INSTITUT QUANTIQUE

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RESEARCH

Thermal transport in quantum materials

Lecture no. 1

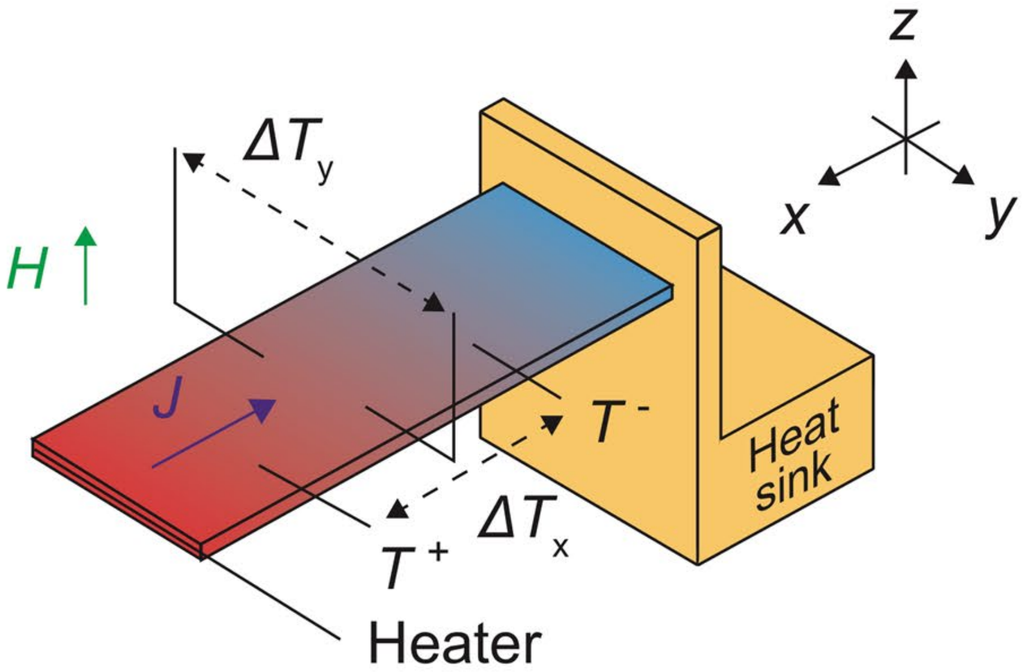
Louis Taillefer

Université de Sherbrooke

CIFAR

Exosup2022 Summer School, Cargèse, June 2022

Measurement of thermal transport



$$\kappa_{xx} = \frac{\dot{Q}}{\Delta T_x\,\alpha}$$

$$\kappa_{xy} = -\,\kappa_{xx}\frac{\Delta T_y}{\Delta T_x}\frac{L}{w}$$

Thermal transport in quantum materials

PART I — Kxx

METALS

1) Electrons & phonons
2) Wiedemann-Franz law in cuprates

SUPERCONDUCTORS

1) Cuprates — *d*-wave + Hc2
2) Iron pnictides — *s*+- or *d*-wave
3) Ruthenate — *d*-wave ?

INSULATORS

1) Nd2CuO4 — phonons
2) Nd2CuO4 — magnons
3) dmit — spinons ?

PART I — Kxx

METALS

Thermal transport in quantum materials

PART I — Kxx

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METALS 1) Electrons & phonons

ELECTRONS 1) Elastic scattering

2) Inelastic scattering — electrons, phonons, spin excitations

3) Wiedemann-Franz law (T = 0 limit)

4) Lorenz ratio (T > 0)

PHONONS 1) Scattering processes — boundaries, impurities, electrons, phonons, …

ELECTRONS

Wiedemann-Franz law

$$\frac{\kappa}{\sigma} = \frac{c_v v^2 \tau/3}{ne^2\tau/m} = \frac{c_v v^2/3}{ne^2/m}$$

NB. Assumes that tau is the same…

SOMMERFELD

$$c_v = \frac{\pi^2}{3} {k_B}^2 g(E_F)\ T = \frac{\pi^2 {k_B}^2 n}{m {v_F}^2}\ T$$

$$\frac{1}{2} m {v_F}^2 = E_F$$

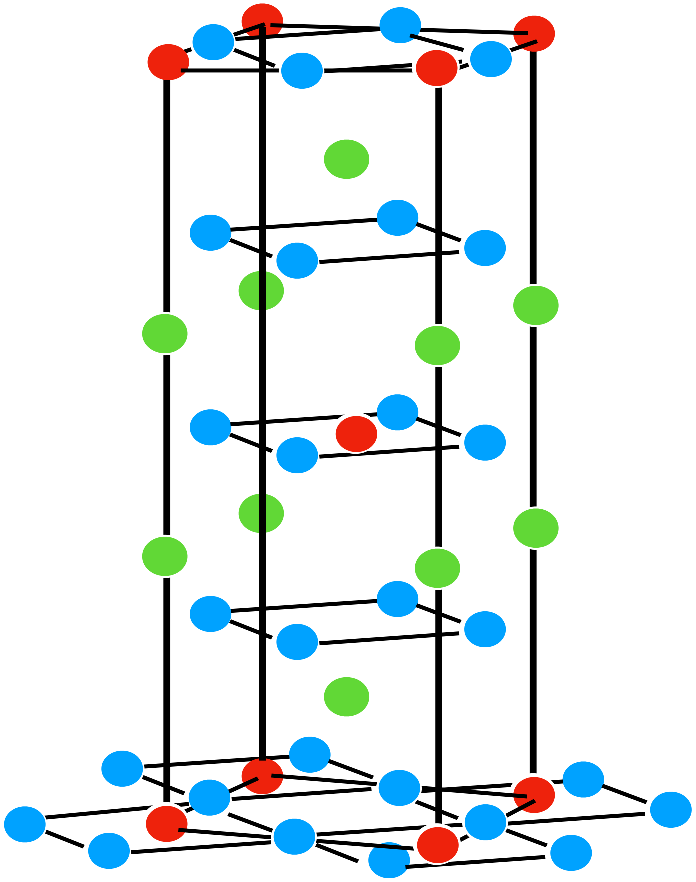
$T \to 0$

$$L = \frac{\kappa}{\sigma T} = \frac{\gamma m v_F^2}{3ne^2} = \frac{\pi^2 {k_B}^2 n}{3ne^2} = \frac{\pi^2}{3} (\frac{k_B}{e})^2 = 2.44 \times 10^{-8}\ \mathrm{W\Omega/K^2}$$

Cuprate superconductors

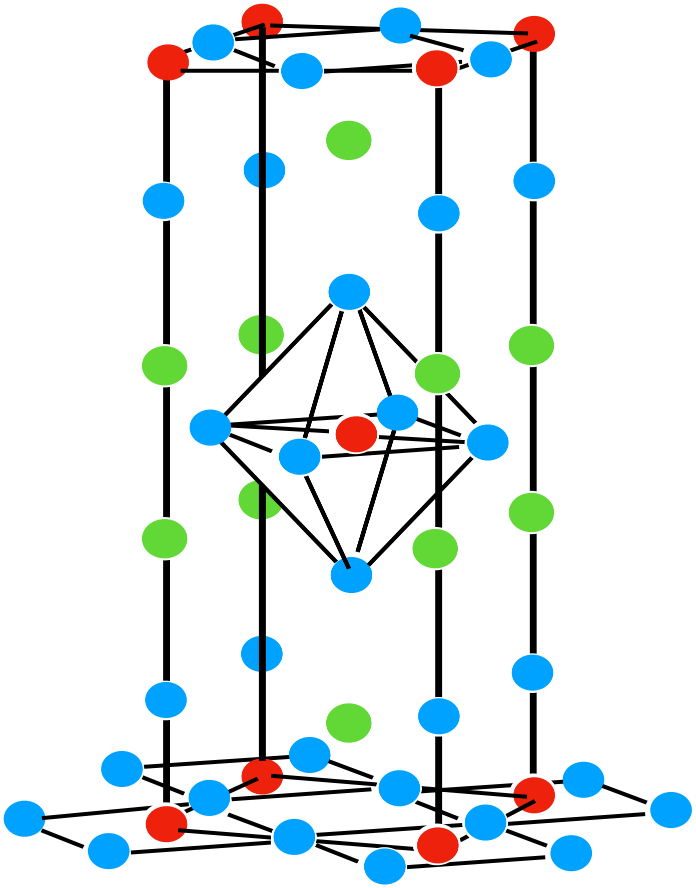
CUPRATES

Nd_2CuO_4



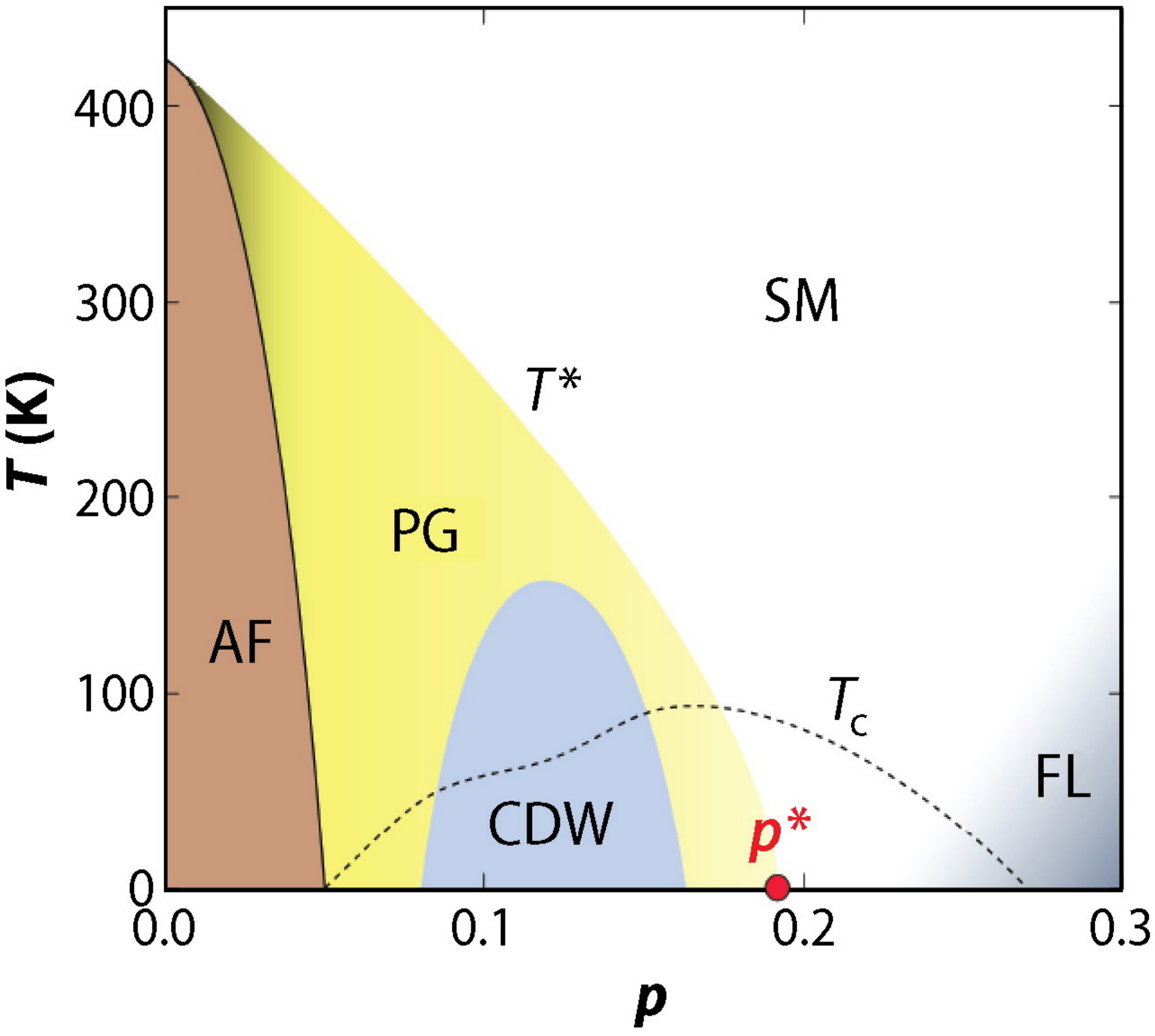
Cu 🔴
O 🔵
Nd 🟢

La_2CuO_4



Cu 🔴
O 🔵
La 🟢

HOLE-DOPED CUPRATES



Six regions

1) Superconductivity

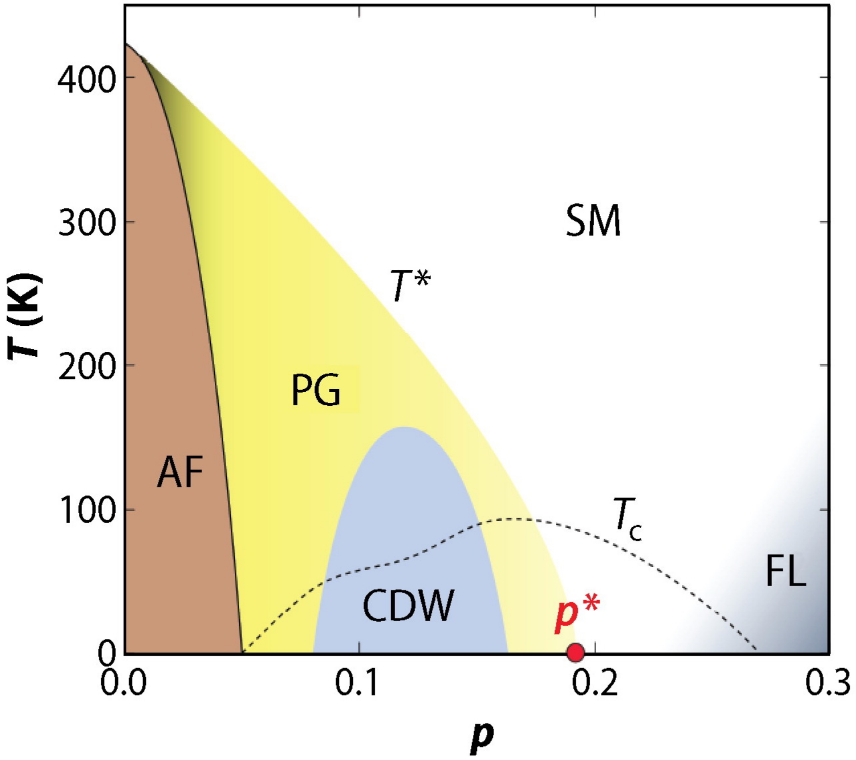
2) Mott insulator

3) Fermi liquid

4) Strange metal

5) Charge order

6) Pseudogap phase



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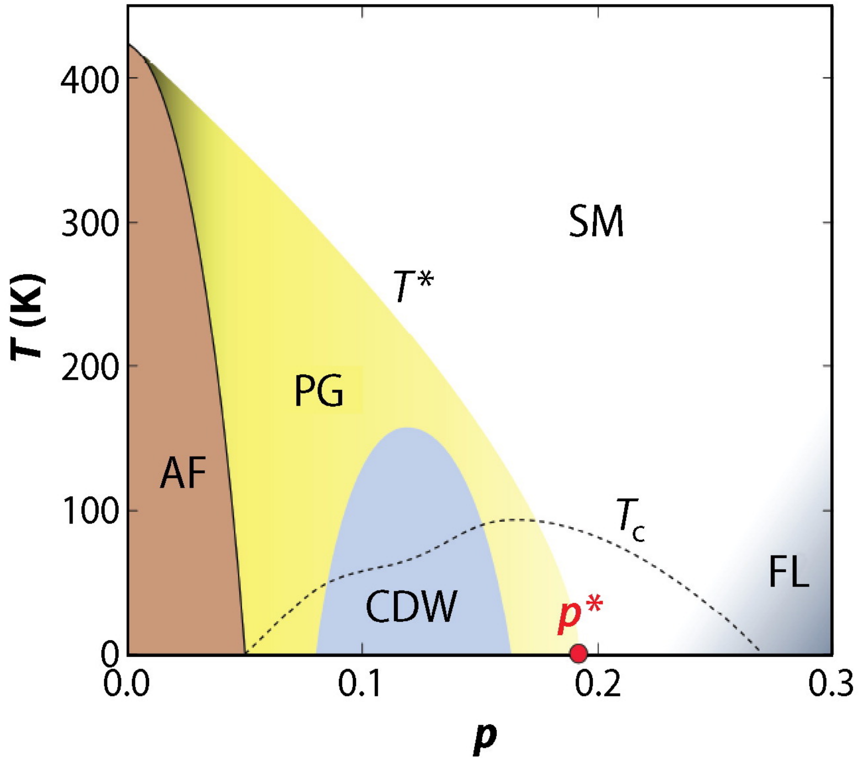


3) Fermi liquid *at high doping*

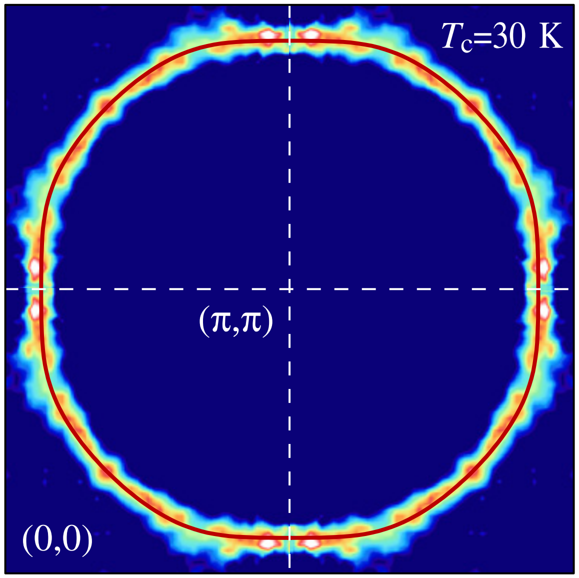
a) Large Fermi surface — ARPES, quantum osc.

b) T^2 resistivity

3) Fermi liquid *at high doping*

a) Large Fermi surface — ARPES, quantum osc.

Fermi surface at $p = 0.3$



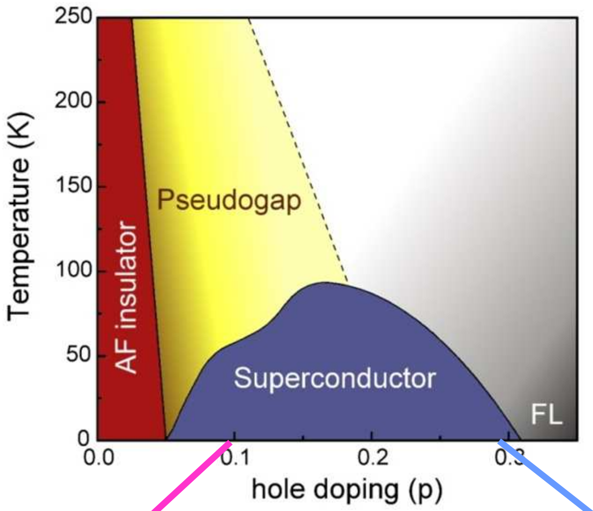
Peets *et al.*, New J. Phys. **9**, 28 (2007)

