Quantum Critical Metals

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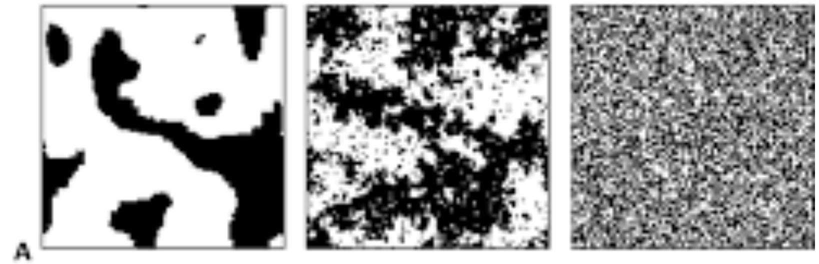
BSF

Weizmann Institute of Science

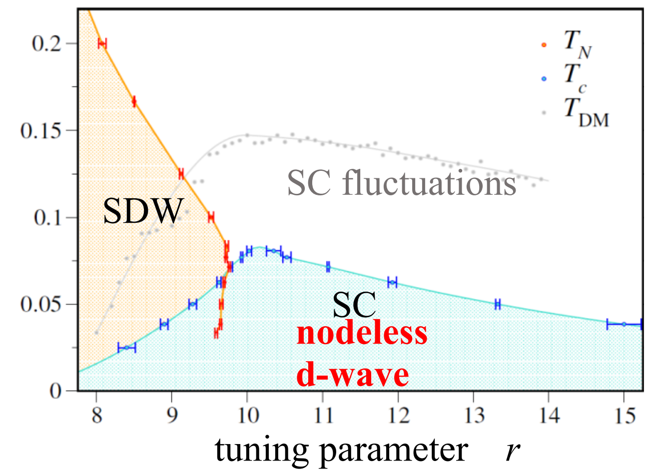
erc European Research Council

Outline

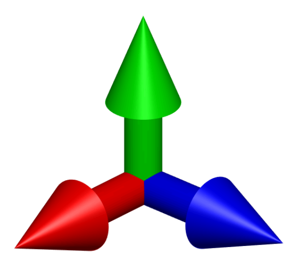
- Review of classical and quantum criticality

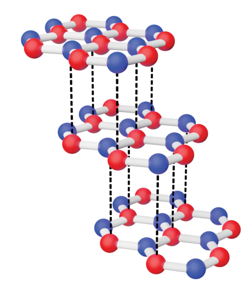


- Quantum critical Fermi surfaces: Analytical and numerical studies



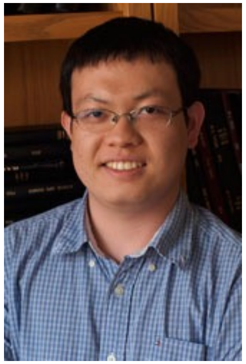
- Exotic superconductivity in graphene multilayers





Sam Lederer (Berkeley)

Yoni Schattner (WIS→Stanford)



Xiaoyu Wang (UChicago →UFL)

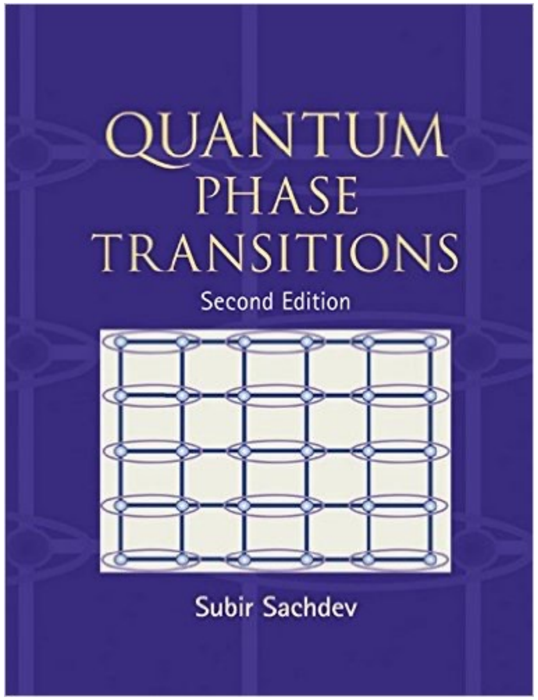
Ori Grossman (WIS)

Johannes Hofmann (WIS)

Tobias Holder (WIS)

Subir Sachdev, Max Metlitski, Steve Kivelson, Simon Trebst, Kai Sun, Rafael Fernandes, Morten Christensen, Andrey Chubukov, Yuxuan Wang, Avi Klein, Max Gerlach, Carsten Bauer, Zi-Yang Meng, Xiao Yan Xu

$T = 0$ **continuous transitions *in insulators* are fairly well understood.**



What happens when a system *with a Fermi surface* goes critical?

Outline

- Classical and Quantum Criticality

- Quantum critical *Fermi surfaces*

- Numerical Quantum Monte Carlo experiments: Results, intermediate conclusions, and outstanding mysteries

Kinetically Constrained

Scale invariance at the critical point

by Douglas Ashton

www.kineticallyconstrained.com

Why do we care about critical phenomena?

- *Emergent* scale invariance

$$\vec{r} \to b\vec{r}:$$

$$G(\vec{r}) = \langle \phi(\vec{r})\phi(0) \rangle \to b^{-2d_\phi} G(\vec{r}/b\ )$$

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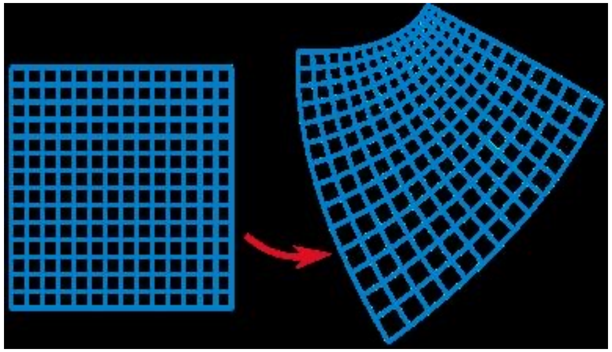
- *Emergent* conformal symmetry



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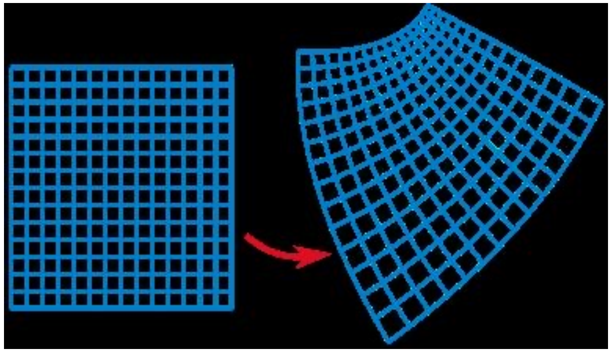

- Theoretical control (renormalization group, Monte Carlo simulations)

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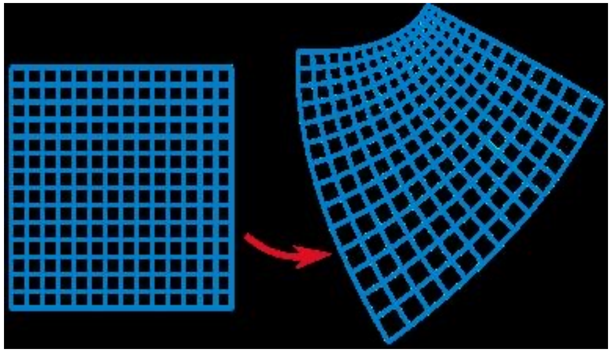


