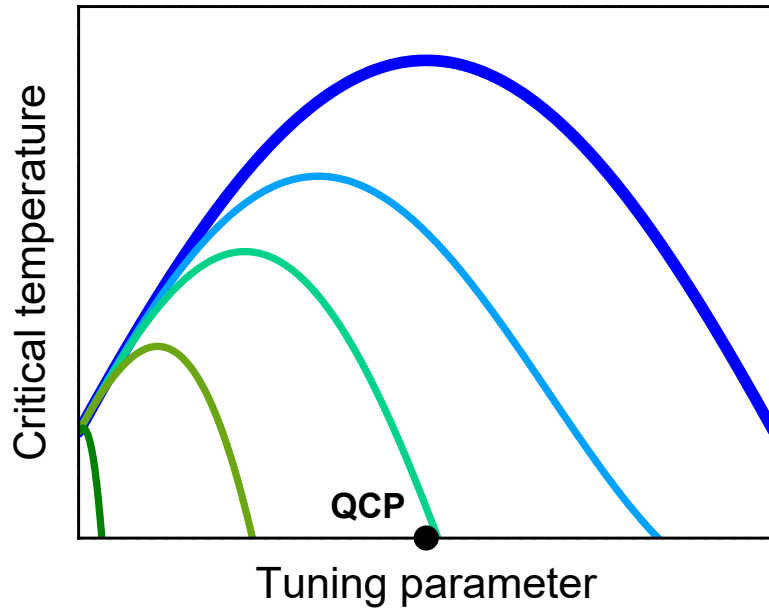


Orbital-limiting fields: 24 mT (5 mT)

Pauli-limiting fields: 15 mT (6.4 mT)

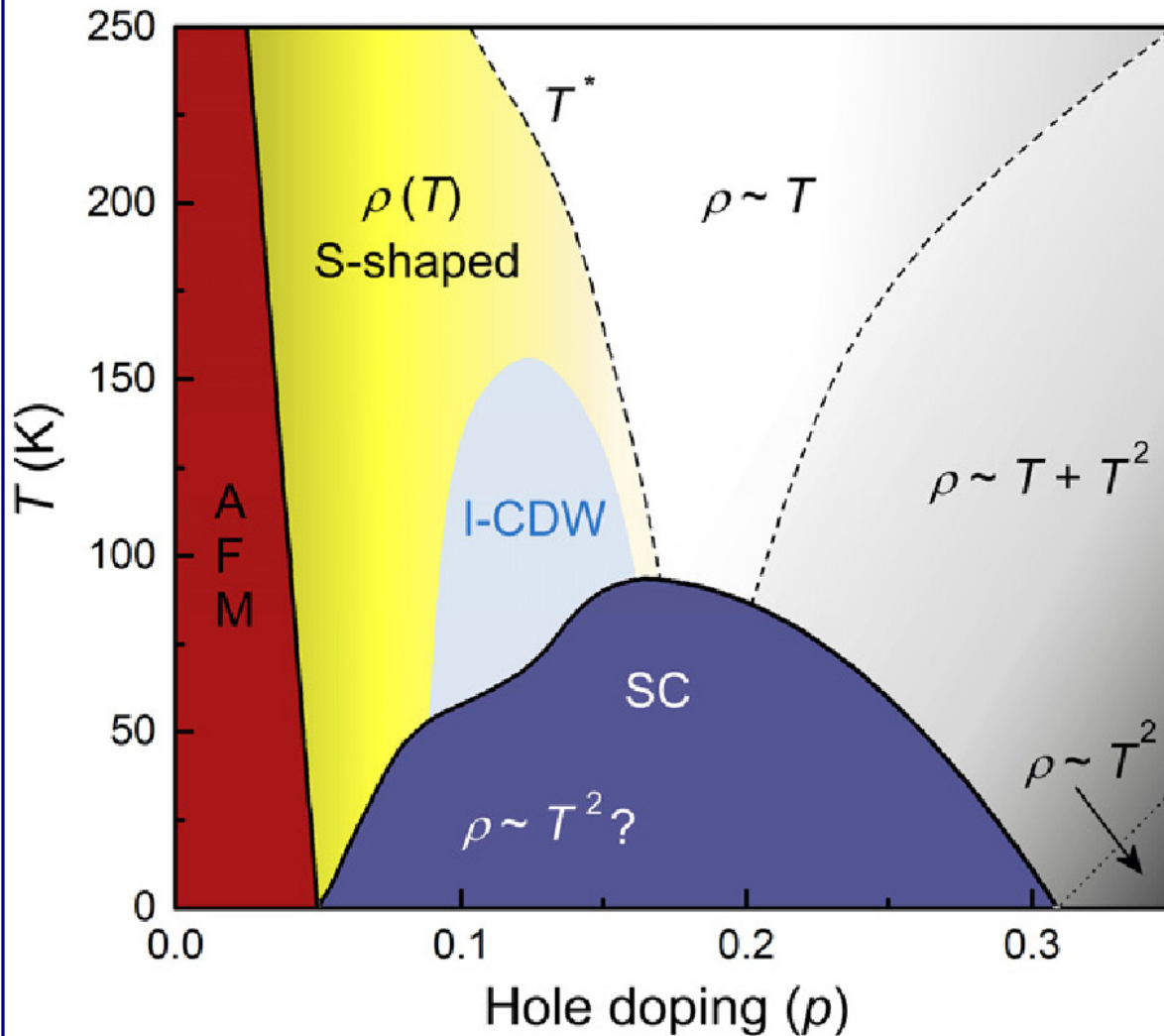
(Nguyen et al., Nat. Commun. 12 (2021) 4341)