Carrier density

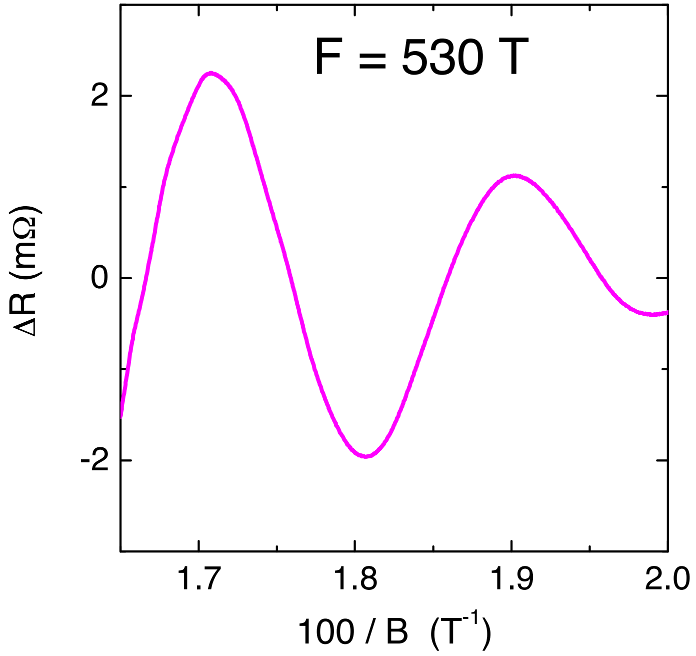
$$\boldsymbol{n = 1+p}$$

Fermi-surface reconstruction Quantum oscillations

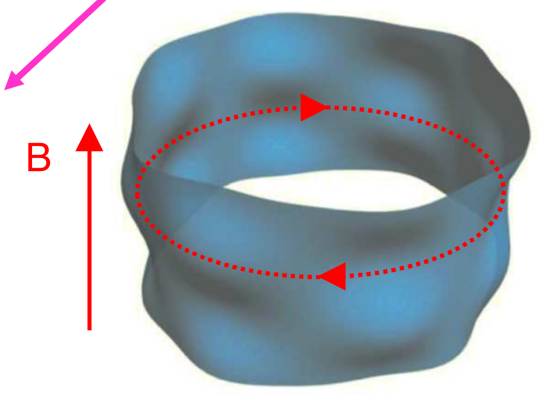
underdoped
$YBa_2Cu_3O_{6.5}$



overdoped
$Tl_2Ba_2CuO_{6+\delta}$

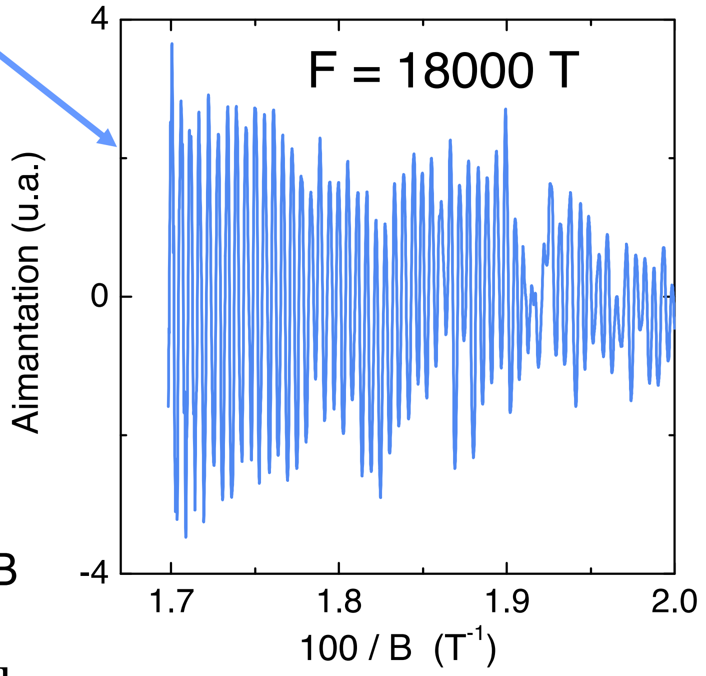


N. Doiron-Leyraud et al, Nature’07



Oscillations periodic in 1/B

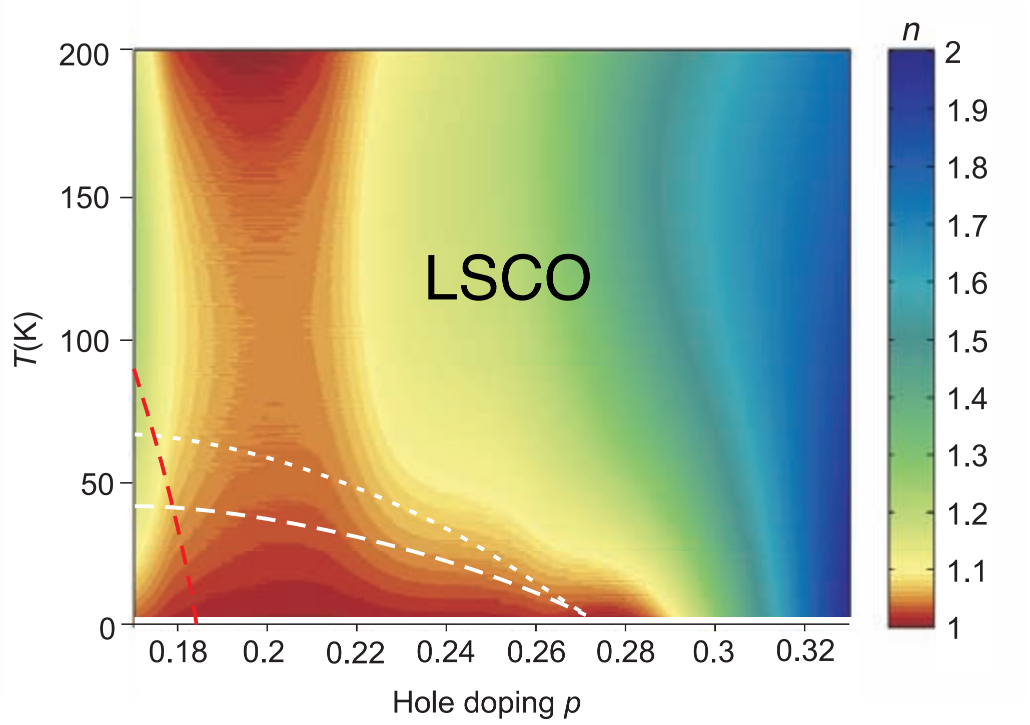
$$F = \frac{\phi_0}{2\pi^2} A_k \quad [\,\mathrm{T}\,]$$

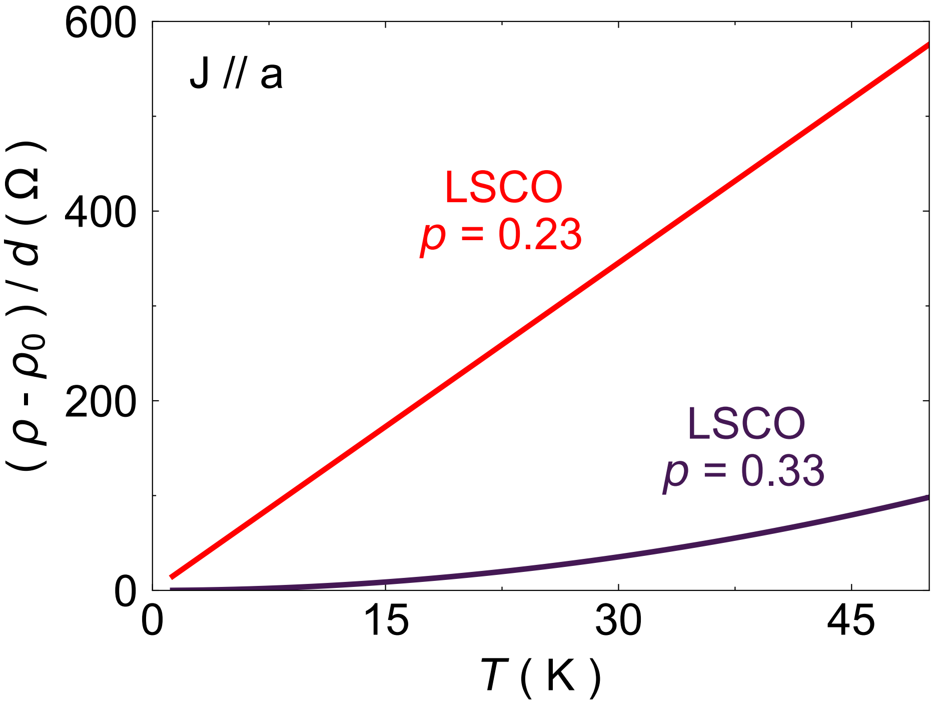


B. Vignolle et al, Nature’08

Courtesy of C. Proust

2009 *T*-linear to *T*-quadratic resistivity in h-doped cuprates LSCO





Cooper *et al*., Science **323**, 603 (2009)

Nakamae *et al*., PRB **68**, 100502 (2003)

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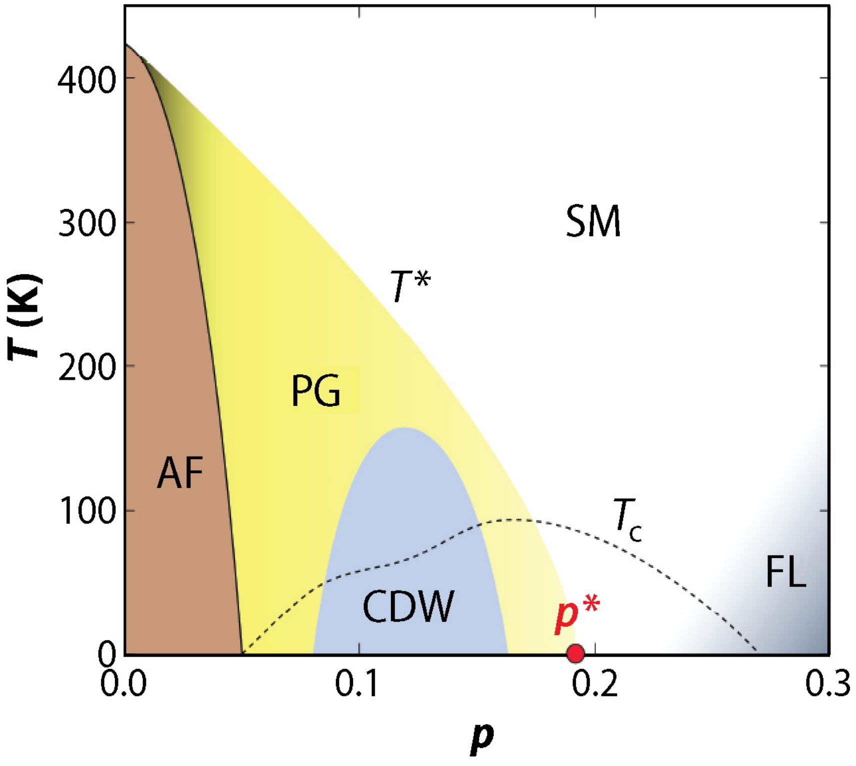

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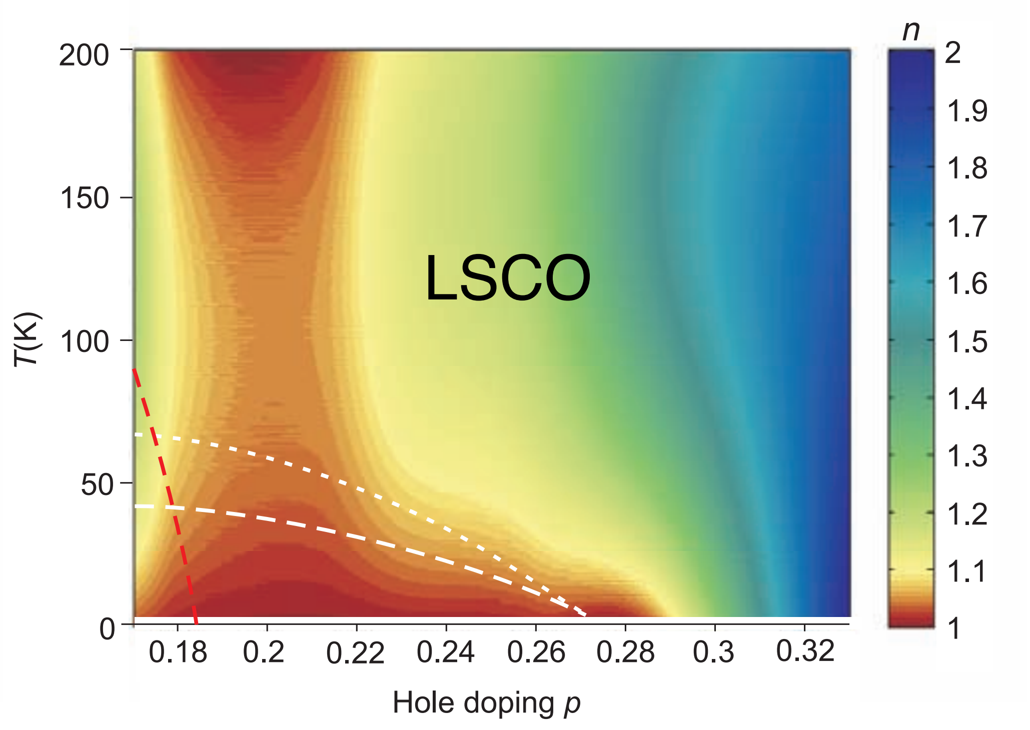
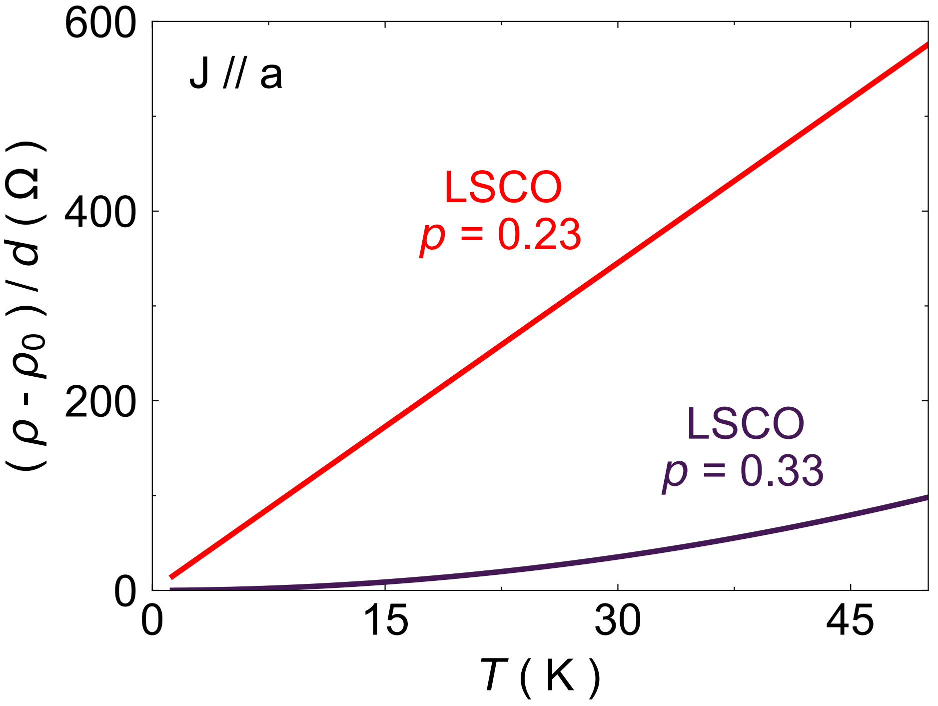


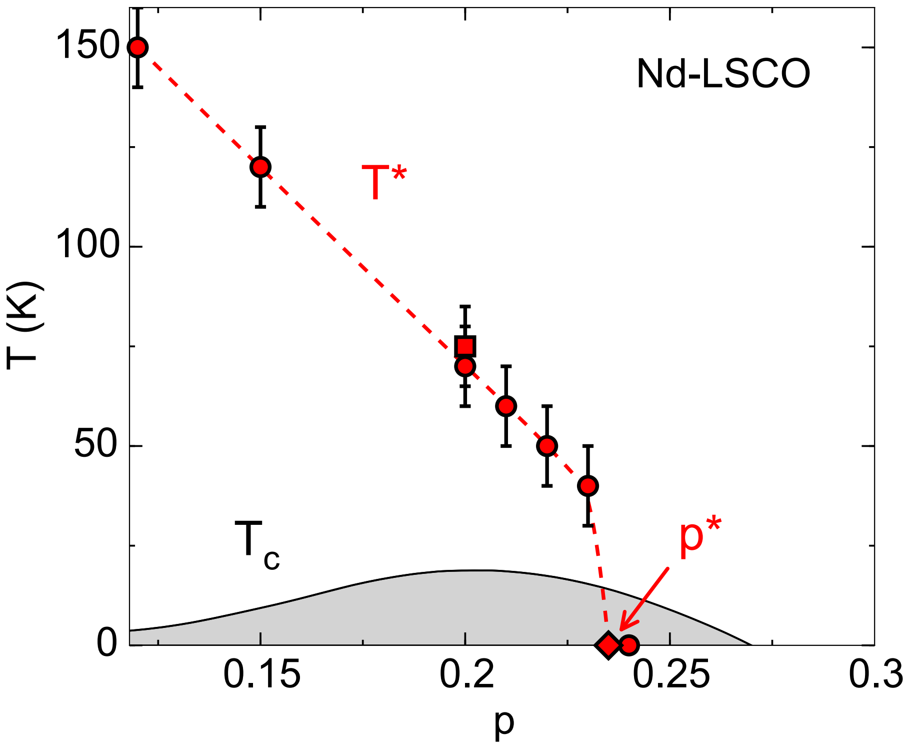


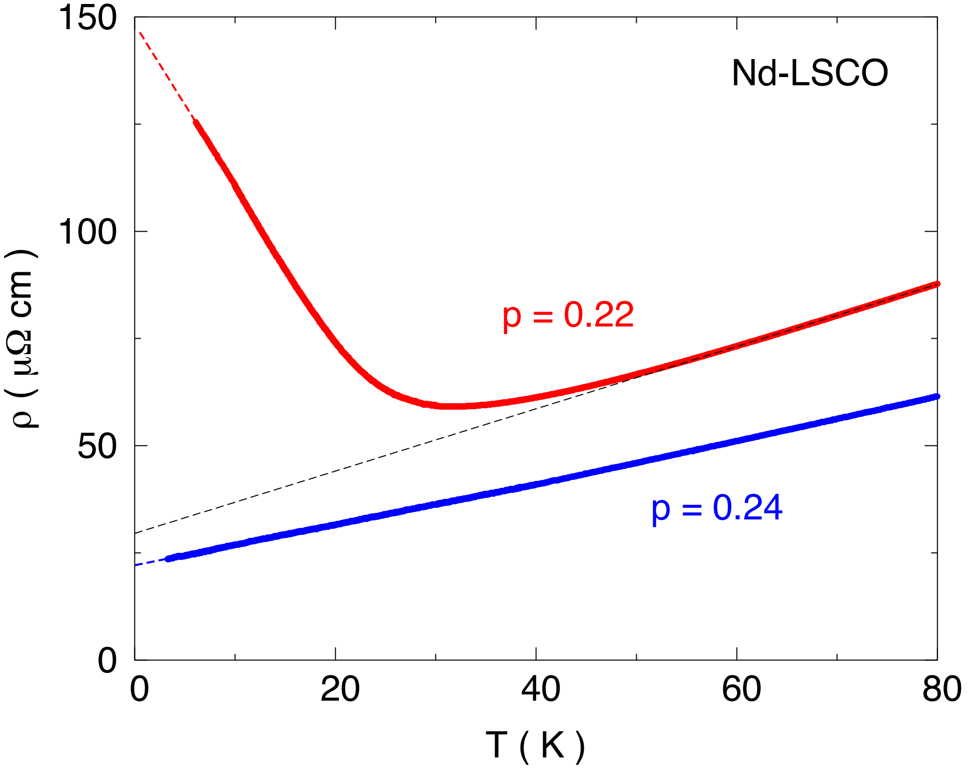
Cooper *et al.*, Science **323**, 603 (2009)

Nakamae *et al.*, PRB **68**, 100502 (2003)

Nd-LSCO

Linear-*T* resistivity & upturn

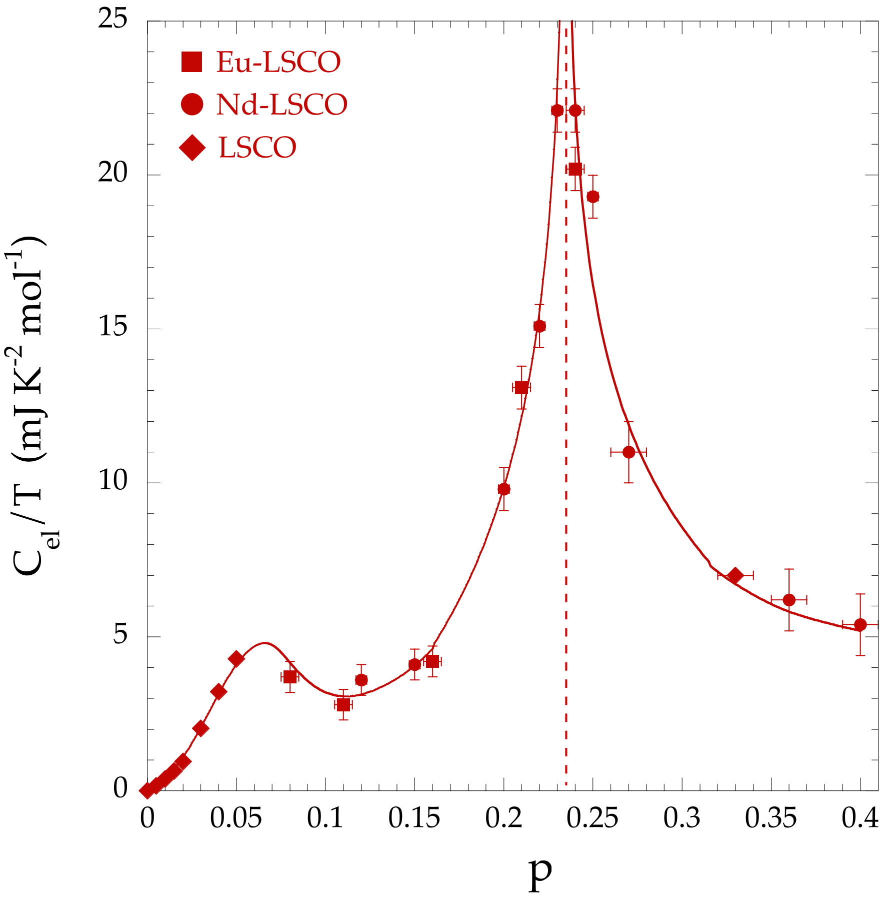




Collignon *et al.*, PRB **95**, 224517 (2017)

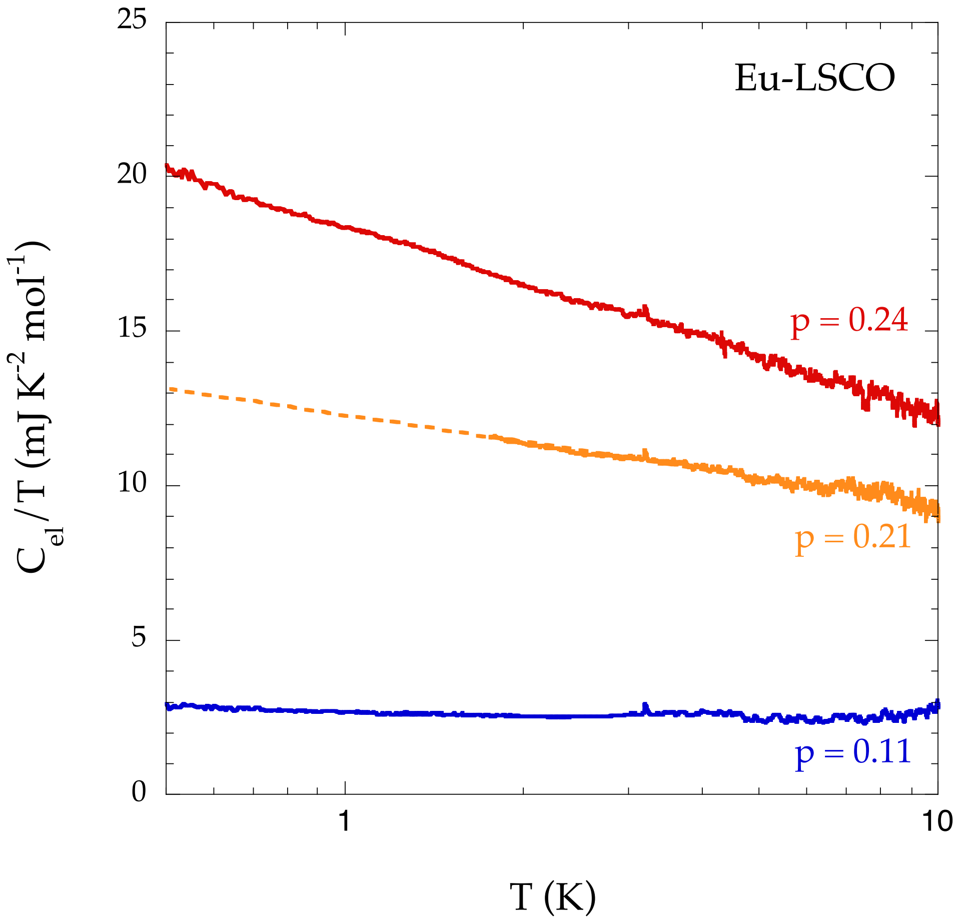
Daou *et al.*, Nature Physics **5**, 31 (2009)