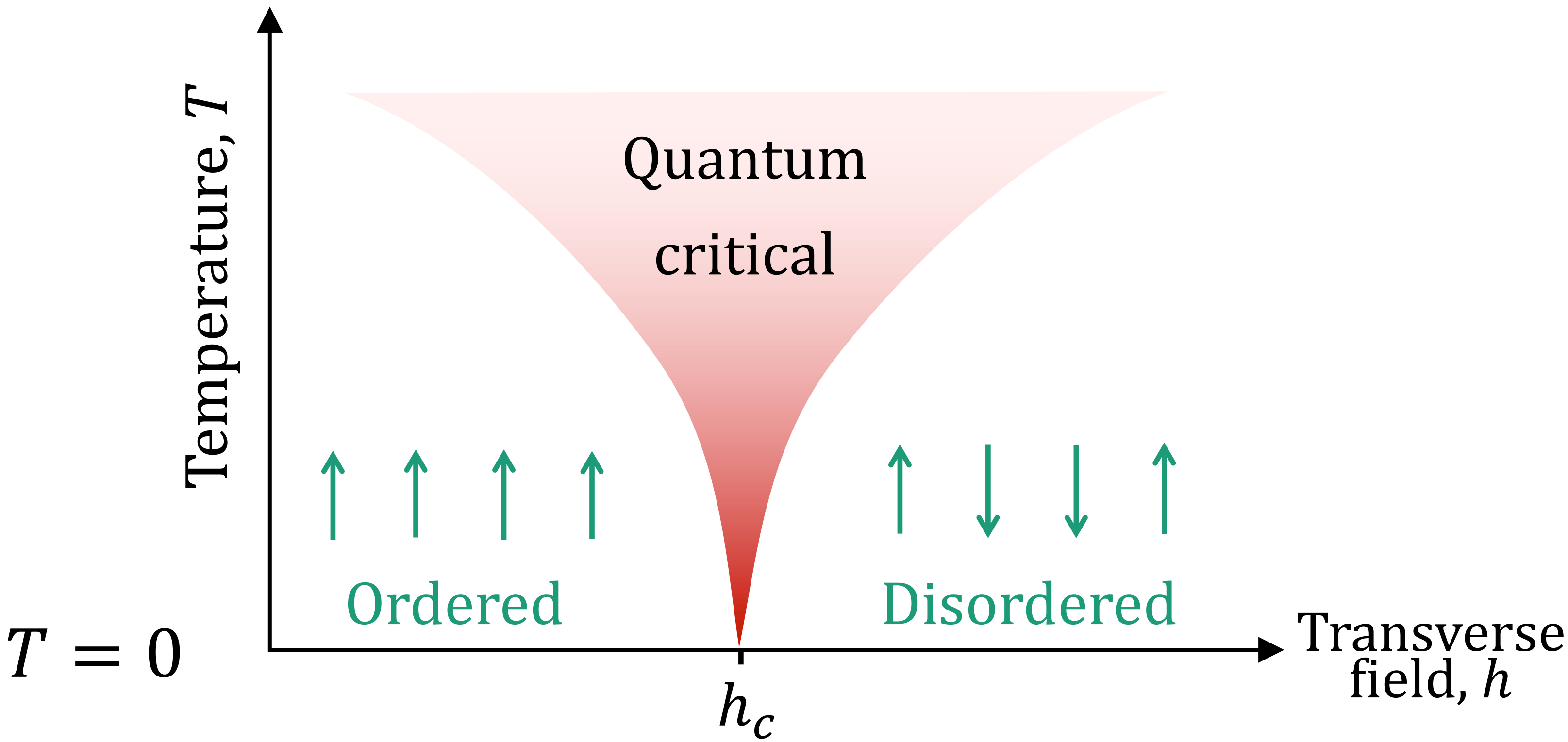
- Theoretical control (renormalization group, Monte Carlo simulations)

- Universality: divergent correlation length, “microscopic” details don’t matter!

Important concept for quantum field theory, too…

Quantum critical phenomena

- Continuous transition at $T = 0$ as a function of Hamiltonian parameter
- Example: the transverse field Ising model

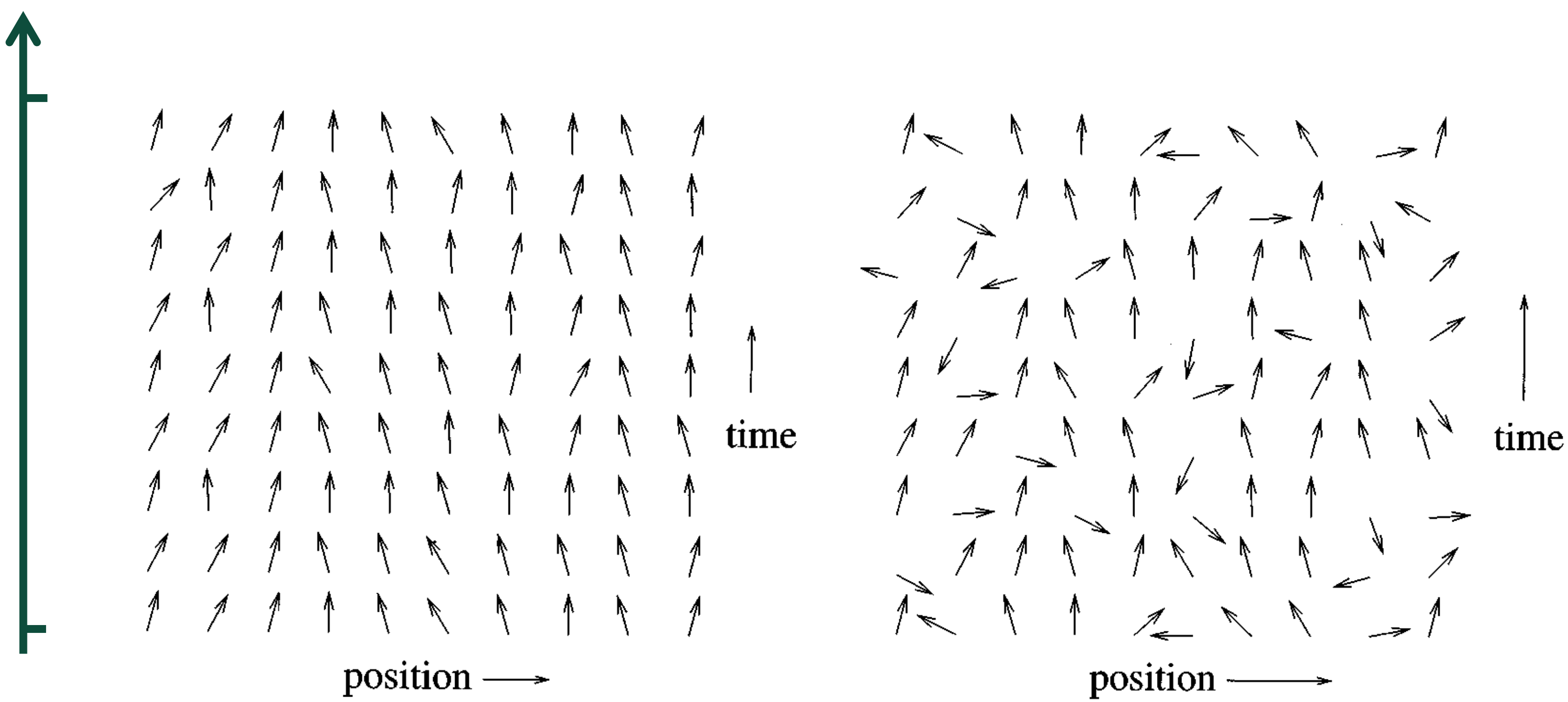


Quantum critical phenomena

d − dimensional quantum ⇔
$d + 1$ − dimensional classical, size $\beta = \hbar / k_B T$ in the “imaginary time,” τ direction