# Magnetic field effect on “dome” structure



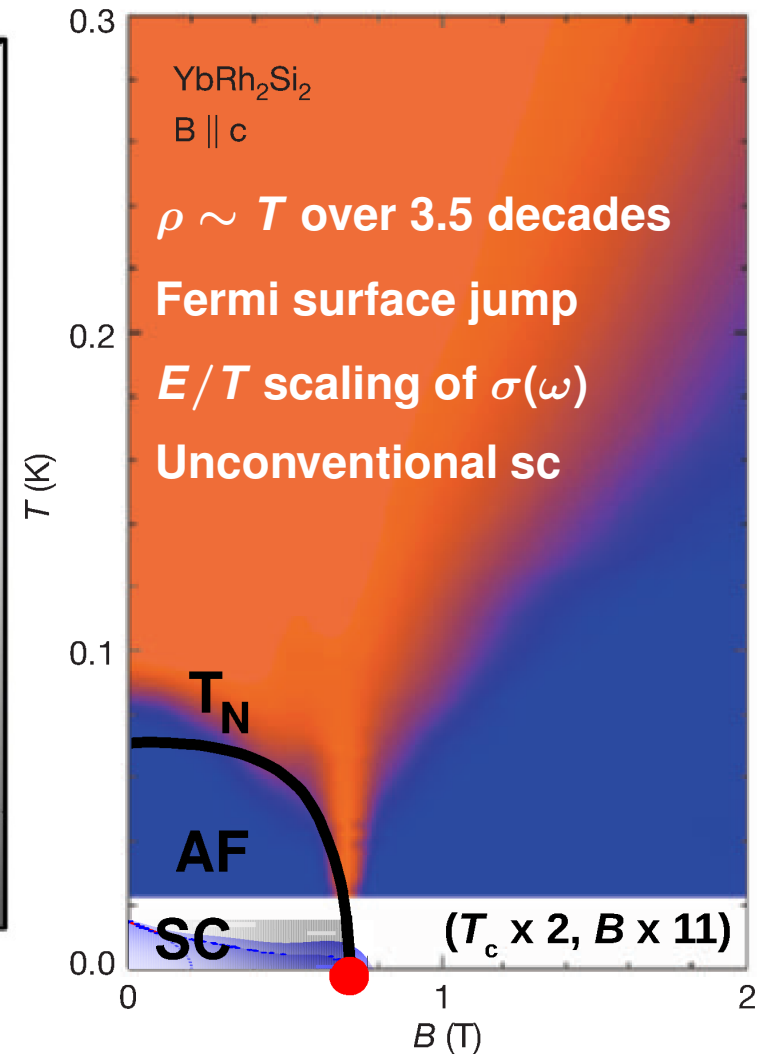
# Advance understanding by connecting the various platforms!

Generic phase diagram of the cuprates



(Proust et al., PNAS 113 (2016) 13654)

Phase diagram of  $\text{YbRh}_2\text{Si}_2$



(Refs. in this talk)

S. Dzsaber\*, G. Eguchi, J. Larrea J.\*, **D. C. MacFarland\***, V. Martelli\*, **D. H. Nguyen**,  
**L. Prochaska**, A. Prokofiev, **A. Sidorenko\***, R. Svagera, **M. Taupin**, M. Waas, X. Yan,  
D. A. Zocco; **A. M. Andrews**, H. Detz, W. Schrenk, G. Strasser; M. Bonta, A. Limbeck; P. Blaha  
*Vienna University of Technology, Austria*

A. Cai\*, H.-H. Lai\*, S. Grefe, K. Ingersent\*, C.-C. Liu, E. M. Nica\*, R. Yu\*, **Q. Si**  
**X. Li, J. Kono**

**E. Bianco, S. Yazdi, E. Ringe**  
*Rice University, USA*

**G. Knebel**, G. Lapertot  
*Université Grenoble Alpes, CEA, Grenoble INP, IRIG, PHELIQS, Grenoble, France*

**E. Schuberth**  
*Technische Universität München, Germany*

T. Shiroka; A. McCollam, L. M. K. Tang, B. Vlaar; F. Weickert, R. McDonald, L. Winter,  
M. Jaime

*Paul Scherrer Institut, High Field Magnet Laboratory Nijmegen, Los Alamos National Laboratory*

O. Rubel

*Department of Materials Science and Engineering, McMaster University*



## Summary

- In heavy fermion compounds the correlation strength can be tuned
- Singularities (can) appear at quantum critical points (QCPs)
- Some heavy fermion compounds are well described by GLW QCPs
- Beyond-GLW QCPs are accompanied by Kondo destruction physics
- AF (!) Kondo destruction QCP in finite  $B$ : Spin triplet pairing?