Two thermodynamic signatures of quantum criticality in cuprates



<u>LSCO</u> : Nakamae *et al.*, PRB **68**, 100502 (2003)

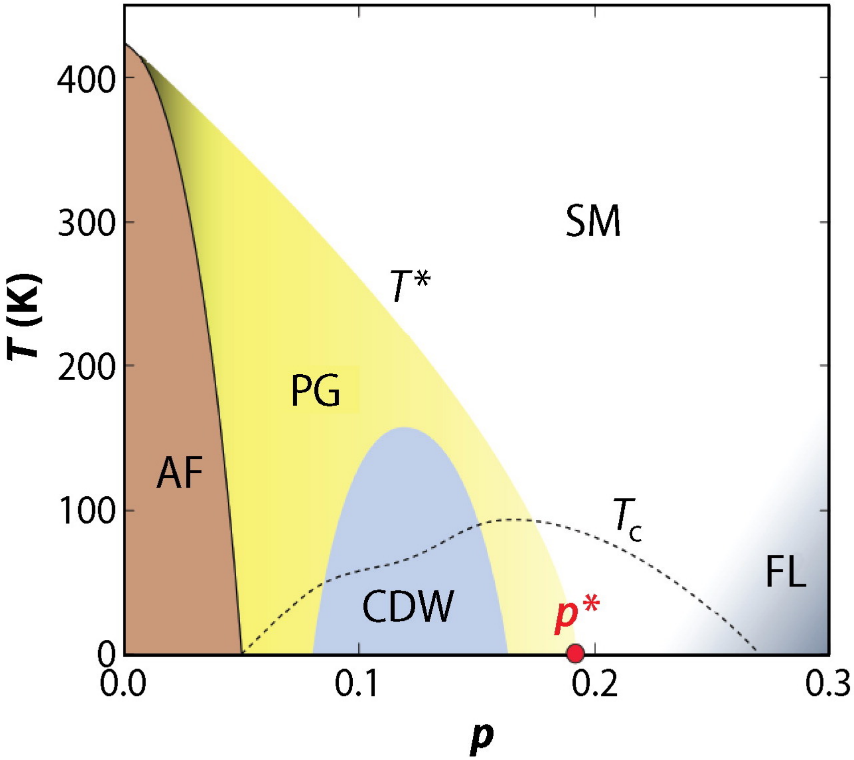
Komiya *et al.*, J. Phys. **150**, 052118 (2009)



$C\,/\,T \sim -\log(T)$

Michon *et al.*, Nature **567**, 218 (2019)

METALS 2) Wiedemann-Franz law in cuprates Tl2201



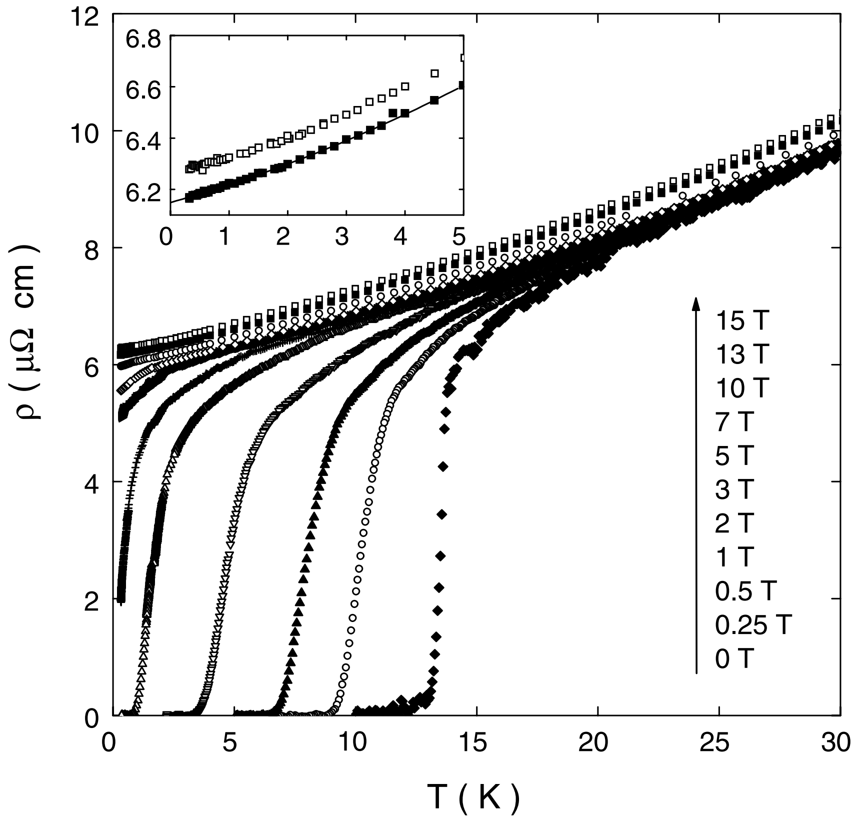
METALS 2) Wiedemann-Franz law in cuprates Tl2201

VOLUME 89, NUMBER 14 PHYSICAL REVIEW LETTERS 30 SEPTEMBER 200

Heat Transport in a Strongly Overdoped Cuprate: Fermi Liquid and a Pure *d*-Wave BCS Superconductor

Cyril Proust,[1,*] Etienne Boaknin,[1] R.W. Hill,[1] Louis Taillefer,[1] and A. P. Mackenzie[2]

$$\rho(T) = \rho_0 + bT + cT^2$$



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Tl2201

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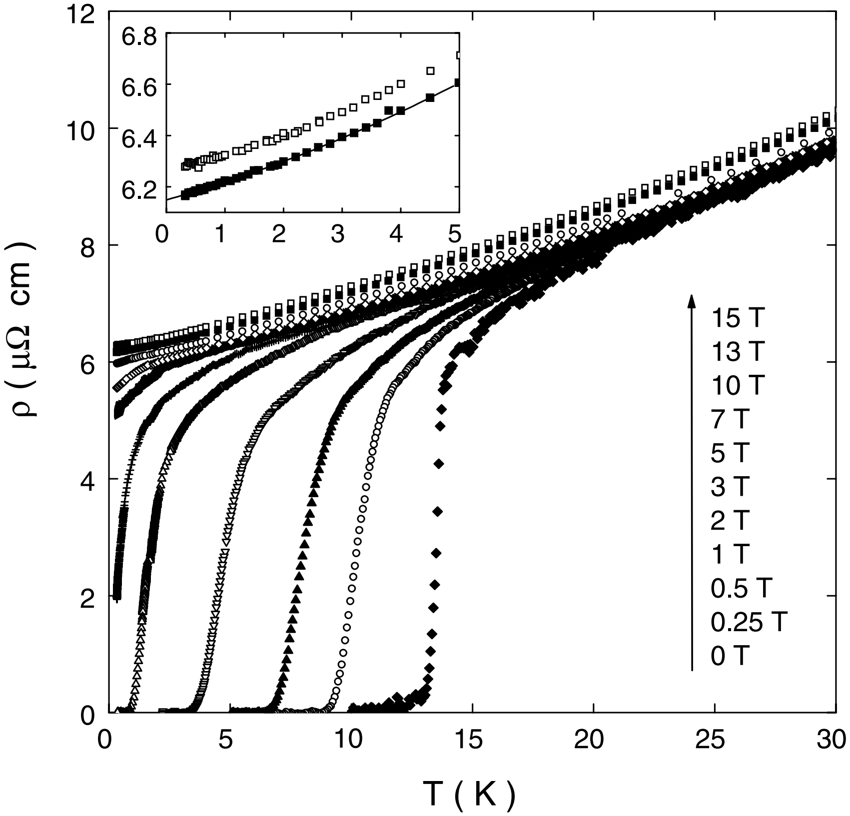
$$\rho(T) = \rho_0 + bT + cT^2$$

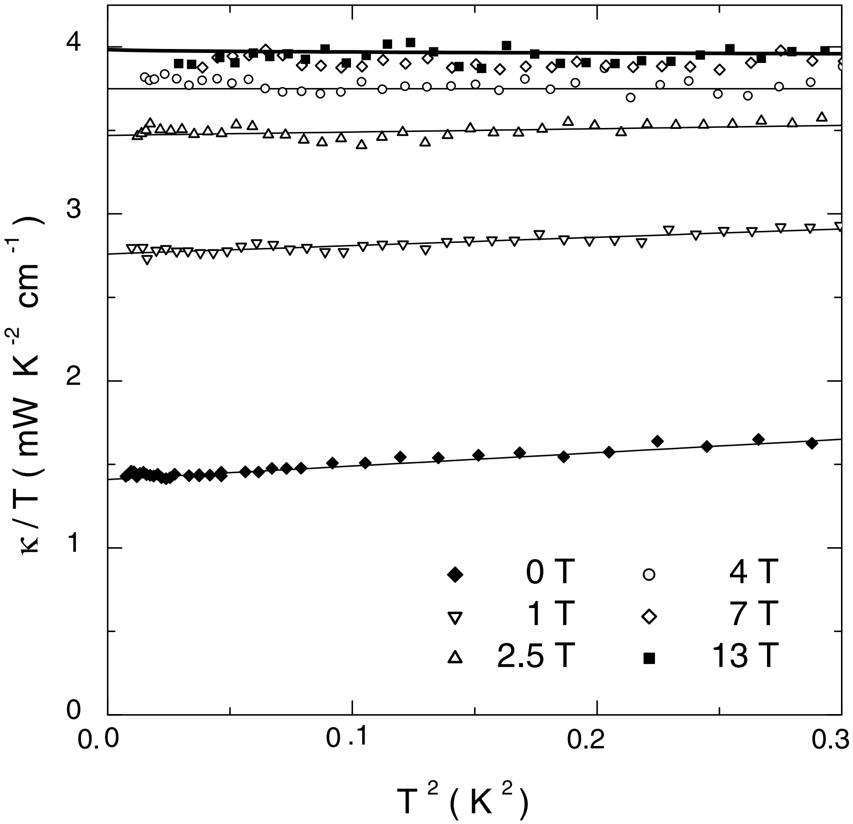
$$\frac{\kappa}{\sigma T} = \frac{\pi^2}{3}\left(\frac{k_B}{e}\right)^2 \equiv L_0$$

$$\kappa_0/\mathrm{T} = 3.95 \pm 0.04\ \mathrm{mW\,K^{-2}\,cm^{-1}}$$

$$\rho_0 = 6.15 \pm 0.03\ \mu\Omega\,\mathrm{cm}$$

$$L = \rho_0\kappa_0/T = 0.99 \pm 0.01 L_0$$





METALS

2) Wiedemann-Franz law in cuprates

Nd-LSCO

The cuprate Nd-LSCO

The pseudogap phase

Six regions

1) Superconductivity

2) Mott insulator

3) Fermi liquid

4) Strange metal

5) Charge order

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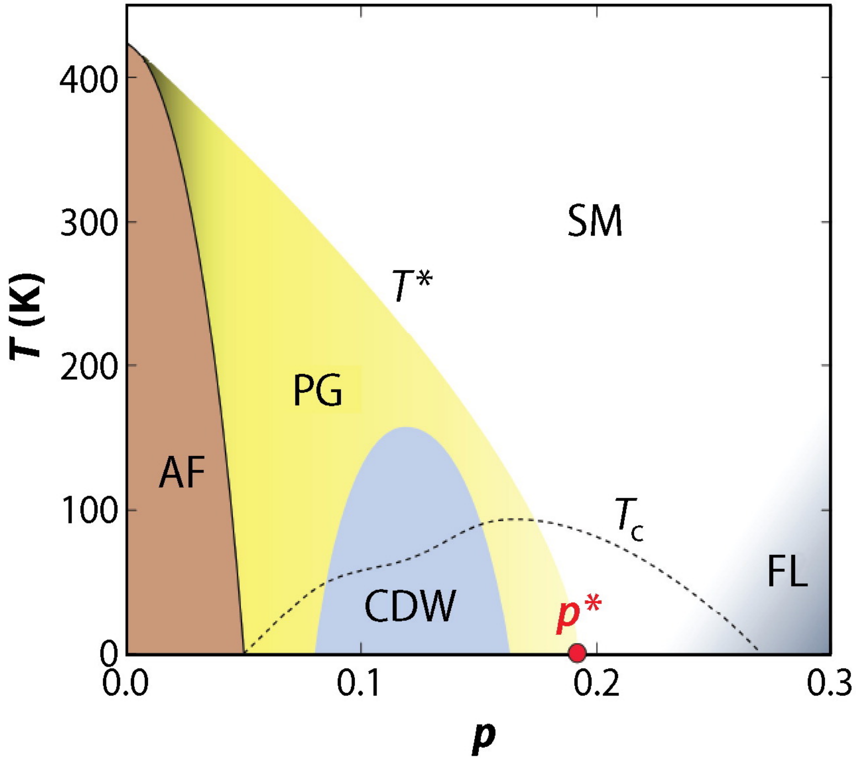

6) Pseudogap phase *below T^*, below p^**

a) ARPES — loss of spectral weight in AN regions

b) STM — FS transformation across p^*

c) Hall number — loss of carrier density

d) NMR — loss of density of states

6) Pseudogap phase *below T^*, below p^**

a) ARPES — loss of spectral weight in AN regions

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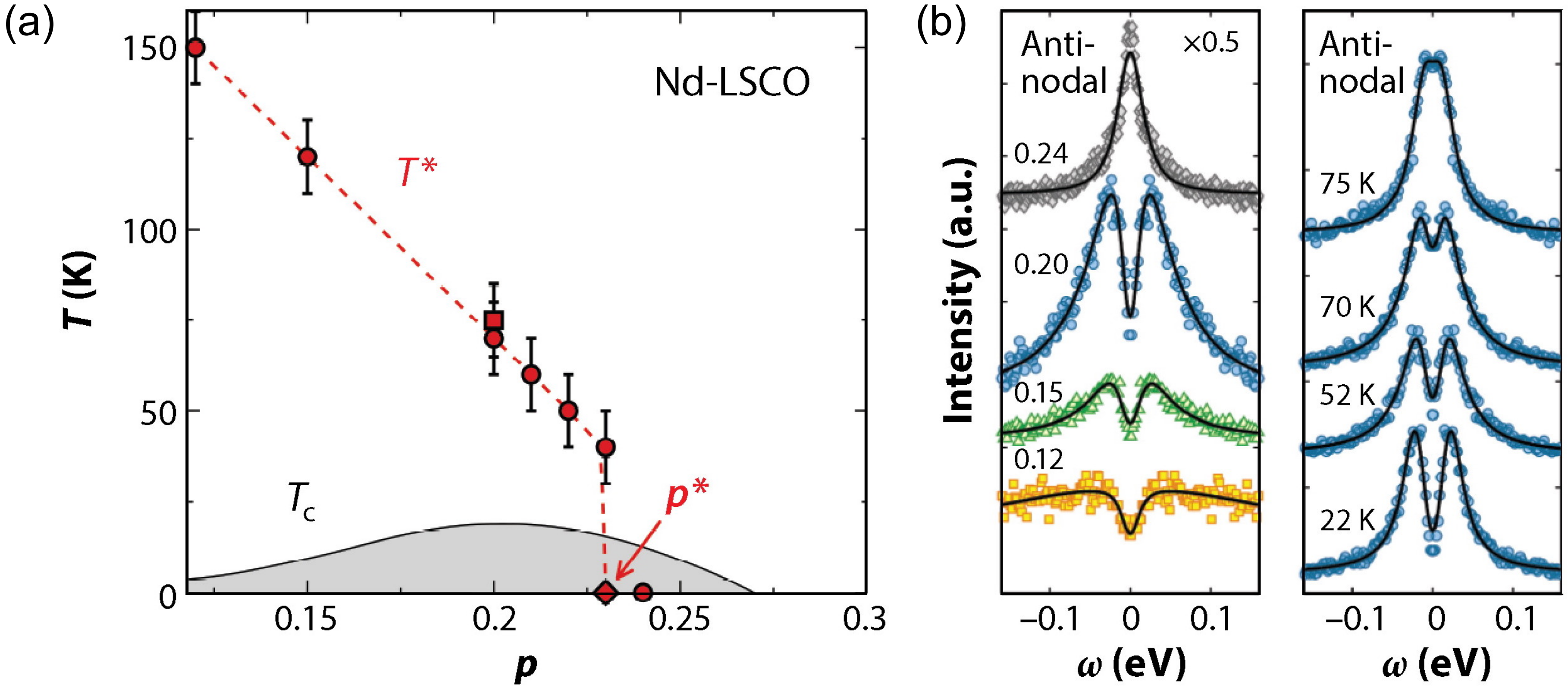
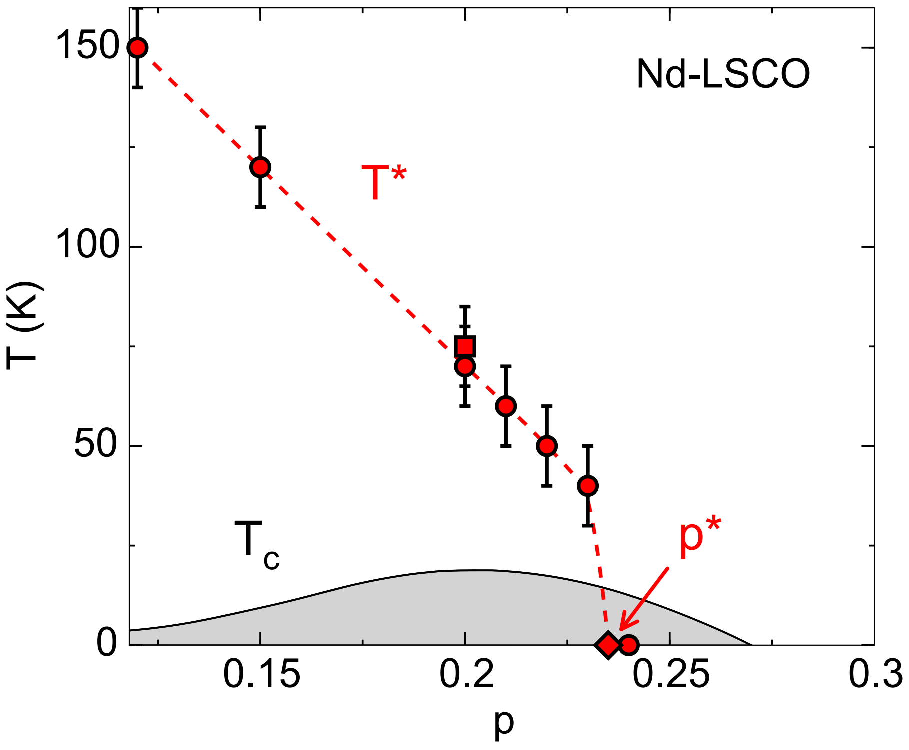
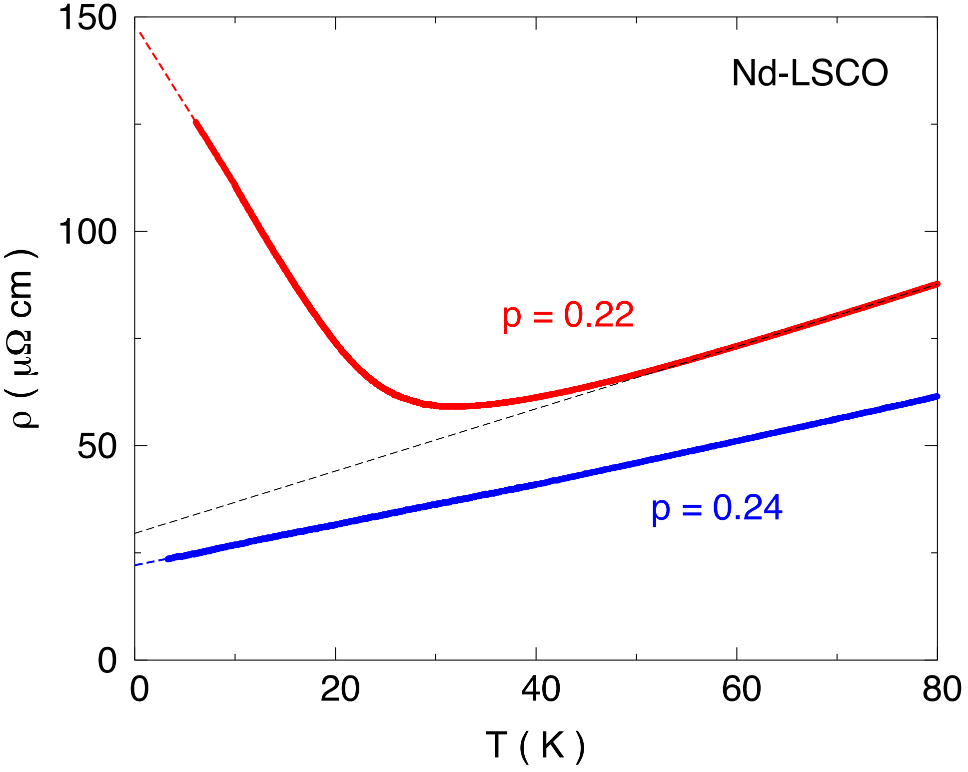