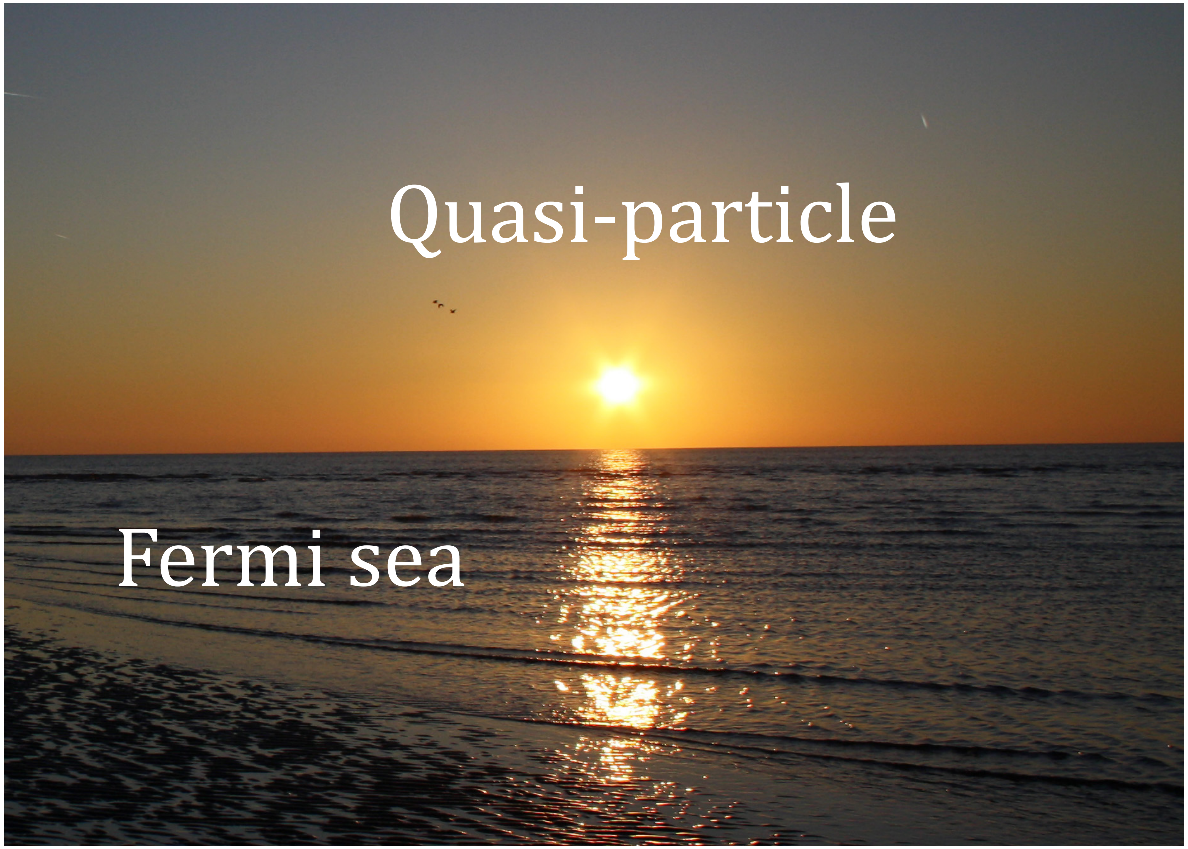
$$\beta = \frac{\hbar}{k_B T}$$

0



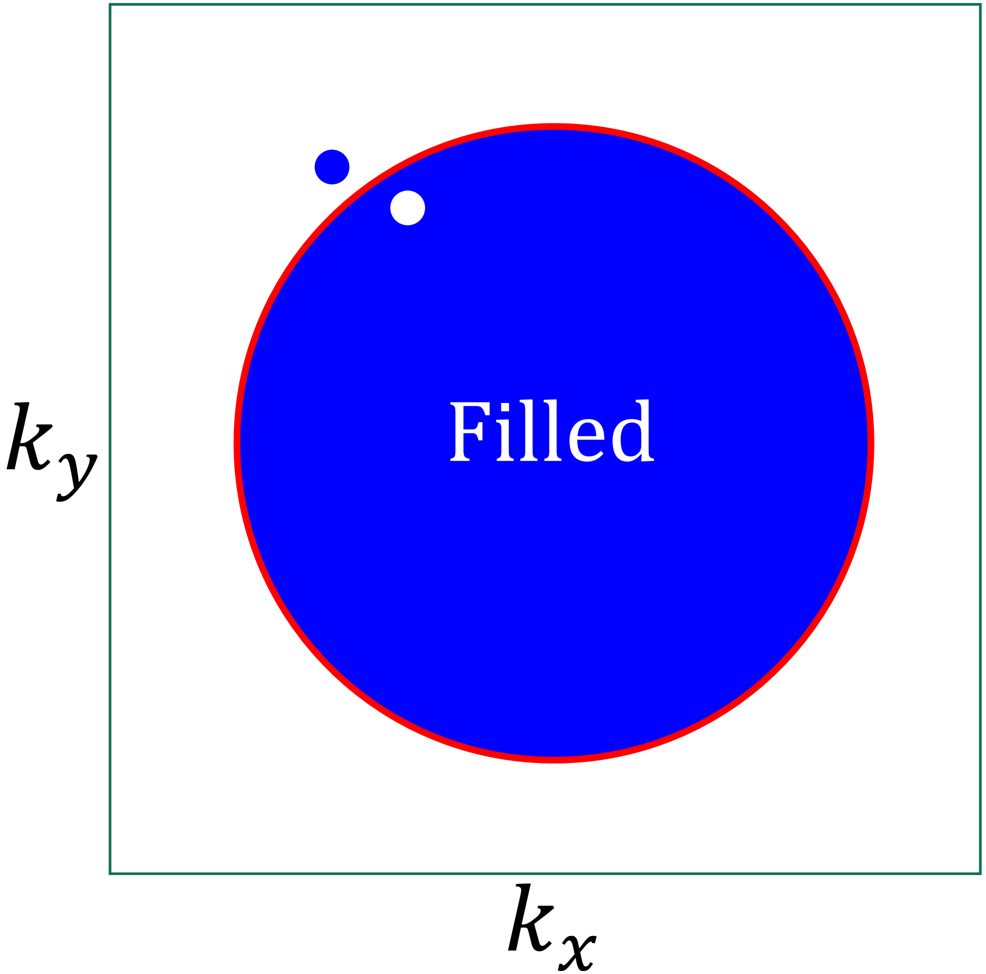
From Sondhi, Girvin, Carini, Shahar, RMP (1997)

Conventional (Fermi liquid) metal

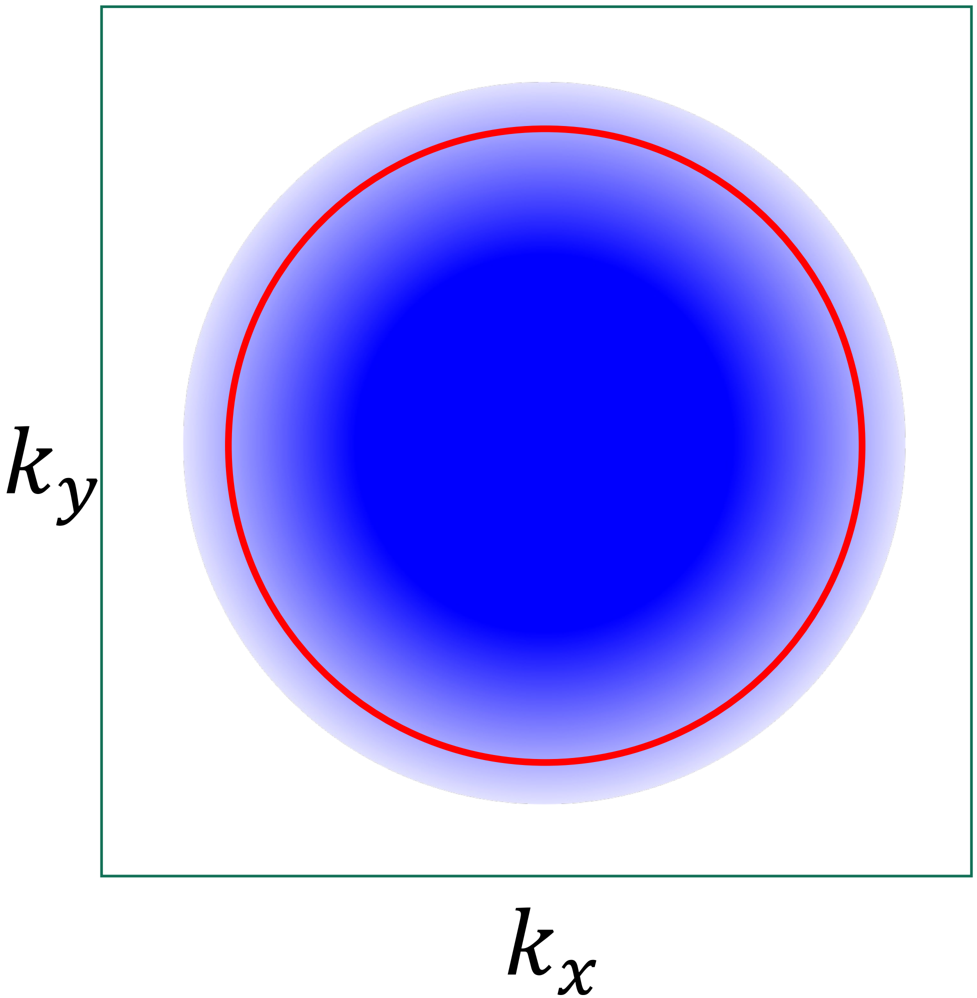


Conventional (Fermi liquid) metal

Fermi surface is quantum mechanical:
no classical analogue

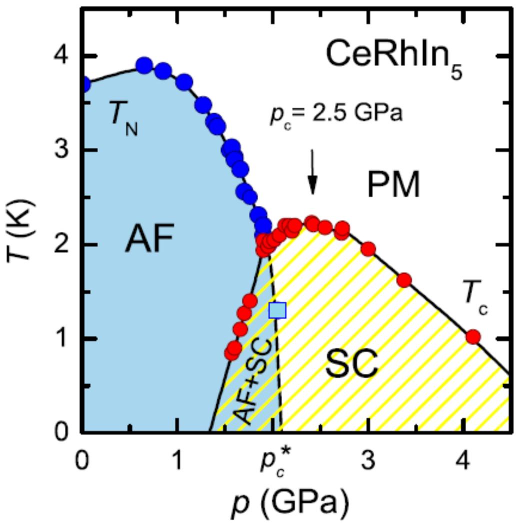


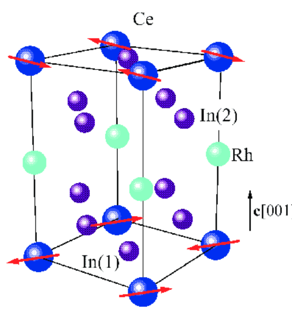
Critical metal: non-Fermi liquid?



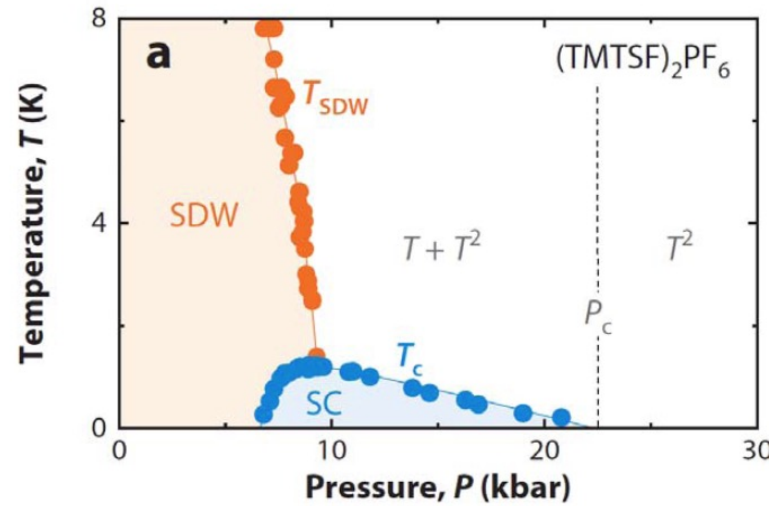
Quantum criticality in unconventional superconductors?