Nd-LSCO

Linear-*T* resistivity & upturn





Collignon *et al.*, PRB **95**, 224517 (2017)

Daou *et al.*, Nature Physics **5**, 31 (2009)

6) Pseudogap phase *below T^*, below p^**

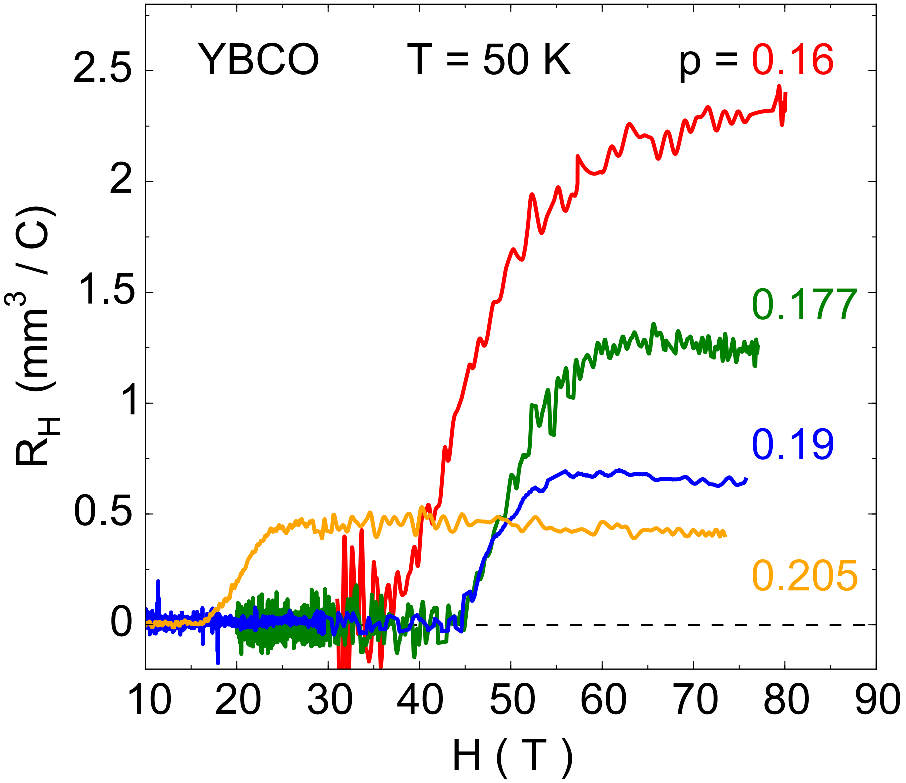
c) Hall number — loss of carrier density

2016

Cuprate superconductors

YBCO

Hall coefficient : drop in carrier density at *p**



Badoux *et al.*, Nature **531**, 210 (2016)

LNCMI Toulouse

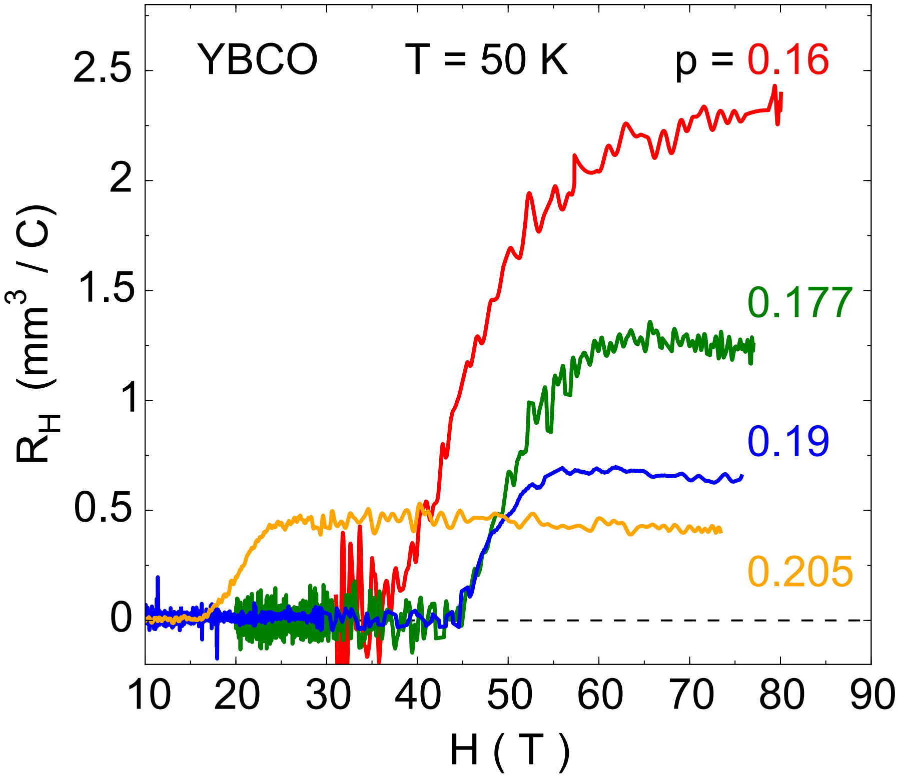
90 T pulsed magnet

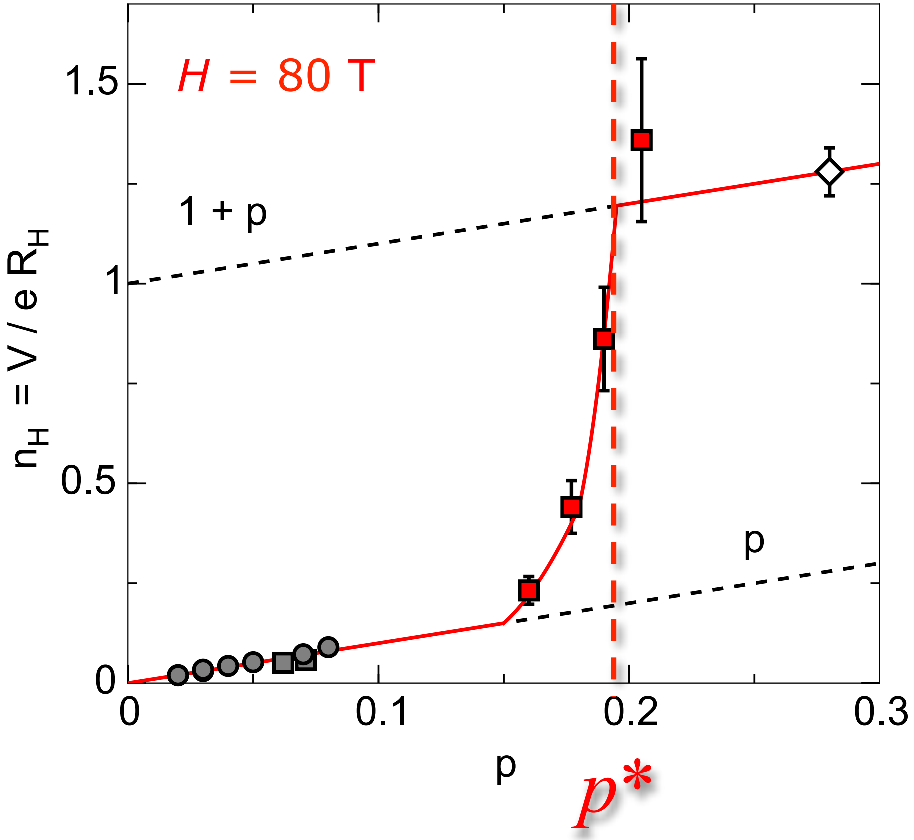
2016

Cuprate superconductors

YBCO

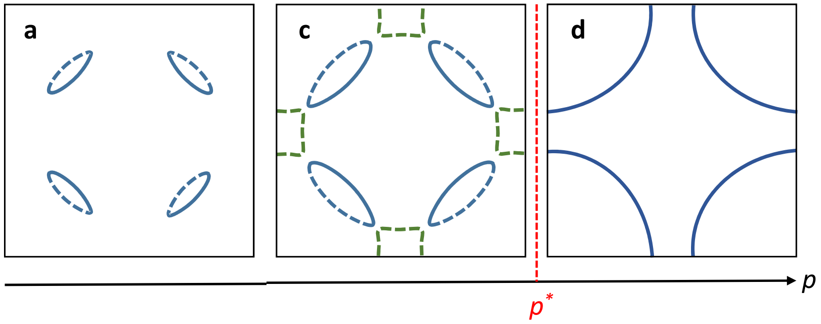
Hall coefficient : drop in carrier density at *p**

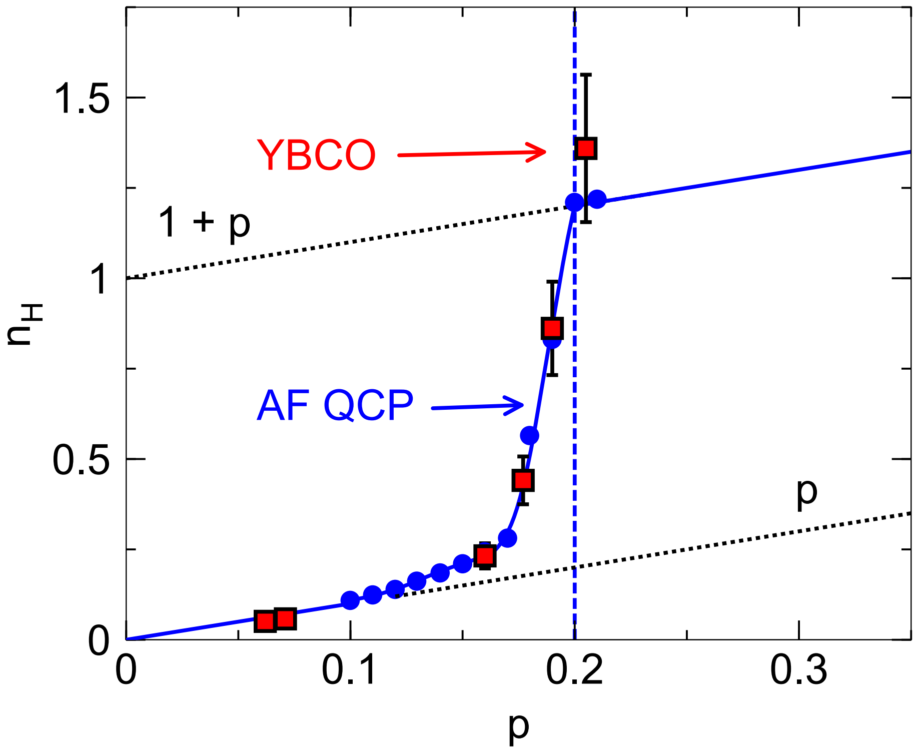




Badoux *et al.*, Nature **531**, 210 (2016)

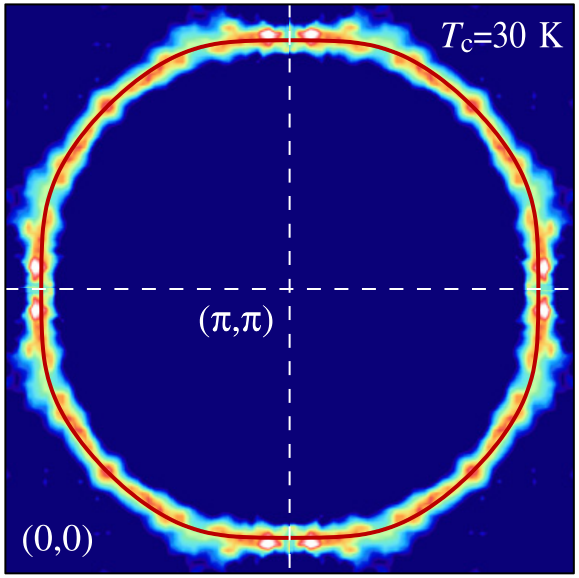
Fermi-surface reconstruction by AF order





Storey, EPL **113**, 27003 (2016)

Fermi surface at p = 0.3

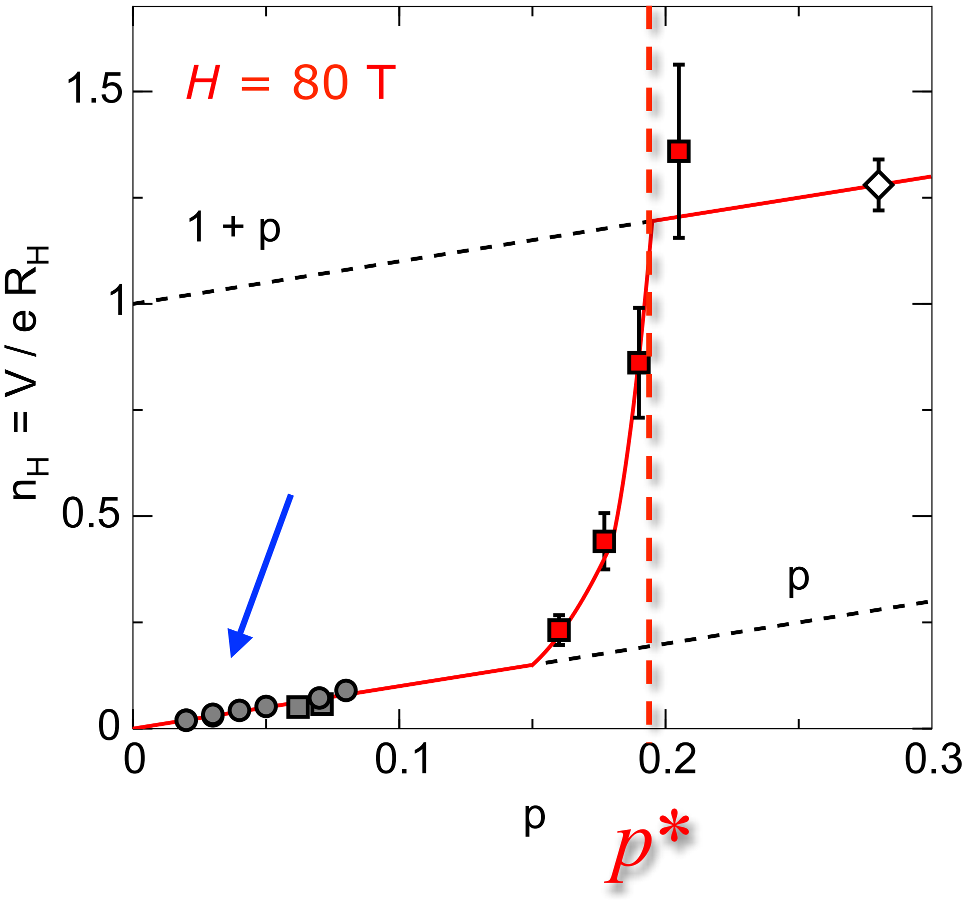


Peets *et al.*, New J. Phys. **9**, 28 (2007)

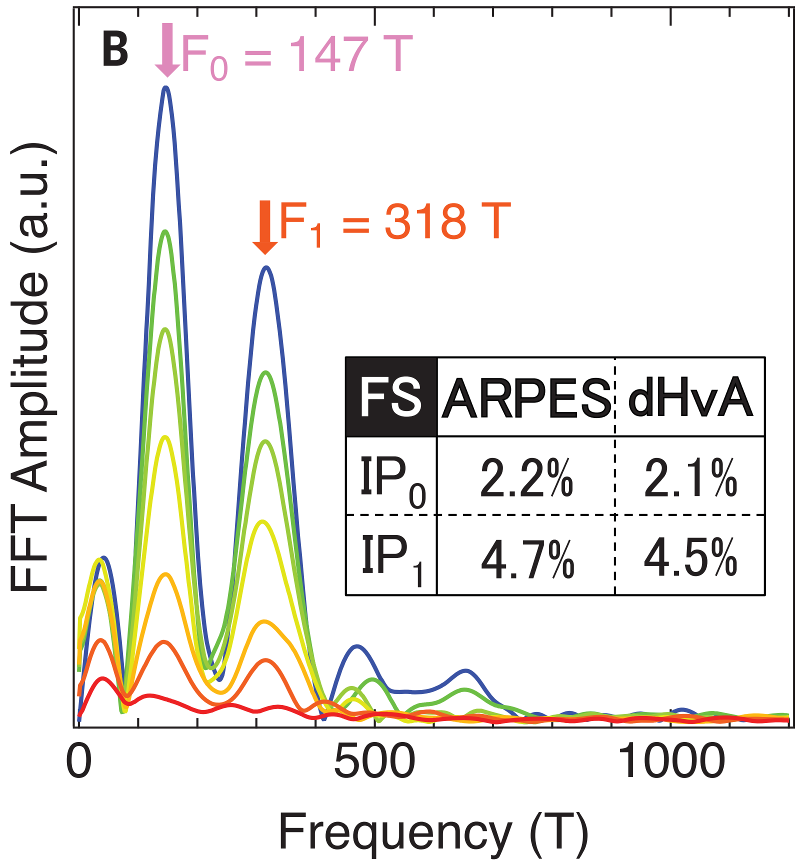
Carrier density

$$n = 1+p \quad \longrightarrow \quad n = p$$

Fermi surface at low *p* - ARPES and quantum oscillations



Badoux *et al.*, Nature **531**, 210 (2016)

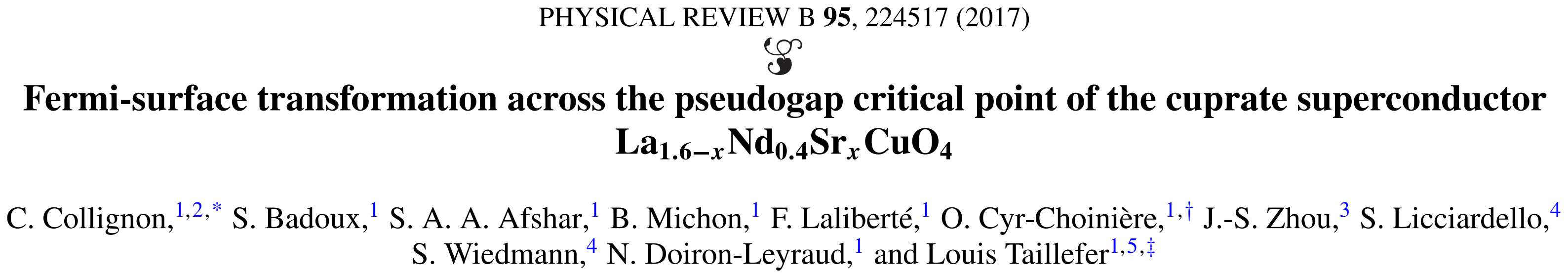


FS	ARPES	dHvA
IP_0	2.2%	2.1%
IP_1	4.7%	4.5%

Kunisada *et al.*, Science **369**, 833 (2020)

Nd-LSCO

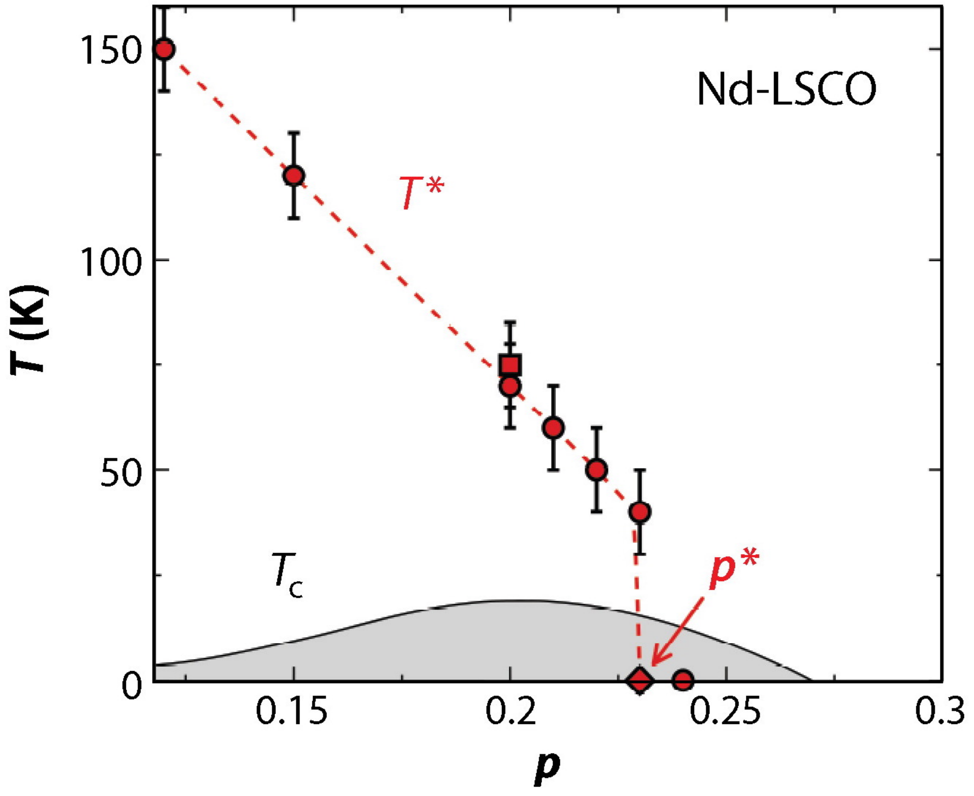
Drop in carrier density

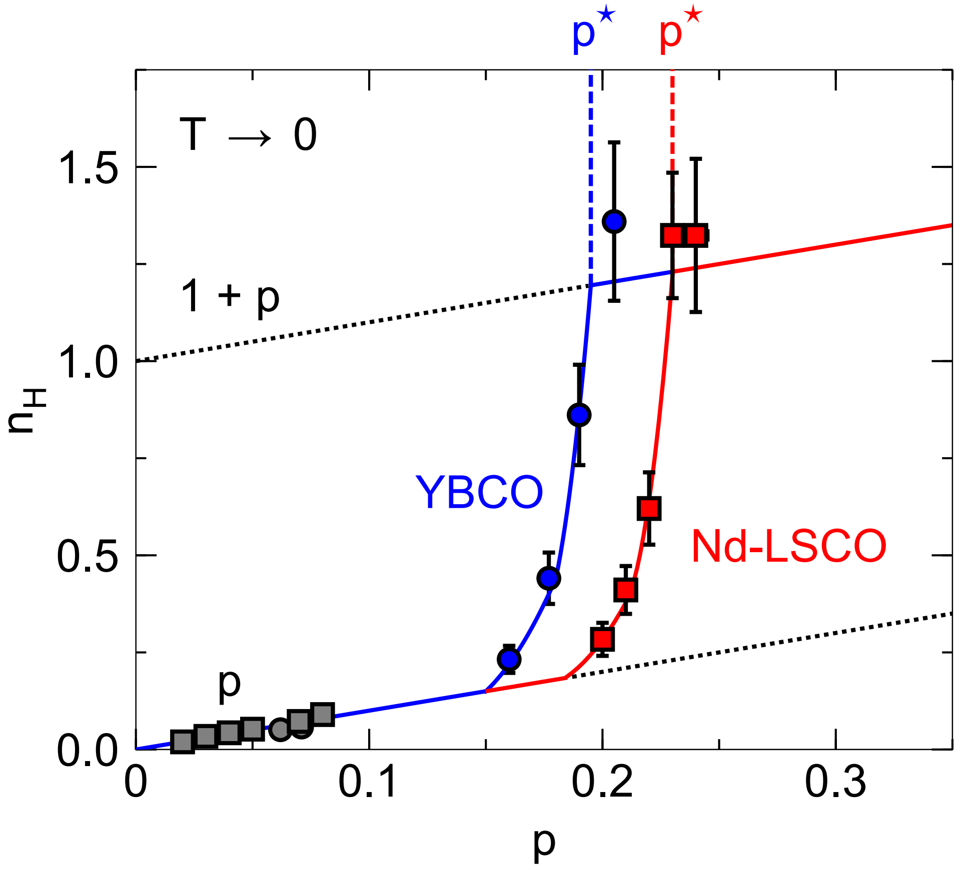


PHYSICAL REVIEW B **95**, 224517 (2017)

Fermi-surface transformation across the pseudogap critical point of the cuprate superconductor $La_{1.6-x}Nd_{0.4}Sr_xCuO_4$

C. Collignon,[1,2,*] S. Badoux,[1] S. A. A. Afshar,[1] B. Michon,[1] F. Laliberté,[1] O. Cyr-Choinière,[1,†] J.-S. Zhou,[3] S. Licciardello,[4] S. Wiedmann,[4] N. Doiron-Leyraud,[1] and Louis Taillefer[1,5,‡]





METALS 2) Wiedemann-Franz law in cuprates Nd-LSCO

Nd-LSCO

Test of the WF law

PHYSICAL REVIEW X **8,** 041010 (2018)

Wiedemann-Franz Law and Abrupt Change in Conductivity across the Pseudogap Critical Point of a Cuprate Superconductor

B. Michon,[1,2] A. Ataei,[1] P. Bourgeois-Hope,[1] C. Collignon,[1] S. Y. Li,[1,*] S. Badoux,[1] A. Gourgout,[1]
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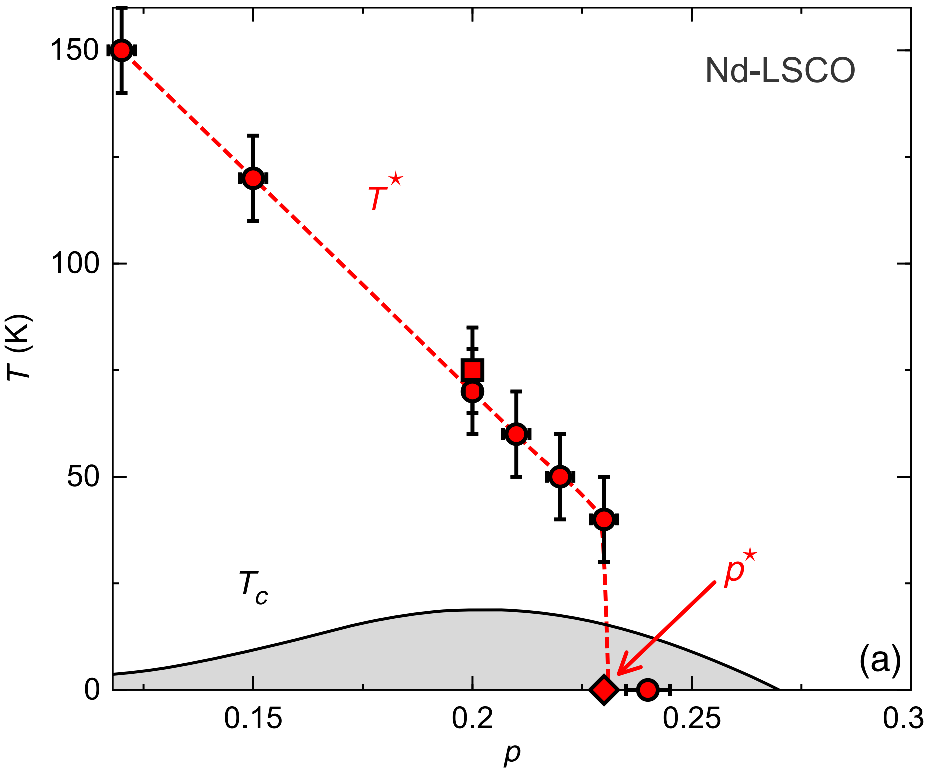
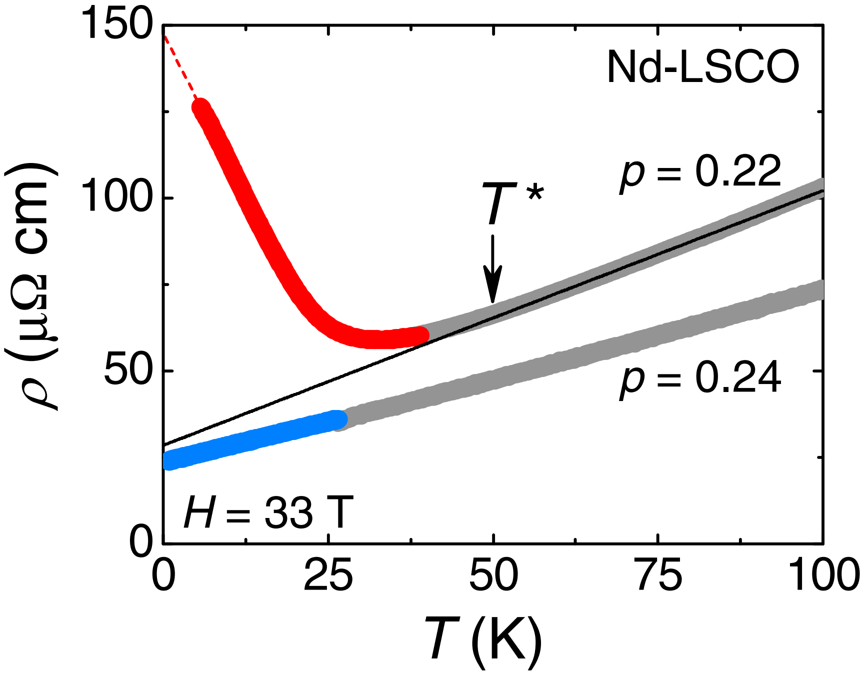
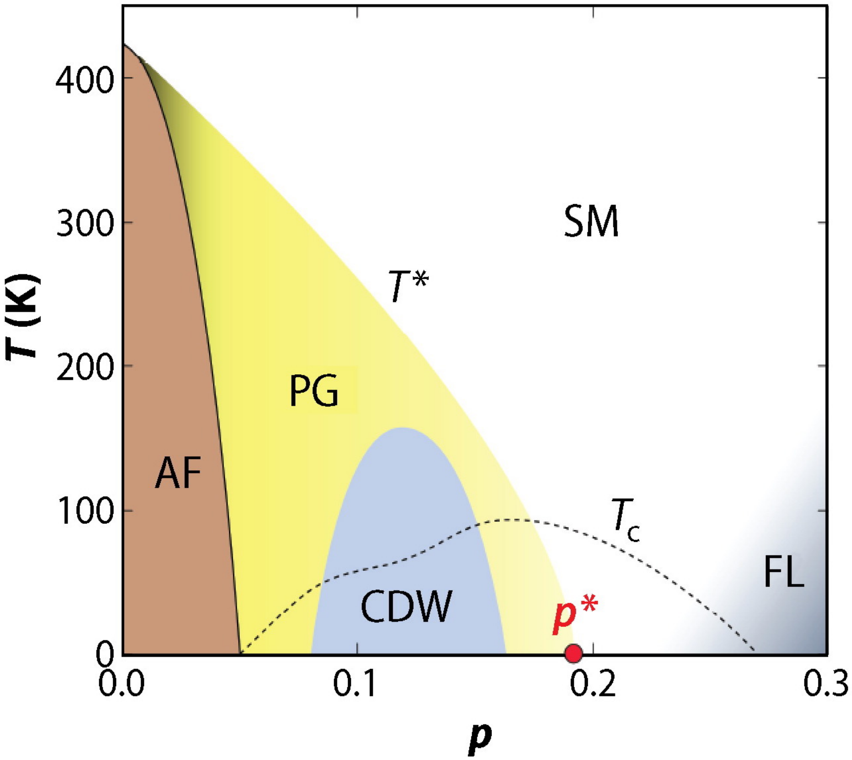
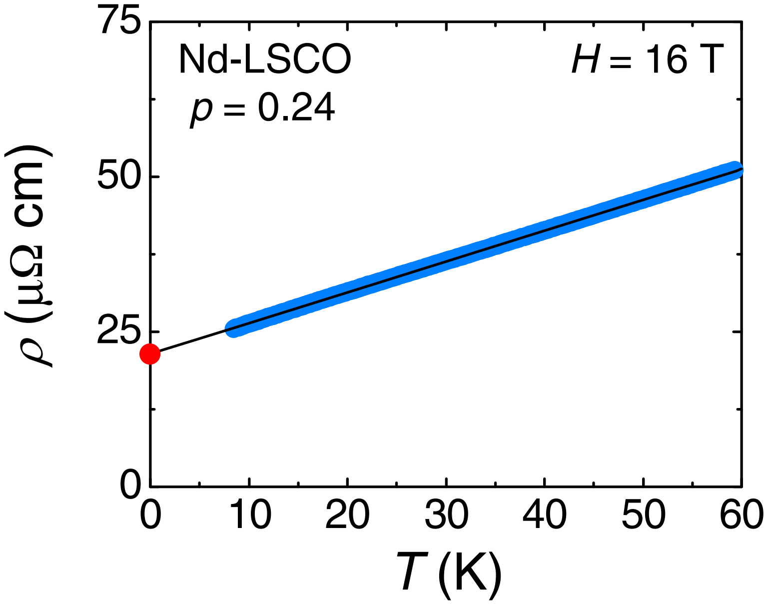




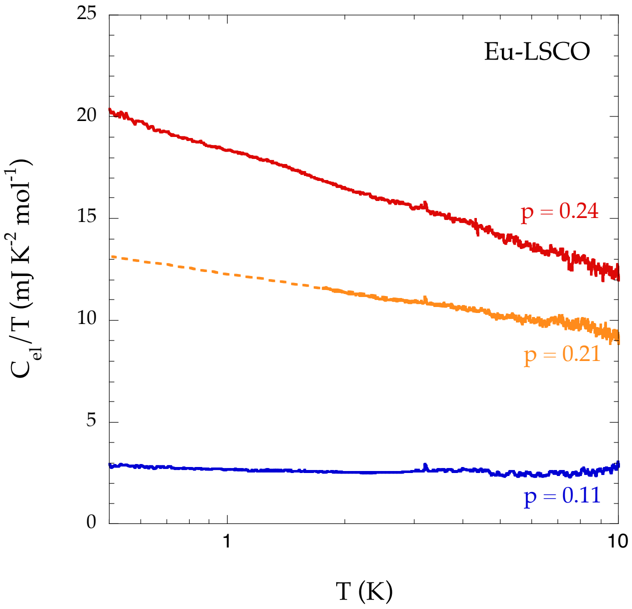
Strange metal phase



T-linear resistivity

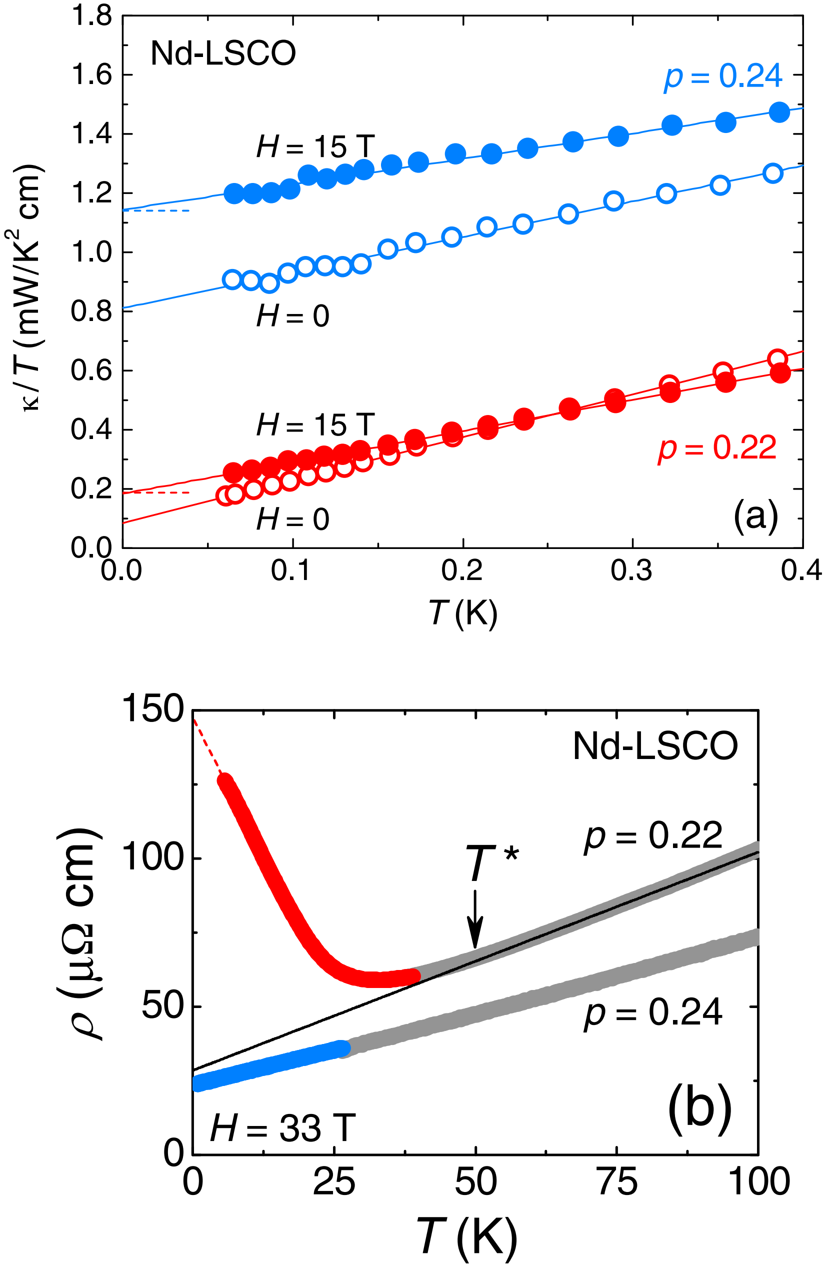


Divergent specific heat



Nd-LSCO

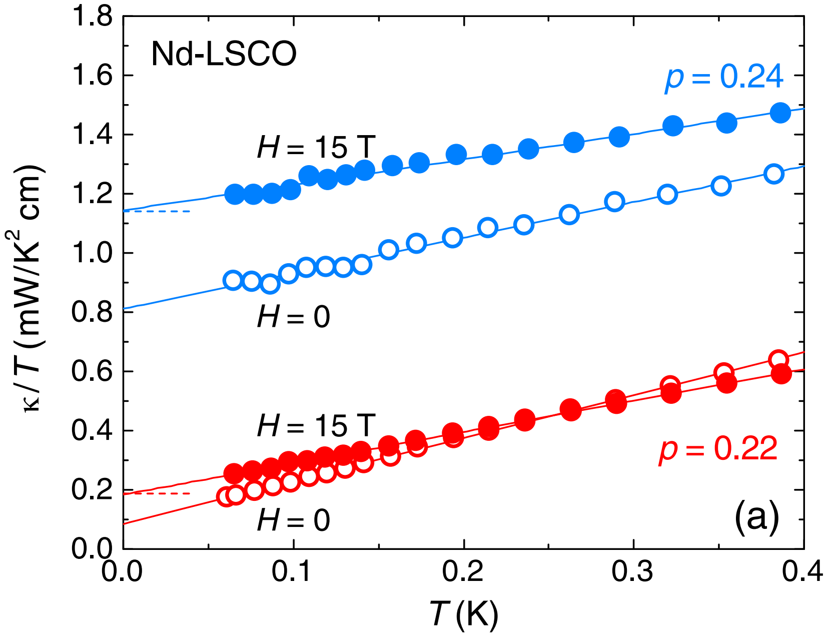
Test of Wiedemann-Franz law

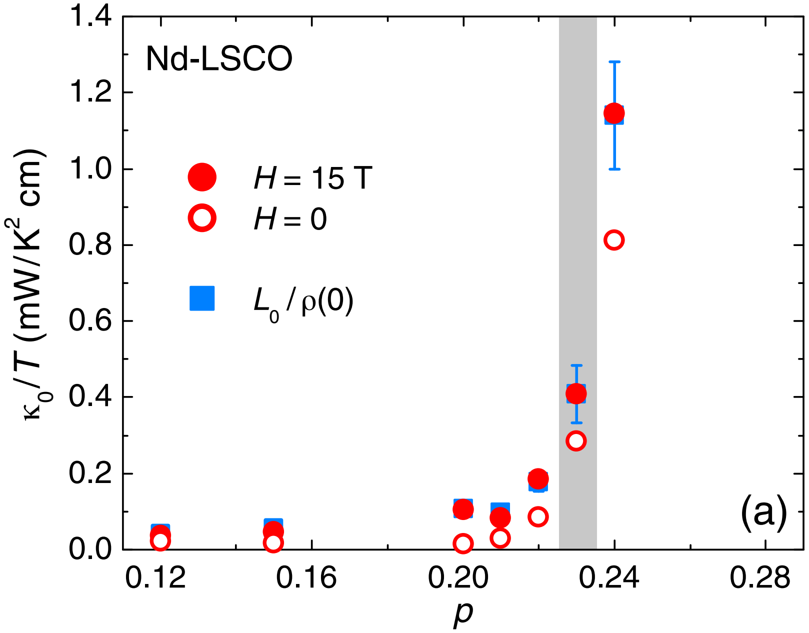


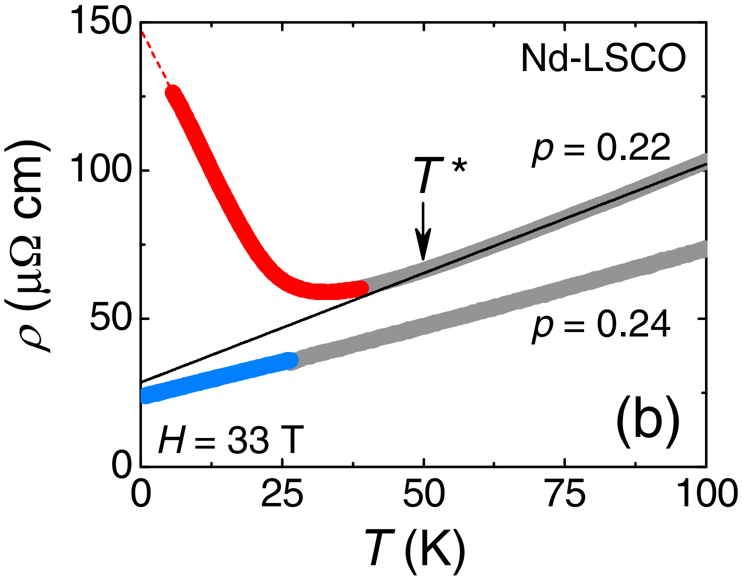
Michon *et al.*, PRX **8**, 041010 (2018)

Nd-LSCO

Test of Wiedemann-Franz law







Michon *et al.*, PRX **8**, 041010 (2018)