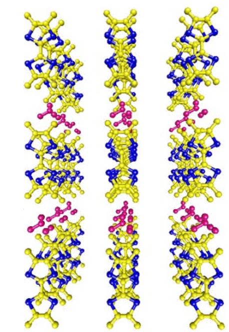
Heavy Fermions



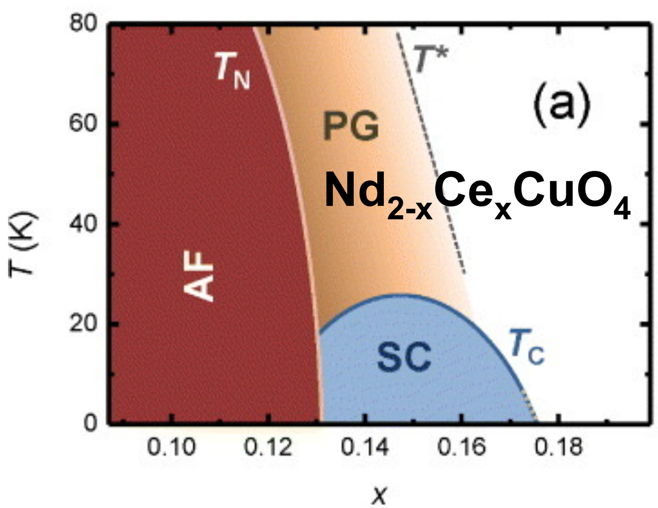


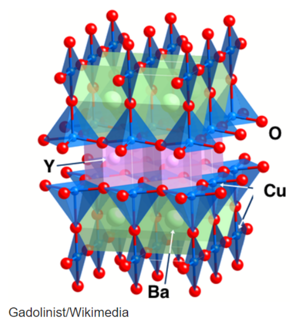
Organic supercondutors



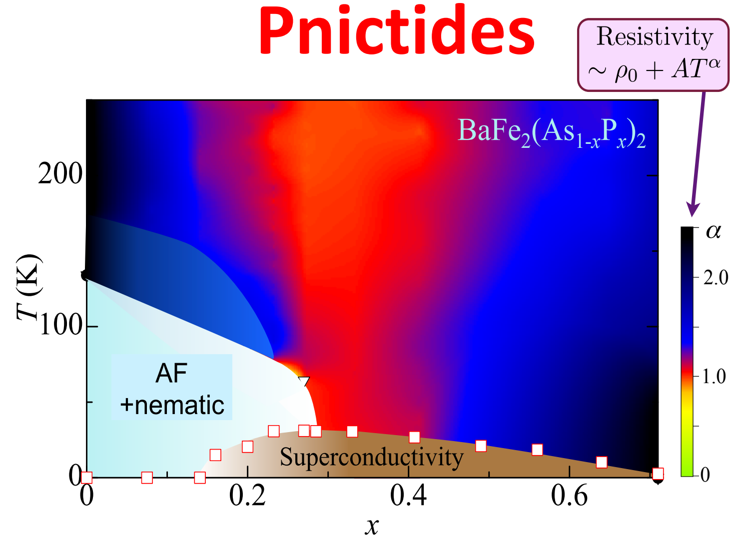


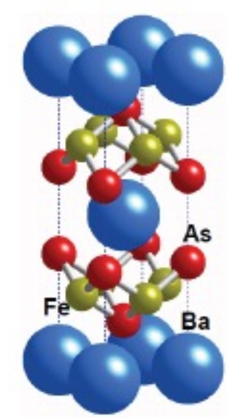
Cuprates





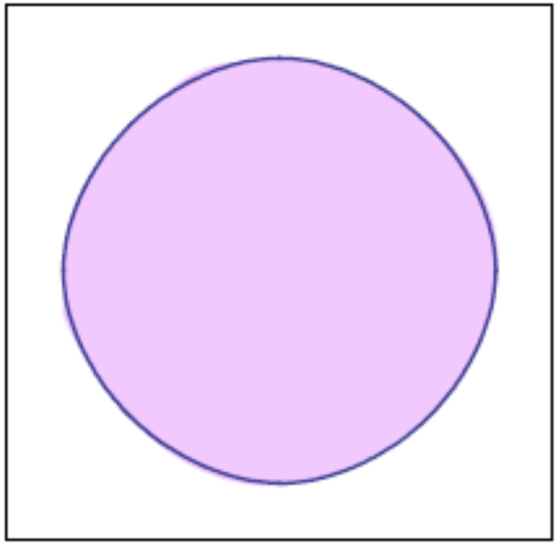
Pnictides





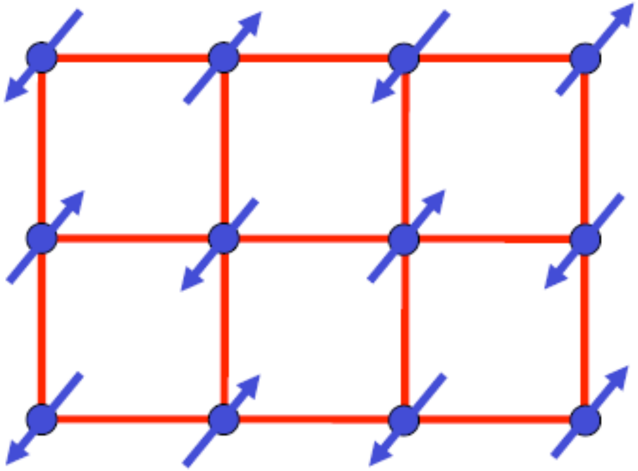
Kasahara,...,Matsuda (2010)

Models for metallic quantum criticality



Fermions

+



Order parameter $\vec{\phi}$

$$S = S_{\text{fermions}} + S_{\phi} + S_{\text{int}}$$

$$S_{\text{fermions}} = \int d^2k d\tau \, \psi_k^{+}(\partial_\tau + \varepsilon_k)\psi_k$$

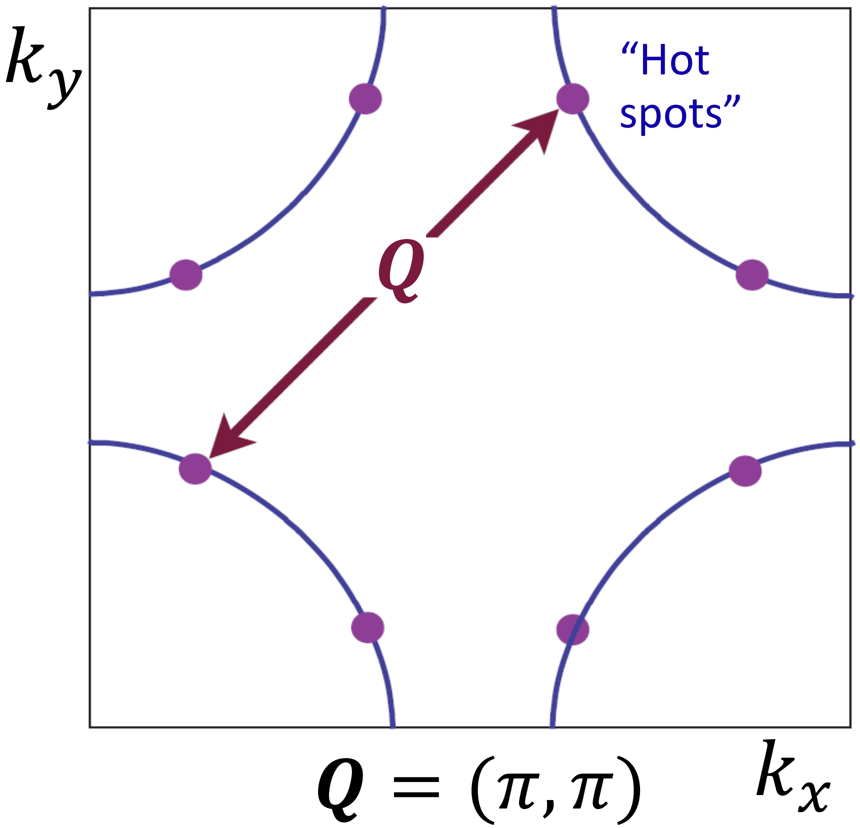
$$S_{\phi} = \int d^2x \, d\tau \left(\nabla\vec{\phi}\right)^2 + r\vec{\phi}^2 + \left(\partial_\tau\vec{\phi}\right)^2/c^2 \ldots$$

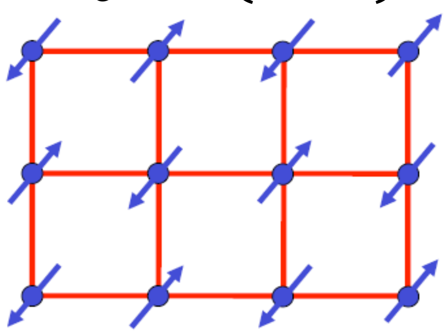
$$S_{\text{int}} = \alpha\int d^2x \, d\tau e^{i\vec{Q}\cdot\vec{x}} \, \vec{\phi}\cdot(\psi^{+}\vec{\sigma}\psi)$$