Nd-LSCO

Drop in conductivity

Nd-LSCO

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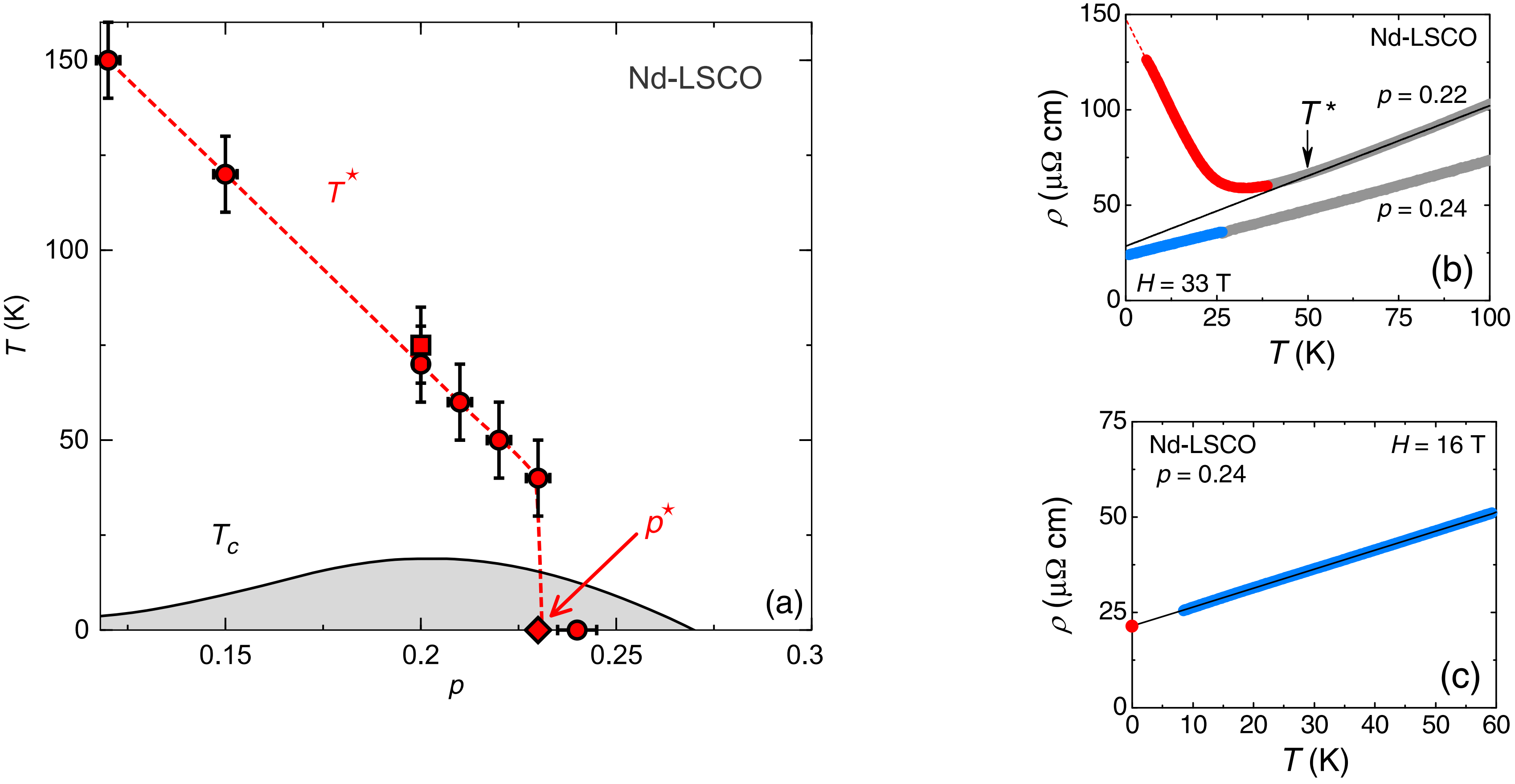
Wiedemann-Franz Law and Abrupt Change in Conductivity across the Pseudogap Critical Point of a Cuprate Superconductor

B. Michon,[1,2] A. Ataei,[1] P. Bourgeois-Hope,[1] C. Collignon,[1] S. Y. Li,[1,*] S. Badoux,[1] A. Gourgout,[1]
F. Laliberté,[1] J.-S. Zhou,[3] Nicolas Doiron-Leyraud,[1,†] and Louis Taillefer[1,4,‡]



PART I — Kxx

SUPERCONDUCTORS

Thermal transport in quantum materials

PART I — Kxx

METALS

1) Electrons & phonons

2) Wiedemann-Franz law in cuprates

SUPERCONDUCTORS

1) Cuprates — *d*-wave + Hc2

2) Iron pnictides — *s*+- or *d*-wave

3) Ruthenate — *d*-wave ?

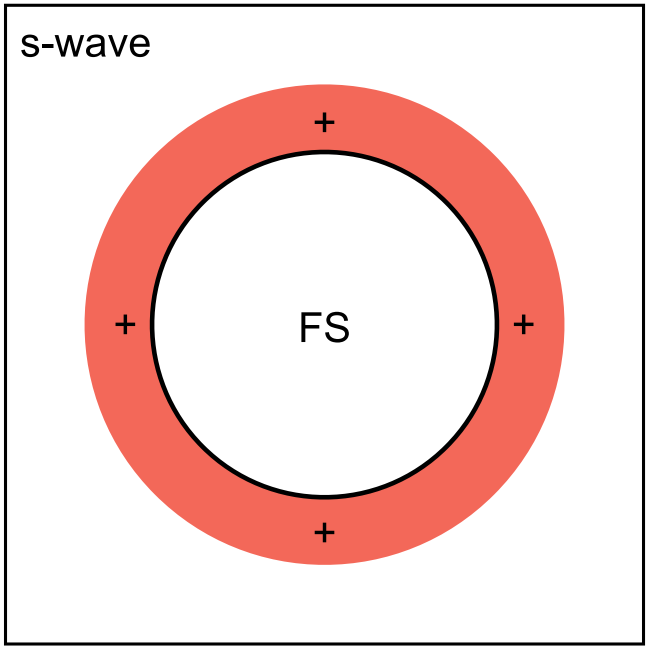
INSULATORS

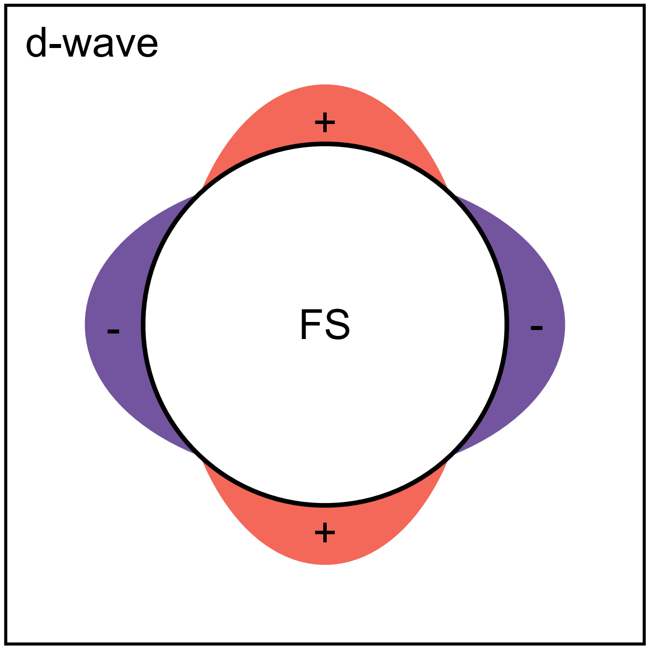
1) Nd2CuO4 — phonons

2) Nd2CuO4 — magnons

3) dmit — spinons ?

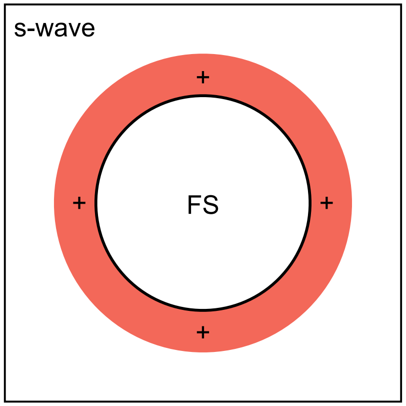
Superconductivity





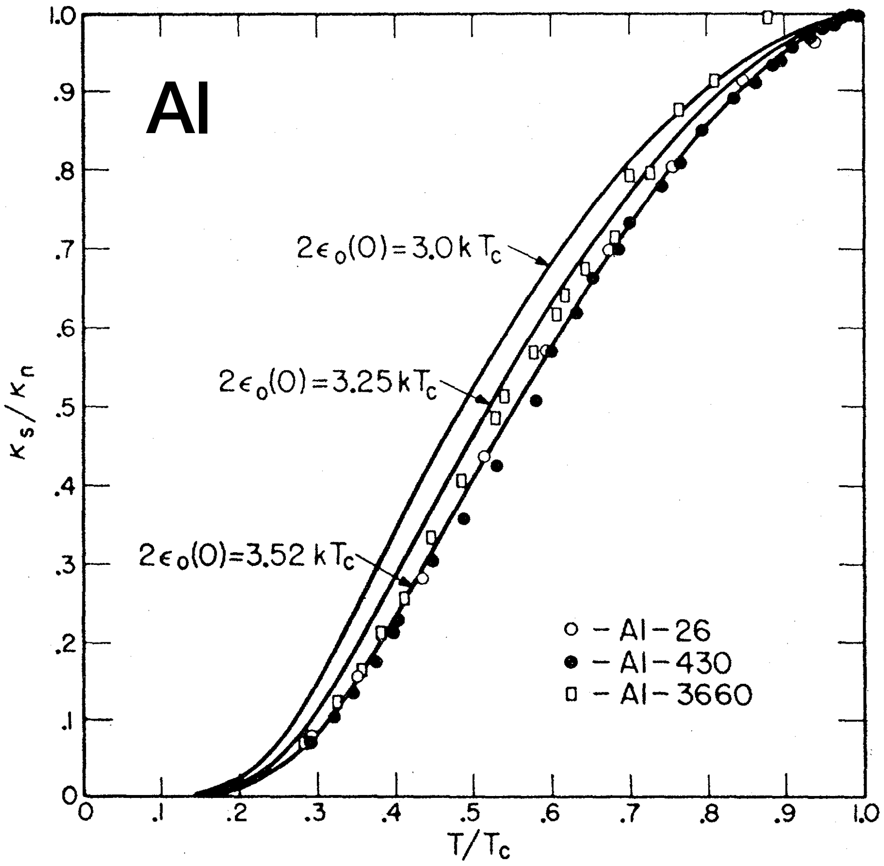
SUPERCONDUCTORS

s-wave



T dependence

Thermal conductivity



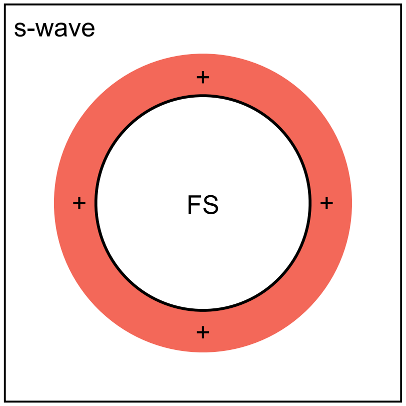
Exponential at low T
— gap

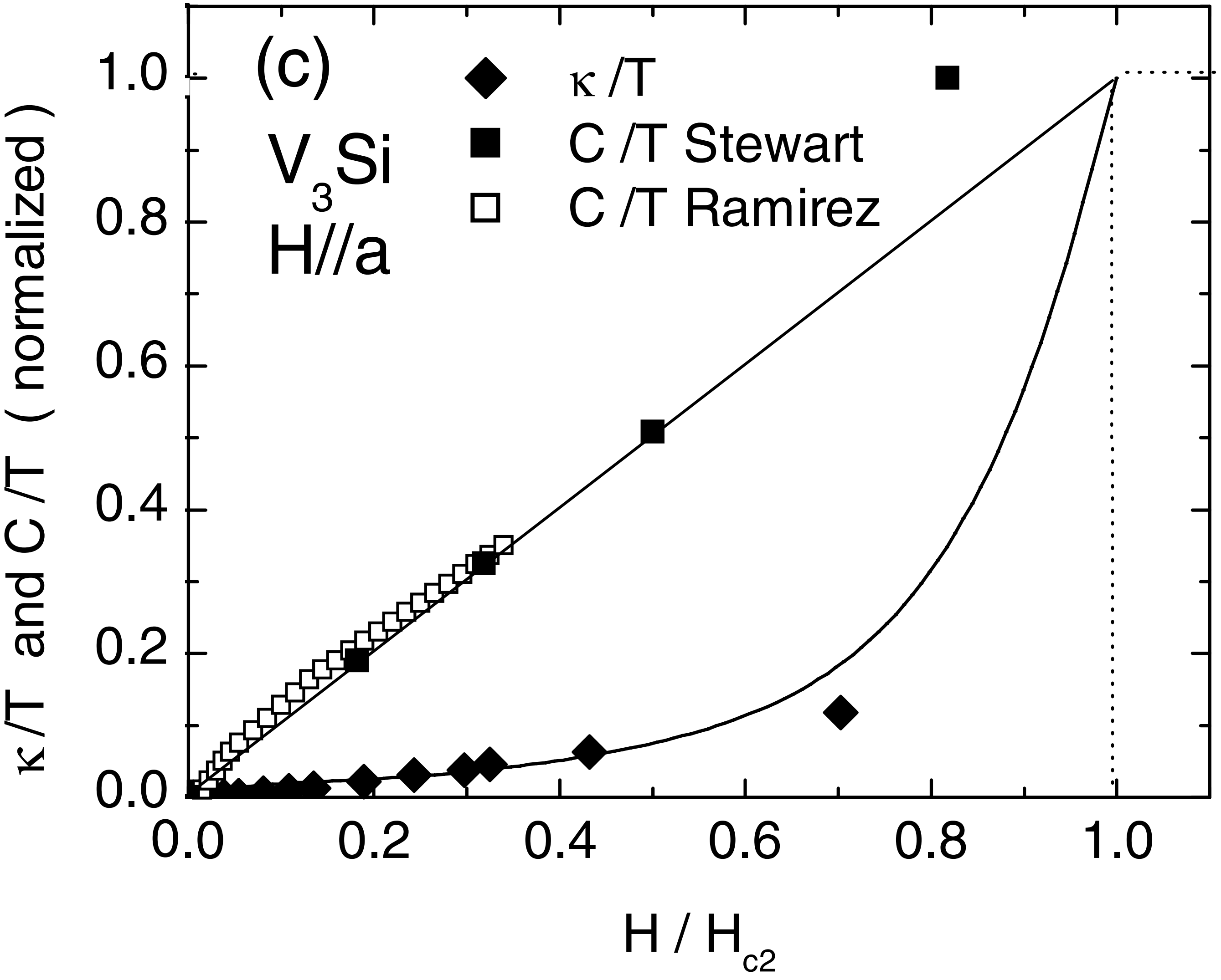
Perfect conductor of charge,
perfect of insulator of heat !

SUPERCONDUCTORS

s-wave

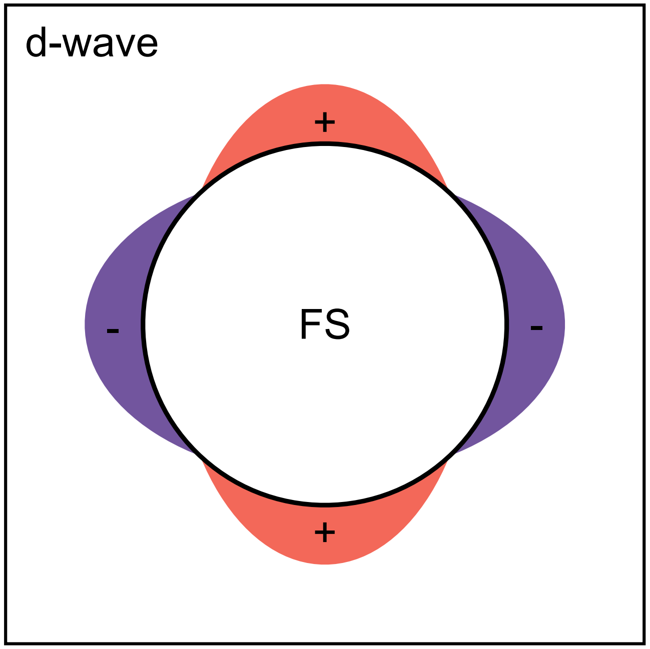
H dependence





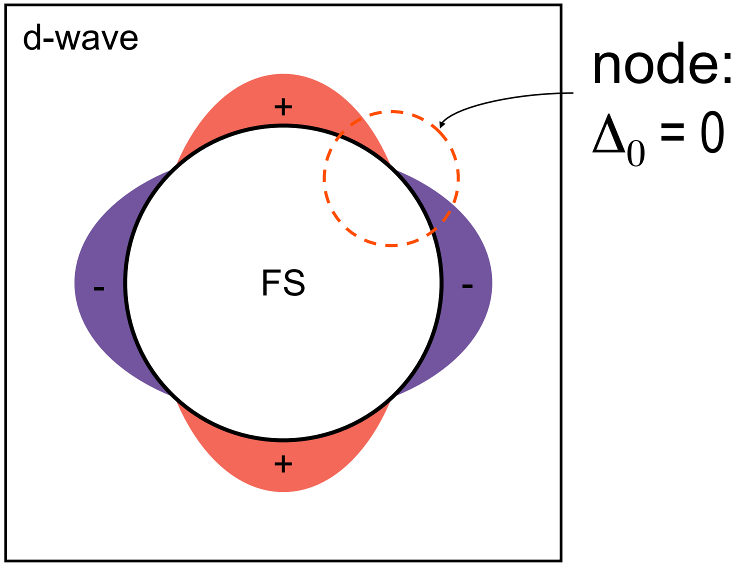
SUPERCONDUCTORS

d-wave

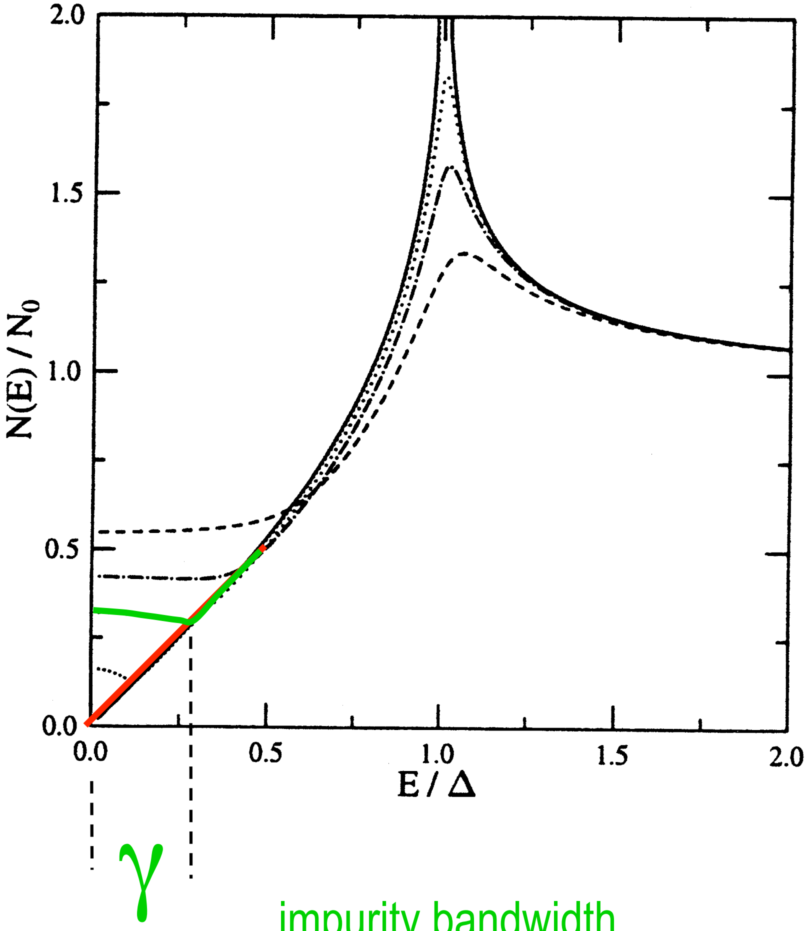


Universal heat conduction

BCS Theory with *d*-wave gap: $\Delta = \Delta_0 \cos(2\phi)$



density of states



impurity bandwidth

clean limit

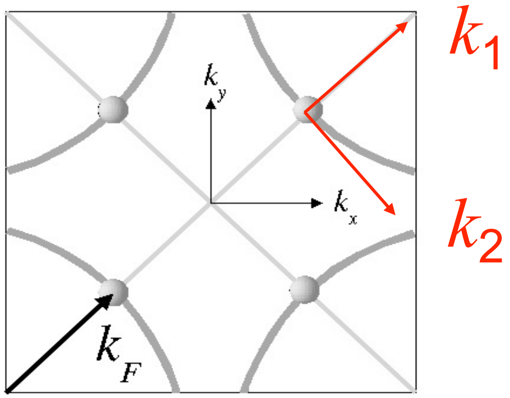
Linear density of states at low energy
- governs all low temperature properties

e.g. $C_e \sim T^2$

impurity effects

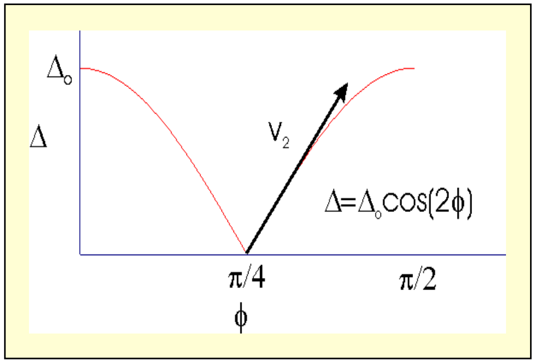
Finite density of delocalised states at zero energy

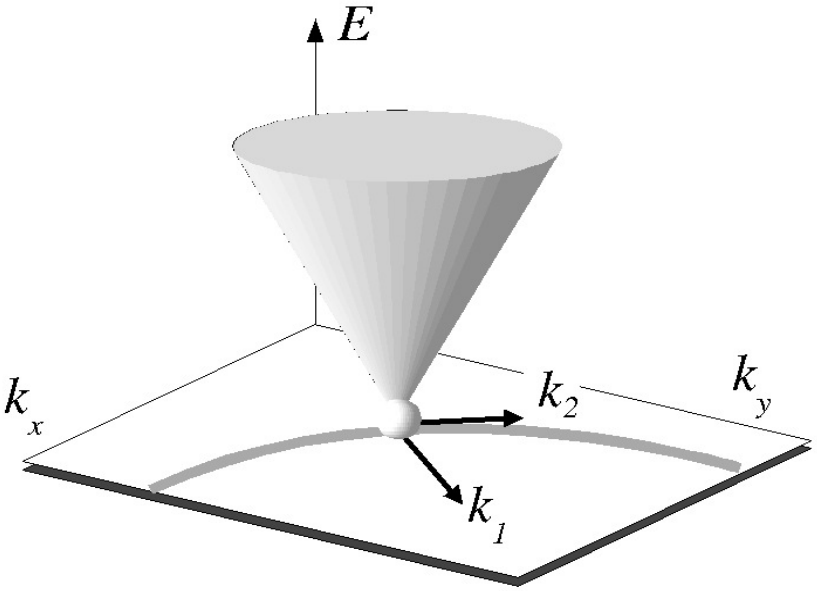
Fermi-liquid theory of nodal quasiparticles



$$E = +\hbar\sqrt{v_F^2 k_1^2 + v_2^2 k_2^2}$$

$$v_2 = \frac{1}{\hbar k_F} \times \left.\frac{\partial\Delta}{\partial\phi}\right|_{\text{node}}$$





Finite residual linear term (RLT)

$$\frac{\kappa_0}{T} = \frac{{k_B}^2}{3\hbar}\frac{n}{c}\left(\frac{v_F}{v_\Delta} + \frac{v_\Delta}{v_F}\right)$$ *d*-wave

Cuprate superconductors

Six regions

1) Superconductivity

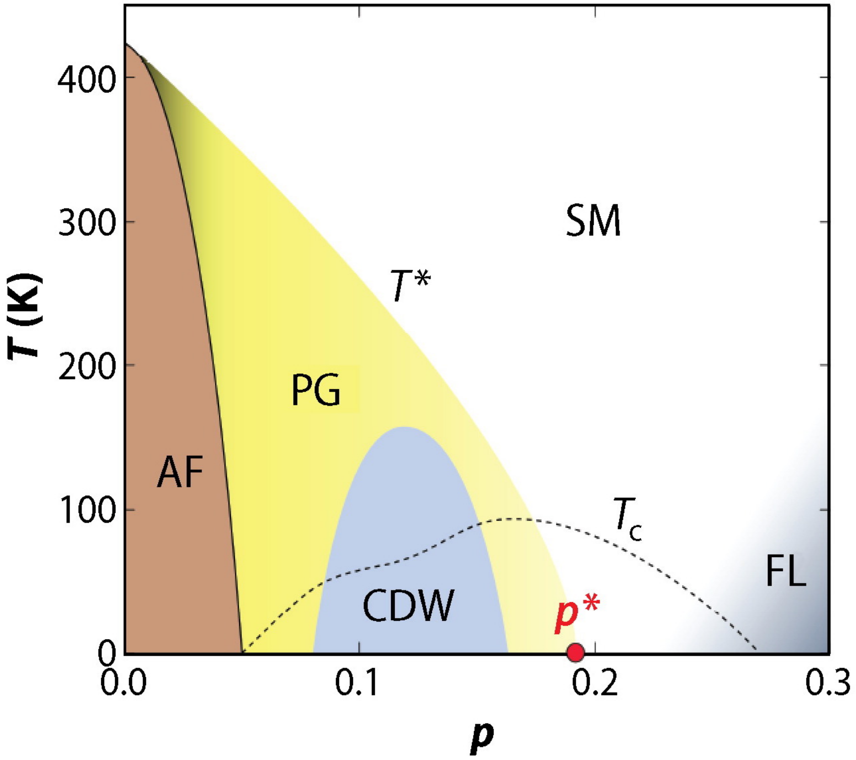
2) Mott insulator

3) Fermi liquid

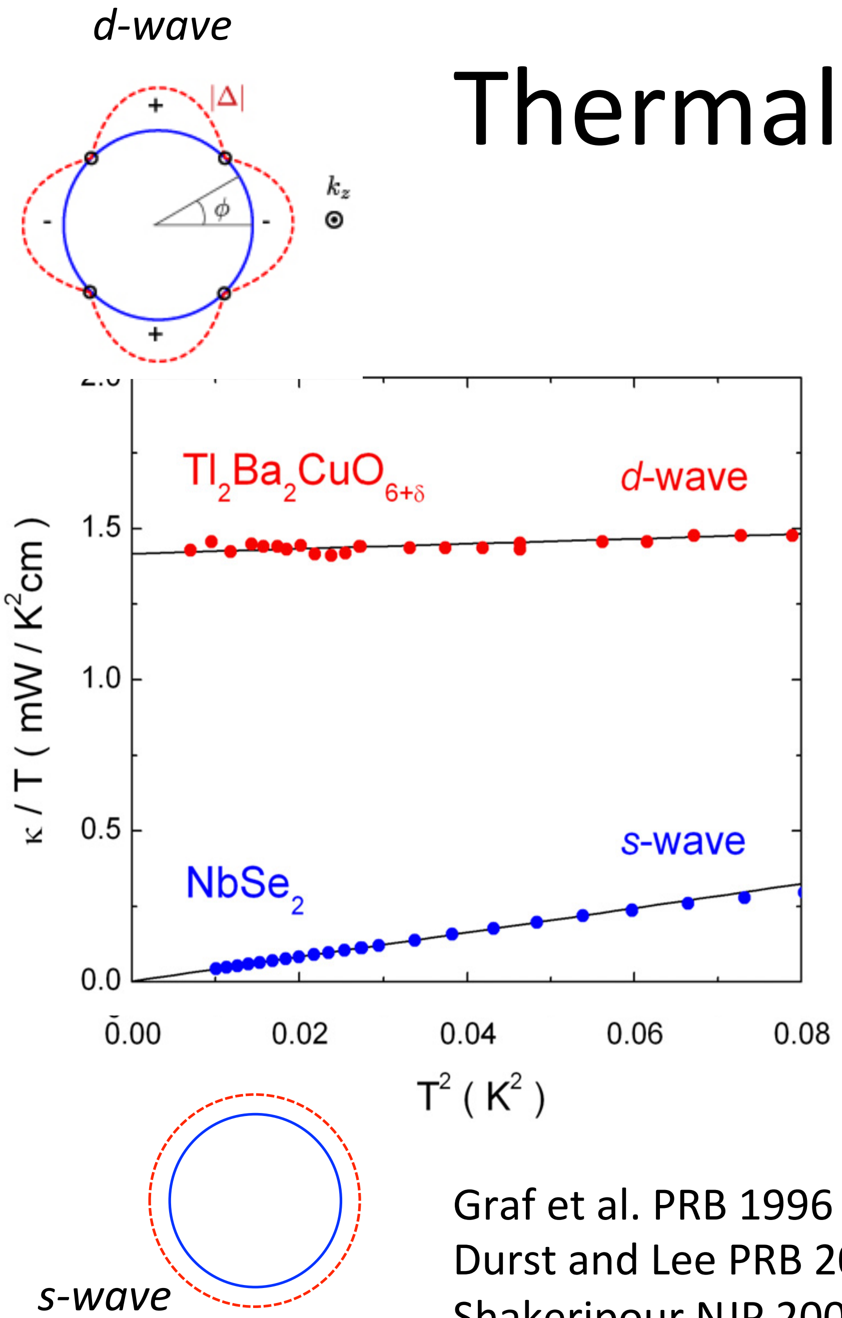
4) Strange metal

5) Charge order

6) Pseudogap phase



Thermal conductivity



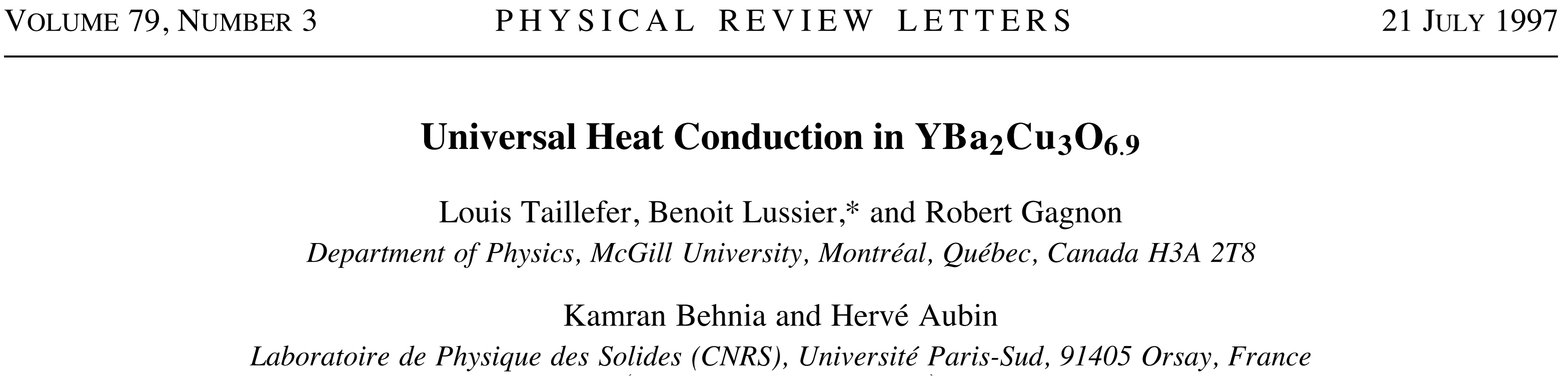
Finite residual linear term (RLT)

$$\frac{\kappa_0}{T} = \frac{{k_B}^2}{3\hbar}\frac{n}{c}\left(\frac{v_F}{v_\Delta} + \frac{v_\Delta}{v_F}\right) \quad d\text{-wave}$$

Graf et al. PRB 1996
Durst and Lee PRB 2000
Shakeripour NJP 2009

SUPERCONDUCTORS *d*-wave YBCO

Low temperature (T=0 limit)

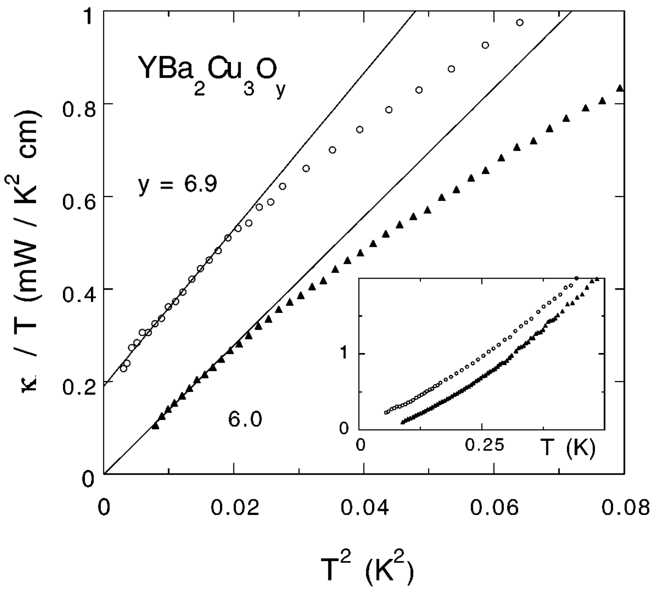


VOLUME 79, NUMBER 3 PHYSICAL REVIEW LETTERS 21 JULY 1997

Universal Heat Conduction in $YBa_2Cu_3O_{6.9}$

Louis Taillefer, Benoit Lussier,* and Robert Gagnon
Department of Physics, McGill University, Montréal, Québec, Canada H3A 2T8

Kamran Behnia and Hervé Aubin
Laboratoire de Physique des Solides (CNRS), Université Paris-Sud, 91405 Orsay, France



Low temperature (T=0 limit)

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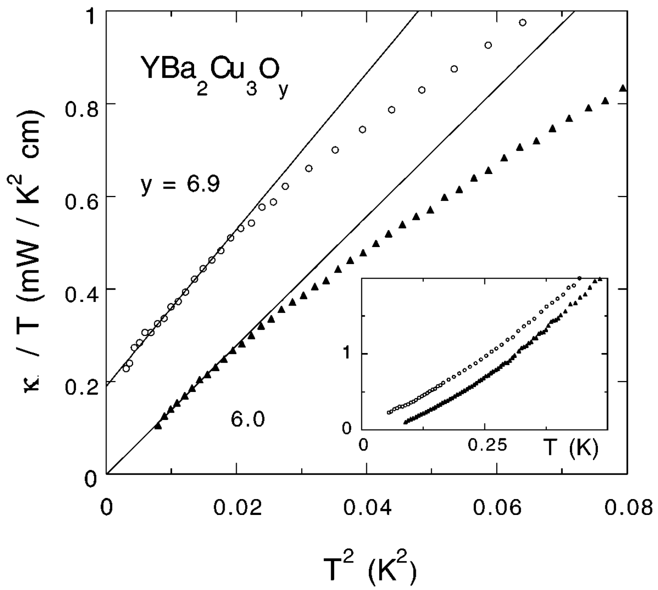
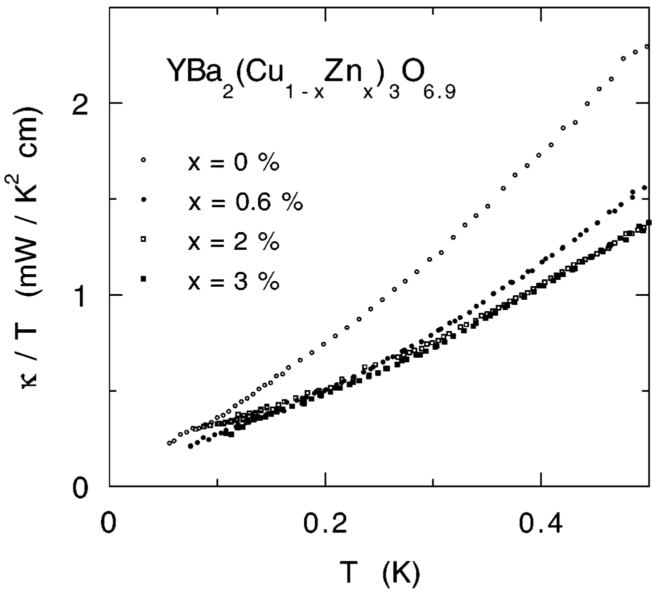
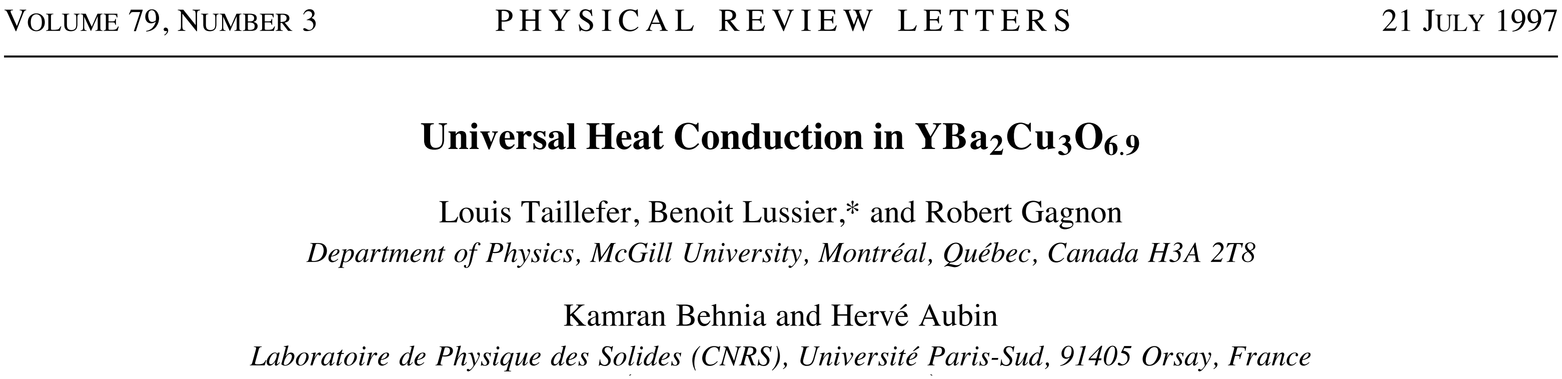




SUPERCONDUCTORS *d*-wave YBCO

Low temperature (T=0 limit)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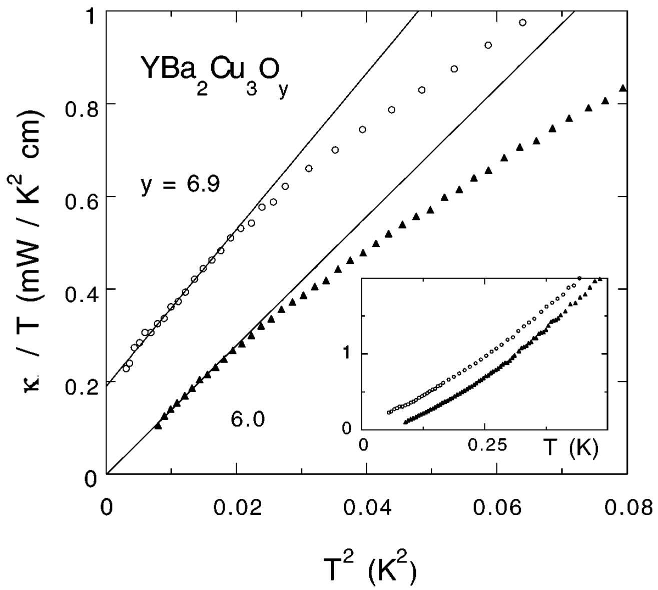
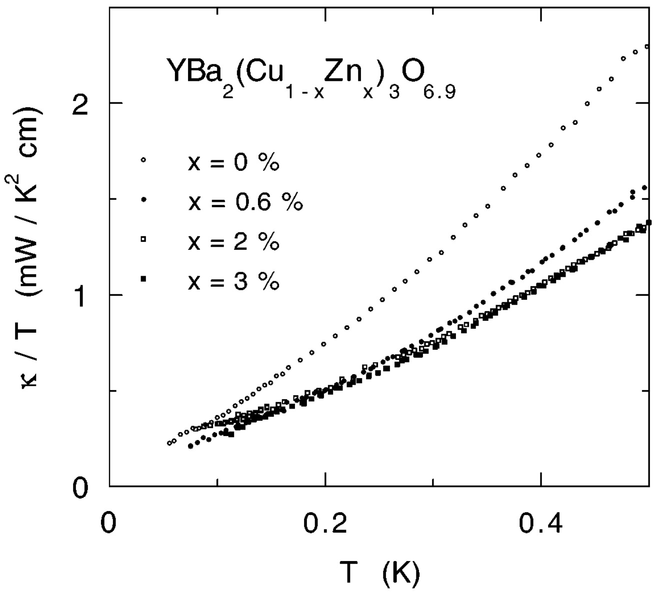
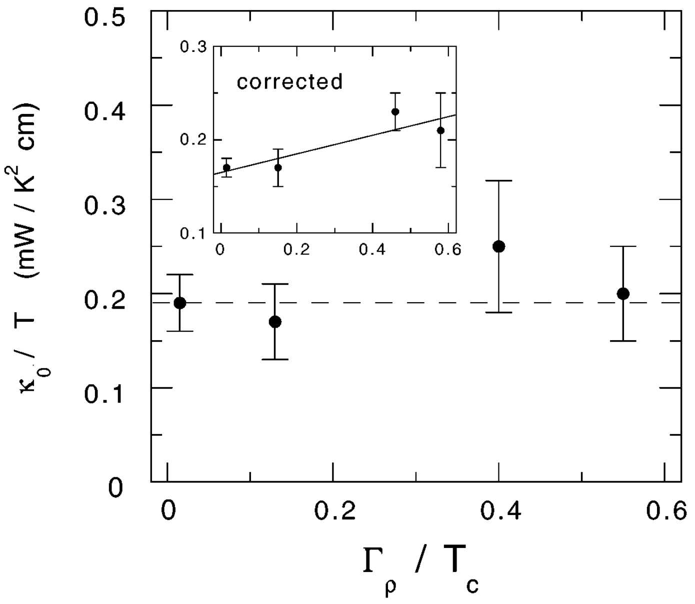






SUPERCONDUCTORS *d*-wave Tl2201

PHYSICAL REVIEW B **75**, 104518 (2007)

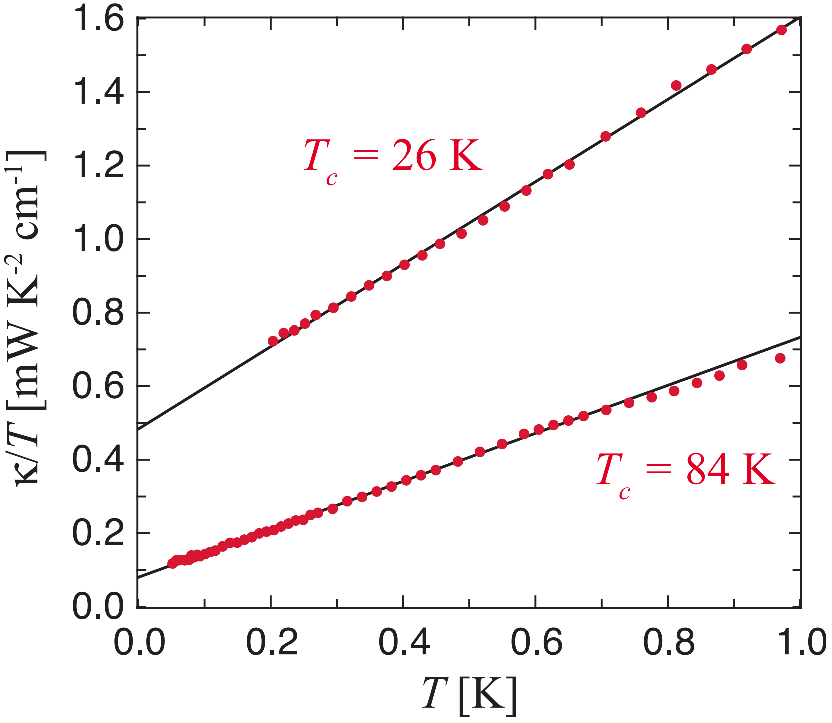
Doping dependence of the superconducting gap in $Tl_2Ba_2CuO_{6+\delta}$ from heat transport

D. G. Hawthorn,* S. Y. Li,[†] M. Sutherland,[‡] Etienne Boaknin, R. W. Hill,[§] C. Proust,[‖] F. Ronning,[¶] M. A. Tanatar,** Johnpierre Paglione,[††] and Louis Taillefer[‡‡]
Department of Physics, University of Toronto, Toronto, Canada M5S 1A7

D. Peets, Ruixing Liang, D. A. Bonn, and W. N. Hardy
Department of Physics and Astronomy, University of British Columbia, Vancouver, Canada V6T 1Z4

$$v_\Delta = \frac{1}{\hbar k_F} \left. \frac{d\Delta}{d\phi} \right|_{node}$$

$$\frac{\kappa}{T} = \frac{\kappa_0}{T} + AT$$



$$\frac{\kappa_0}{T} = \frac{k_B^2}{3\hbar}\frac{n}{c}\left(\frac{v_F}{v_\Delta} + \frac{v_\Delta}{v_F}\right) \qquad \frac{\kappa_0}{T} \simeq \frac{k_B^2}{6}\frac{n}{c}k_F\frac{v_F}{\Delta_0}$$

T_c (K)	p	κ_0/T (mW K^{-2} cm^{-1})	Δ_0 (meV)	Δ_0/k_BT_c
84[a]	0.188	0.08	40	5.6
76	0.203	0.15	22	3.4
72	0.209	0.14	23	3.8
68	0.214	0.12	28	4.8
27	0.252	0.34	9.5	4.1
26[a]	0.253	0.48	6.7	3.0

[a] Denotes the same sample measured at two dopings.