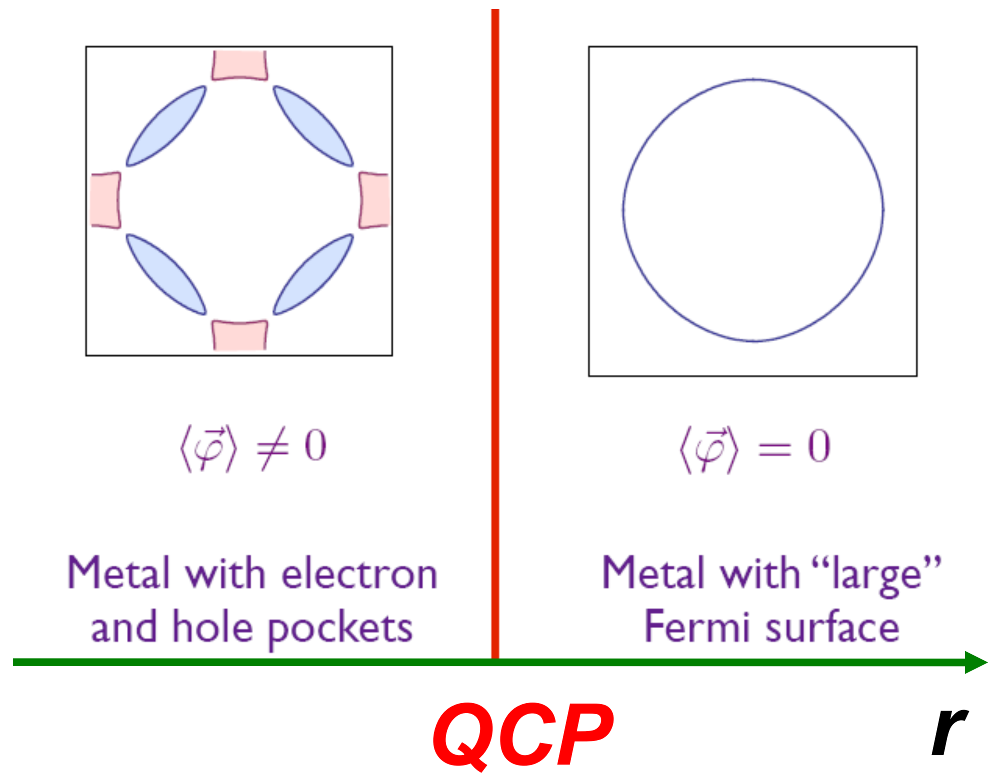
α – "Yukawa" coupling

Two types of metallic quantum critical points

Antiferromagnetic QCP

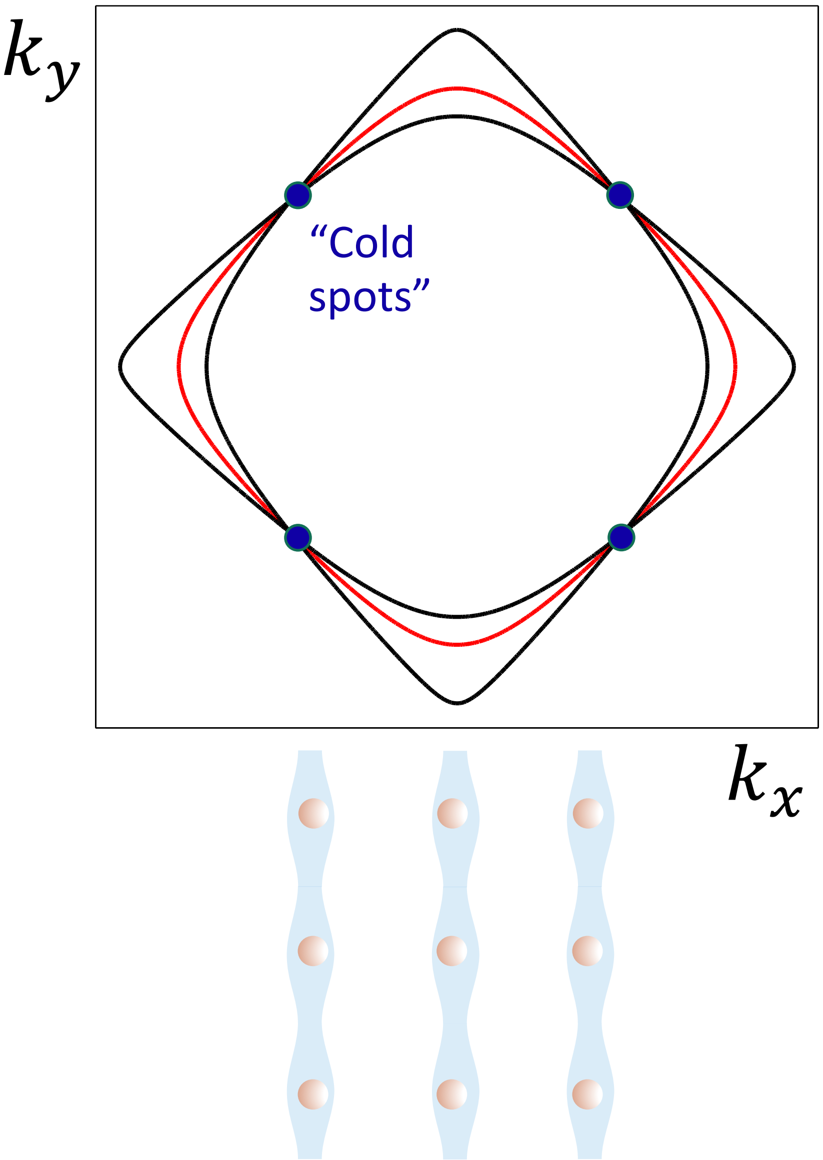






Two types of metallic quantum critical points

Ising-nematic QCP



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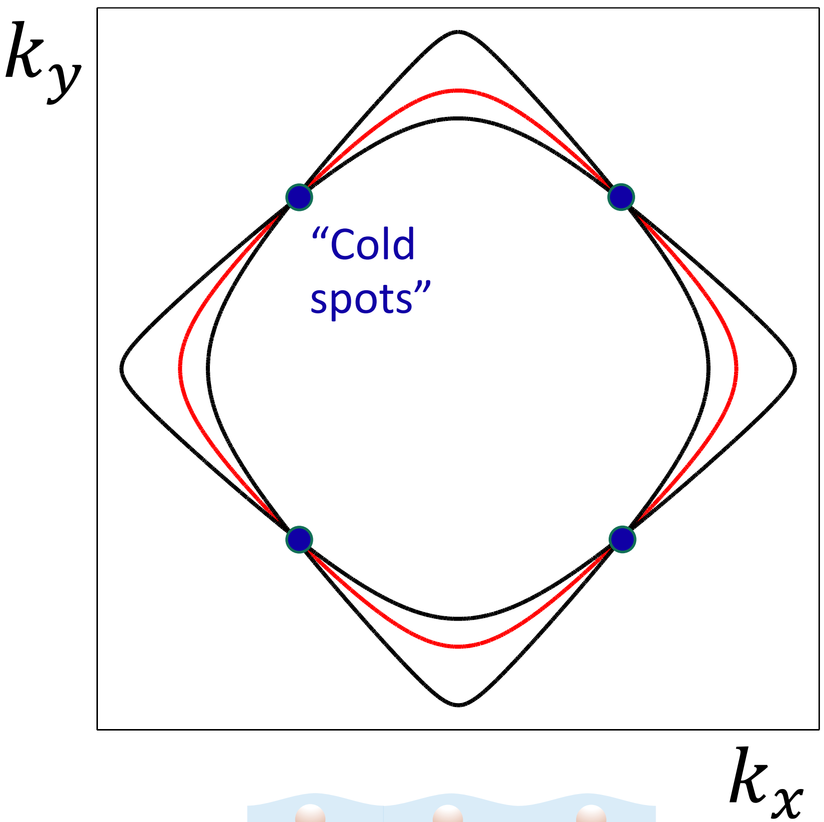
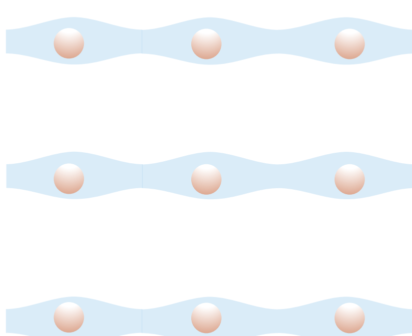
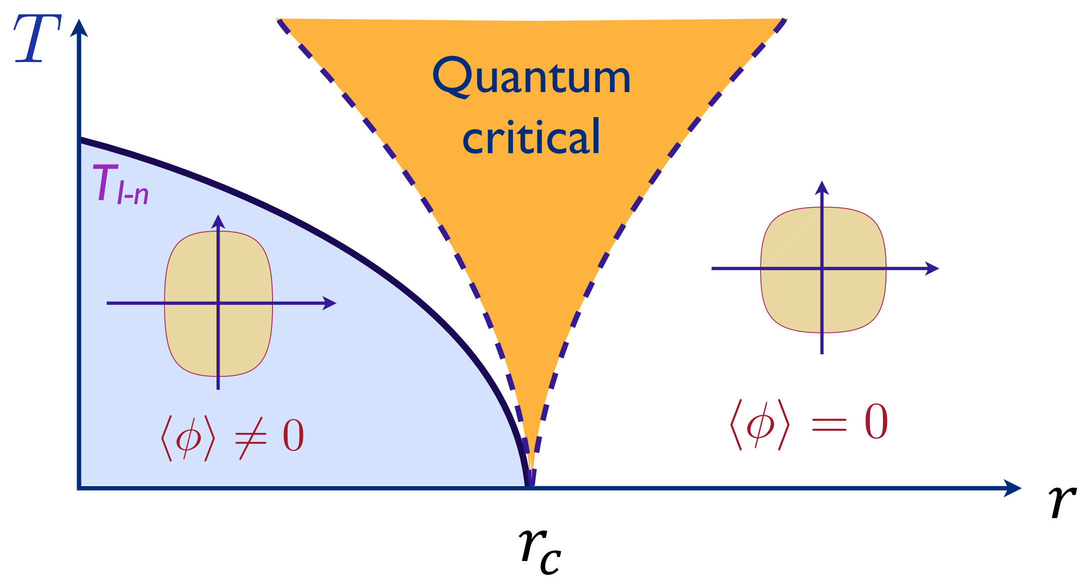
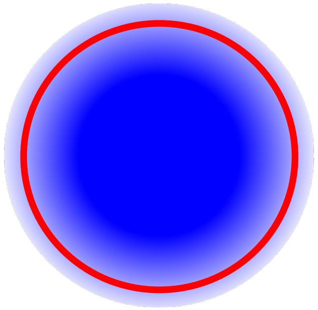






Metallic Quantum Criticality: Open Questions

- Critical exponents?
- Destruction of Fermi Liquid theory?
- QCP “masked” by enhanced superconductivity/other order?