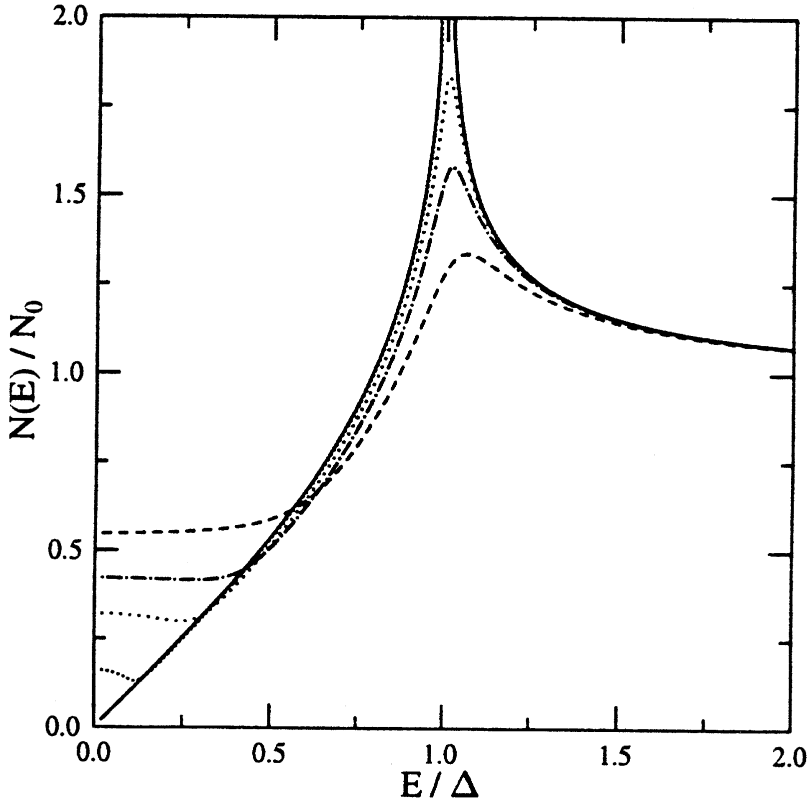
SUPERCONDUCTORS

d-wave

Field dependence — Doppler shift

$$E_H = a\hbar\sqrt{2/\pi}\,v_F\sqrt{H/\Phi_0}$$

density of states



C ~ \sqrt{H}

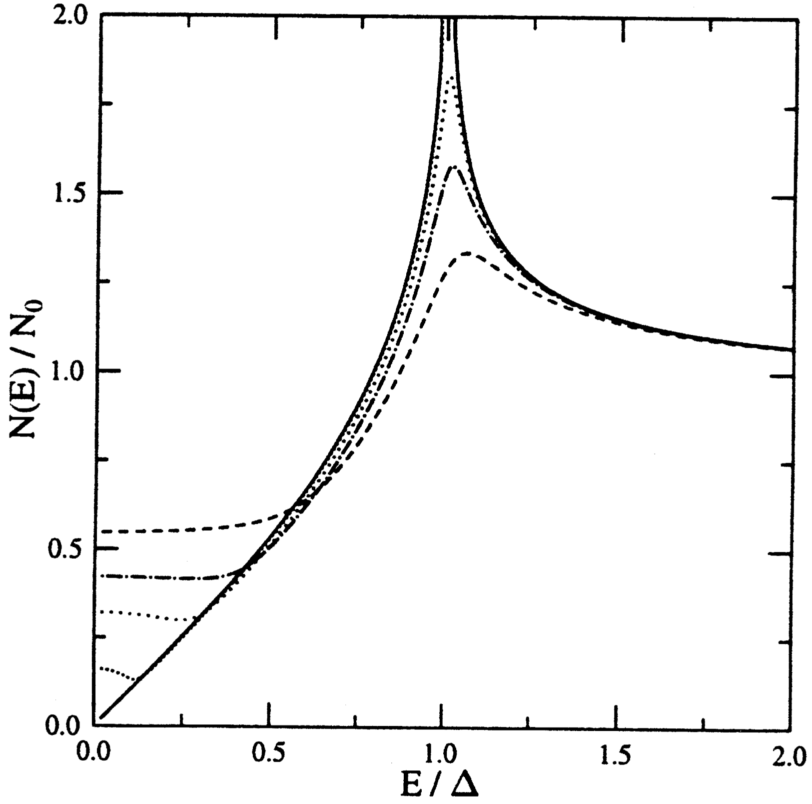
SUPERCONDUCTORS

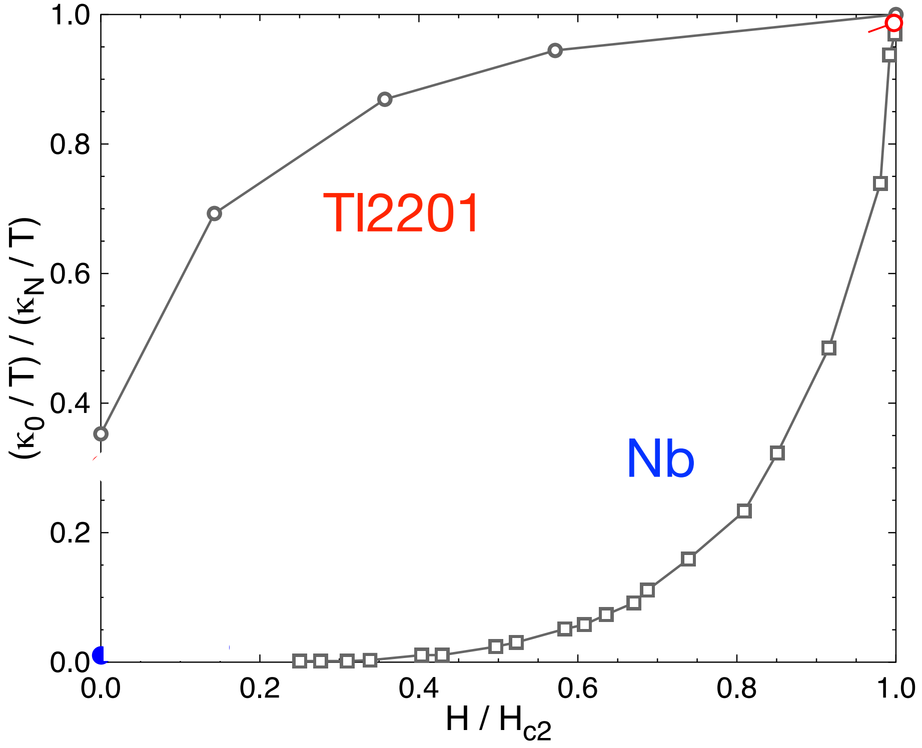
d-wave

Field dependence — Doppler shift

C ~ \sqrt{H}

density of states



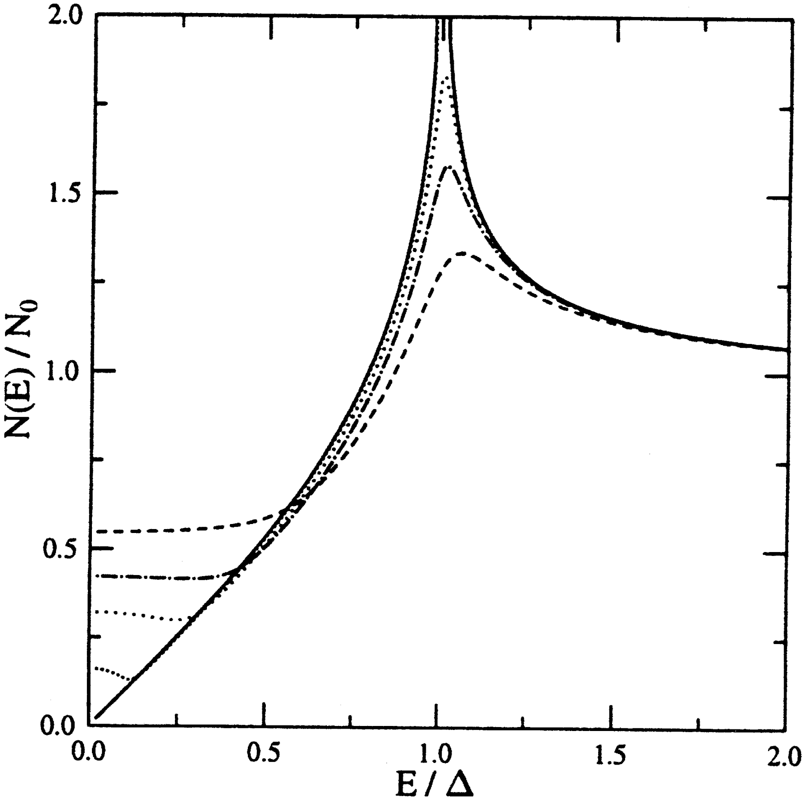


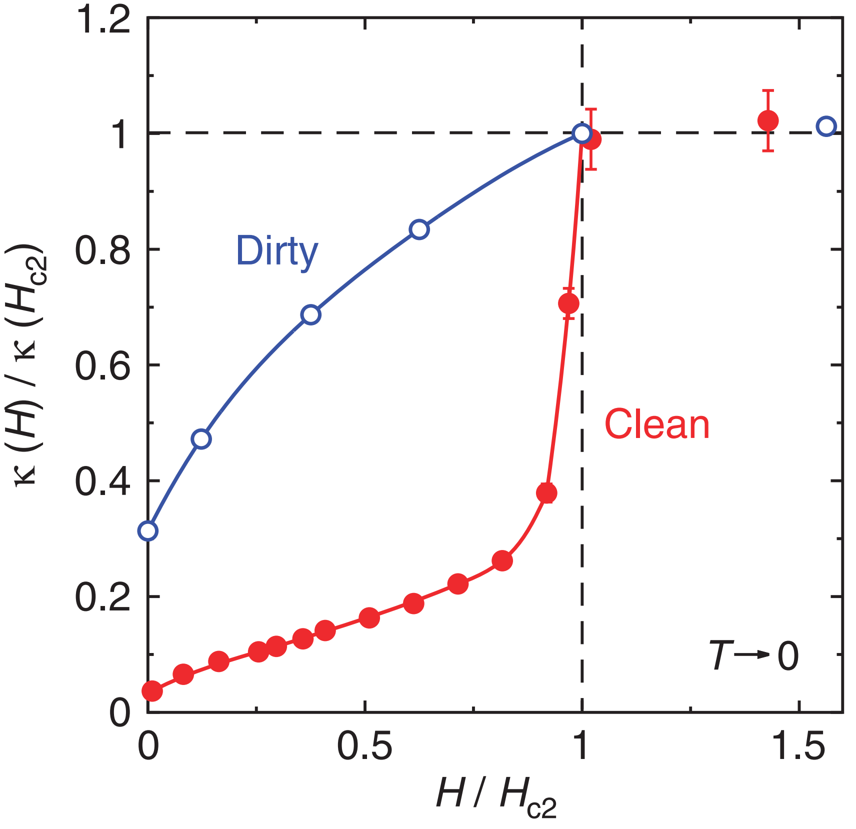
SUPERCONDUCTORS *d*-wave YBCO

Field dependence — Doppler shift

C ~ \sqrt{H}

density of states

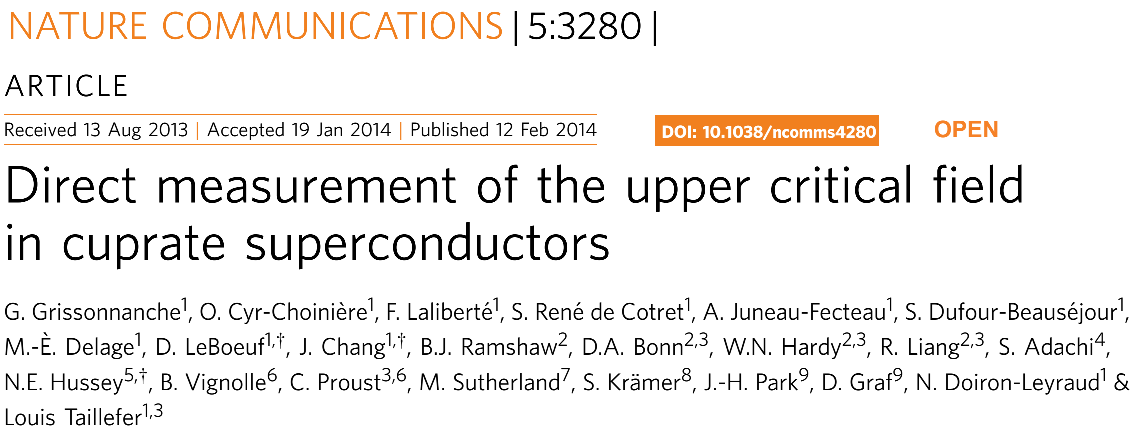




SUPERCONDUCTORS

Detection of upper critical field H_{c2}

YBCO



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OPEN

Direct measurement of the upper critical field in cuprate superconductors

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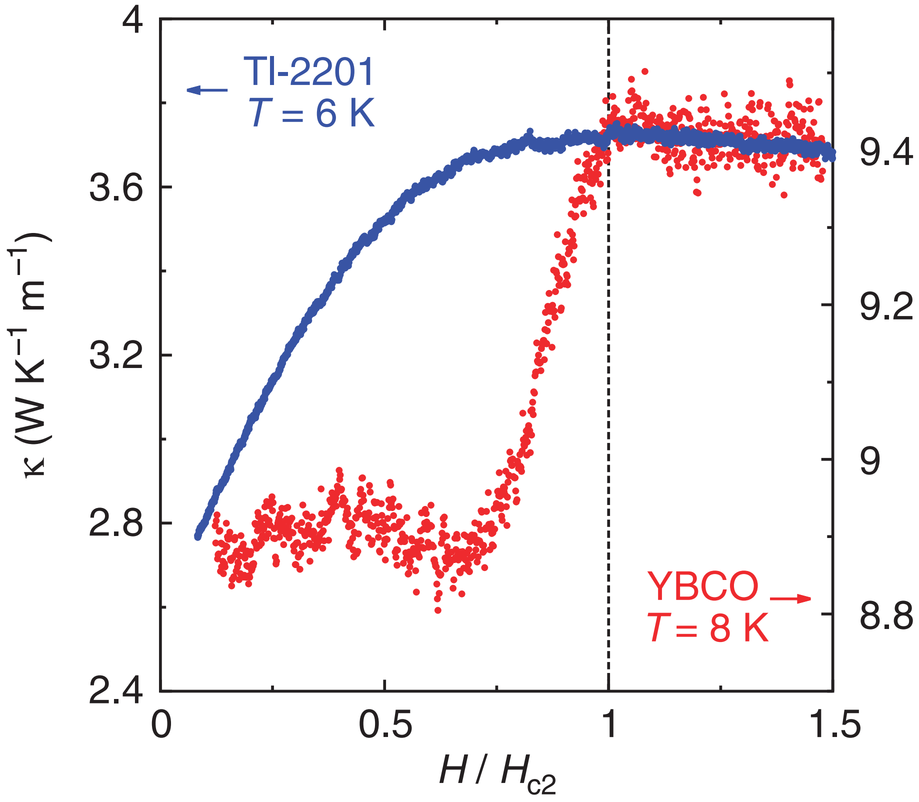

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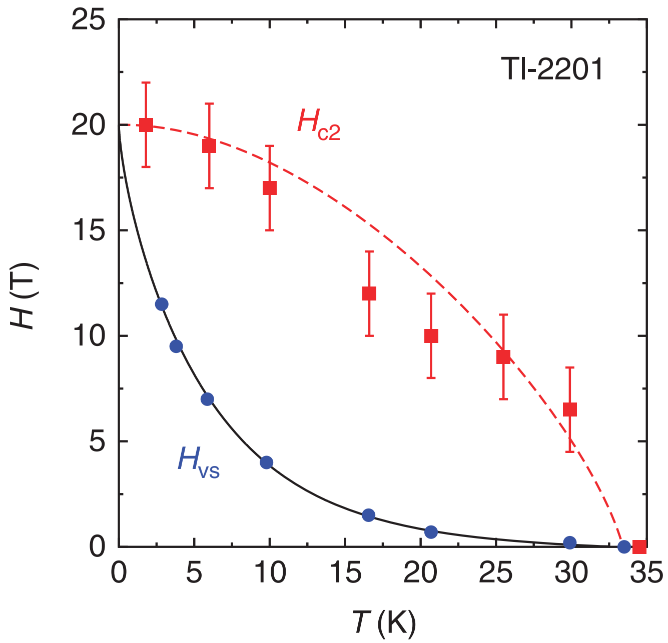

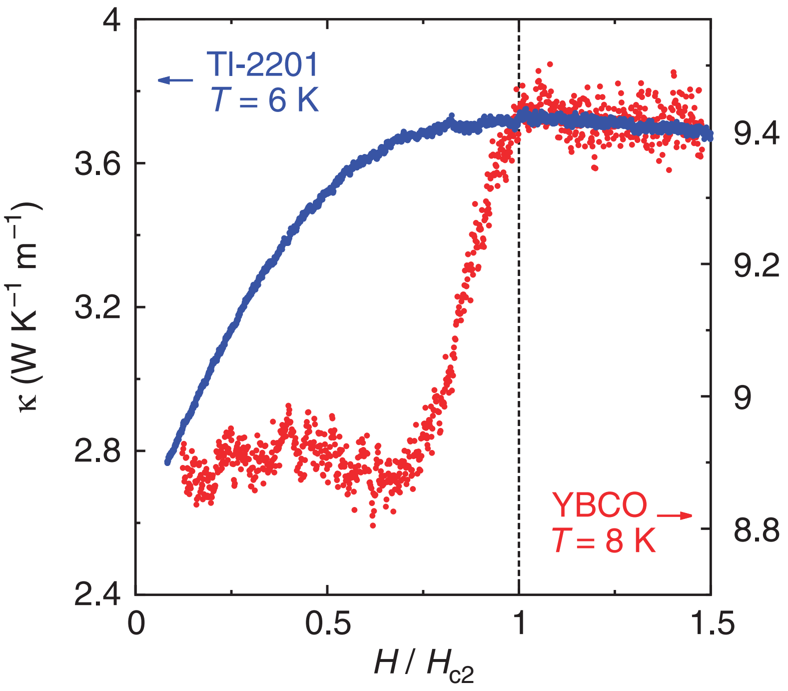

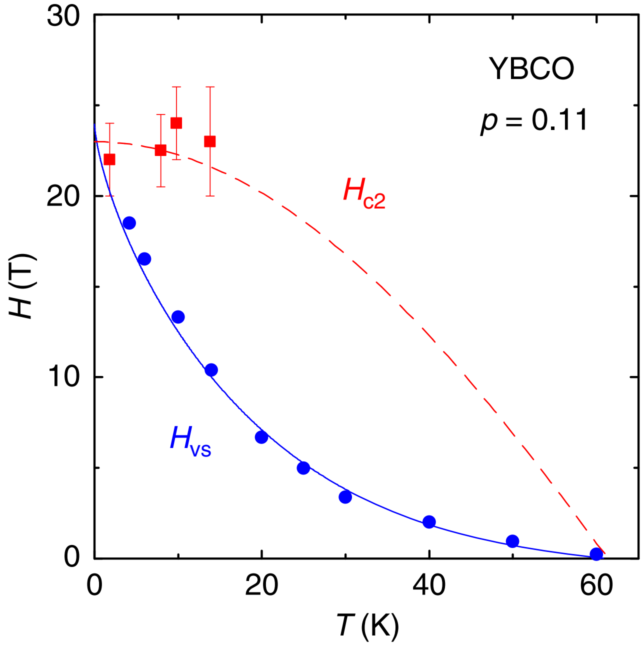

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YBCO