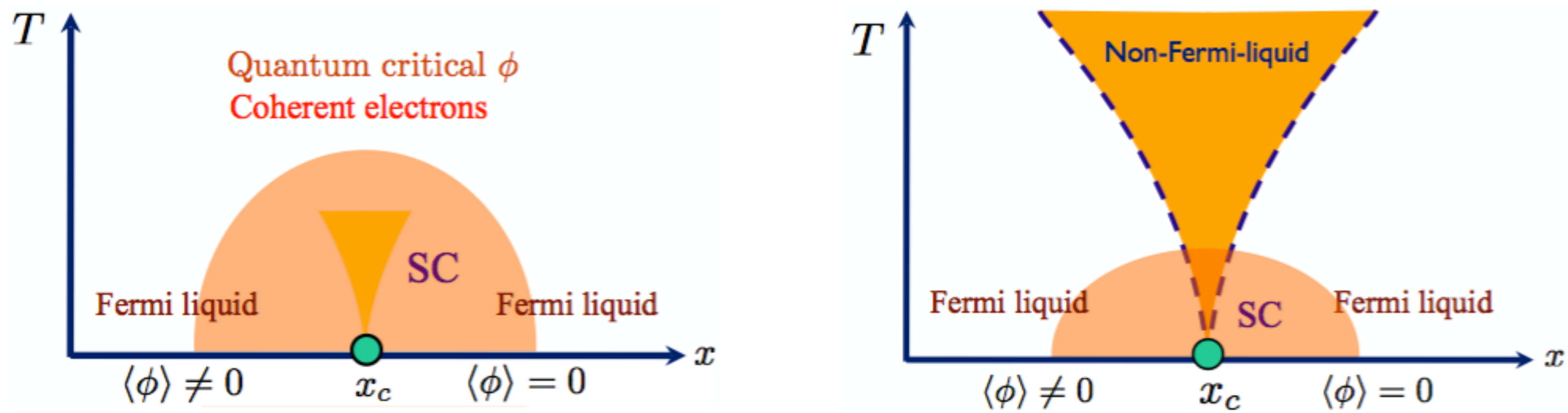


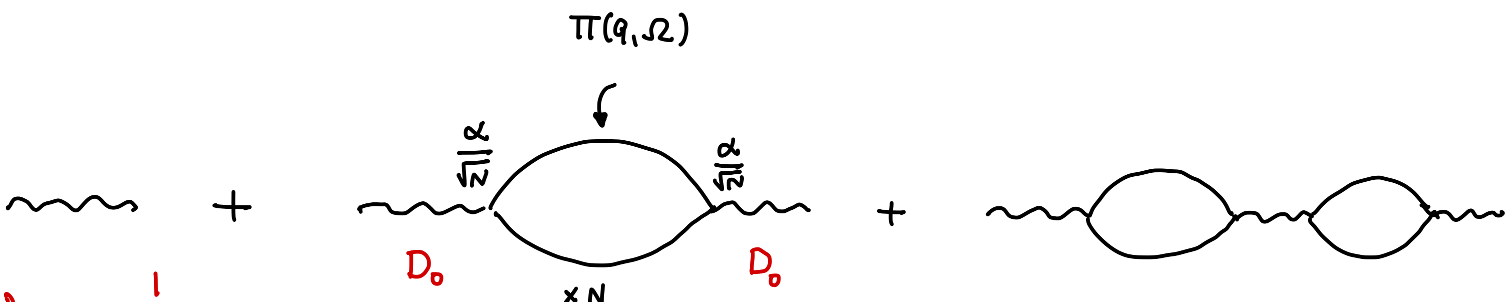
Figure from: Max Metlitski, David Mross et. al. (PRB, 2014)

Strongly coupled problem!

$$S_{\text{fermions}} = \sum_{i=1}^{N} \int d^2k d\tau\, \psi^+_{i,\boldsymbol{k}} (\partial_\tau + \varepsilon_{\boldsymbol{k}}) \psi_{i,\boldsymbol{k}}$$

$$S_{\text{int}} = \frac{\alpha}{\sqrt{N}} \sum_{i=1}^{N} \int d^2x\, d\tau e^{i\vec{Q}\cdot\vec{x}}\, \vec{\phi} \cdot (\psi^+ \vec{\sigma} \psi)$$

$$D(q,\Omega) = \langle \vec{\phi}_{q,\Omega} \cdot \vec{\phi}_{-q,-\Omega} \rangle =$$  $=$



$$D_0(q,\Omega) = \frac{1}{r + q^2 + \frac{\Omega^2}{c^2}}$$

$$+ \dots = \frac{D_0(q,\Omega)}{1 - \alpha^2 \Pi_0(q,\Omega)}$$

$$\Pi_0(q,\Omega) = -\nu_0 \cdot \frac{|\Omega|}{|q|}$$

"Landau damping"

$$S_{\text{fermions}} = \sum_{i=1}^{N} \int d^2k\, d\tau\, \psi^{+}_{i,\boldsymbol{k}} (\partial_\tau + \varepsilon_{\boldsymbol{k}}) \psi_{i,\boldsymbol{k}}$$

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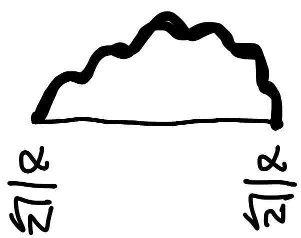
Electron self energy to :

$\Sigma(\boldsymbol{k}, \omega) =$



$$= -i \frac{\alpha^2}{N} v_0 \,\mathrm{sgn}(\omega) \begin{cases} \left(\frac{|\omega|}{\omega_0}\right)^{1/2}, & |\vec{Q}| \neq 0 \quad (\text{@ hot spot}) \\ \left(\frac{|\omega|}{\omega_0}\right)^{2/3}, & |\vec{Q}| = 0 \end{cases}$$

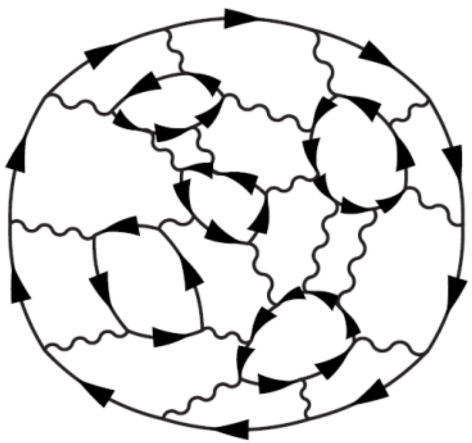
$\Sigma(\omega) \sim \omega^{\beta}$ with $\beta < 1$: ***Non Fermi liquid***

Breakdown of the large N expansion

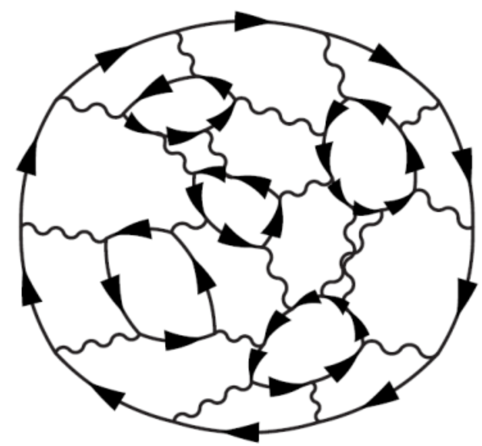
Diagrams that are "naively" subleading in $1/N$ diverge!

Correct counting of power of $1/N$: all *planar diagrams* are of leading order

$O(N^0)$



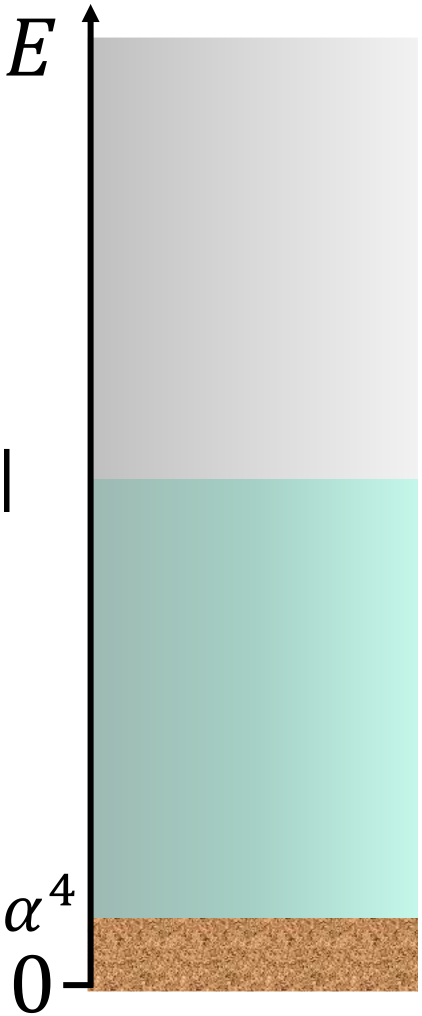
$O(N^{-2})$



S-S. Lee, PRB 80, 165102 (2009)

Weak coupling, d=2 (Ising nematic)

$\nu(\varepsilon_F)\alpha^2 \ll 1$



Wilson-Fisher

$z = 1$, FL

$\Omega_b \propto |\alpha|$

Landau damped bosons, coherent electrons

$z = 3$

$\Omega_{NFL} \sim \nu(\varepsilon_F)\alpha^4$

NFL? $(\Sigma(\omega = \Omega_{NFL}) \sim \omega)$

Superconductor? $(T_c \sim \Omega_{NFL})$

Determinant Quantum Monte Carlo (QMC)

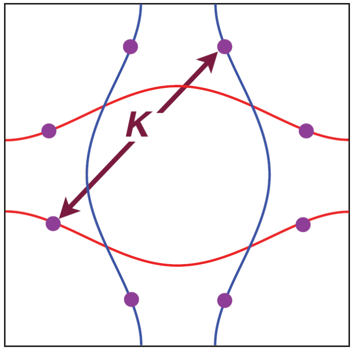
Effective bosonic action: $e^{-S_{\text{eff}}[\phi]} = e^{-S_0[\phi]}\det(M[\phi])$
M-fermion action matrix

$e^{-S_{\text{eff}}[\phi(\vec{x},\tau)]}$ can be negative (or complex): **"Sign Problem"**

Many actions describing QCPs in metals are sign problem free:

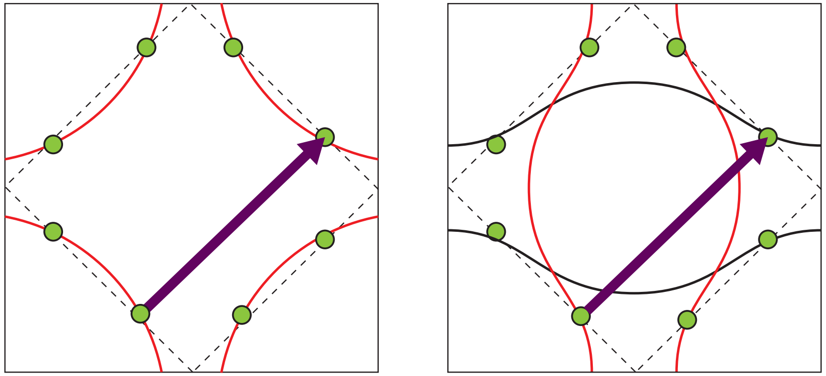
$$\mathbf{Im}\left(e^{-S_{\text{eff}}}\right) = 0,\ \mathbf{Re}\left(e^{-S_{\text{eff}}}\right) \geq 0$$

- **Ising Nematic criticality:**
  $\det(M) = \det(M_\uparrow)\det(M_\downarrow) = |\det(M_\uparrow)|^2 \geq 0$
- **SDW criticality:**
  Two bands, inter-band "hot spots" :
  Effective "time reversal" $\Longrightarrow$ sign free



EB, Metlitski, Sachdev, Science (2012)

Absence of sign problem for AFM QCP



$$e^{-S_{eff}[\phi]} = \det[M(\phi)]$$

$$M[\vec{\phi}(x,\tau)] = \partial_\tau + H_0 + \alpha e^{i\vec{Q}\cdot\vec{x}}(\vec{\phi}\cdot\vec{\sigma}) \otimes \mu_x$$

$\mu_z = \pm 1$: band index

"Time-reversal like" symmetry:

$$U = i\sigma_y \mu_z K: \quad [U, M] = 0, \quad U^2 = -1$$

Kramers' theorem: eigenvalues of *M* in conjugate pairs

$\det(M) \geq 0$: **No sign problem!**

Sufficient condition for absence of sign problem:
C-J. Wu and S-C. Zhang, PRB ***71****, 155115 (2005)*

Determinant Quantum Monte Carlo (QMC)

- Unbiased, numerically exact
  (sources of error: statistical sampling errors, Trotter errors: both controlled)

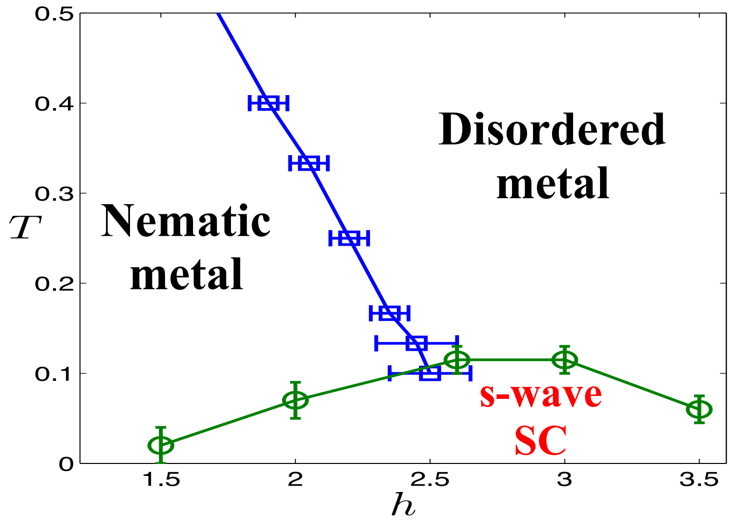
- Finite systems (here $L \leq 24$)

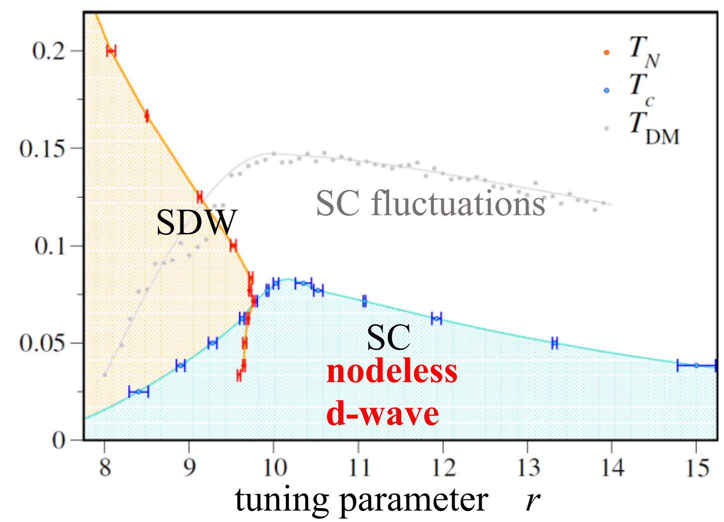
- Finite temperatures (here $\mathrm{T} \geq 0.05t \approx E_F/80$)

- Thermodynamic quantities, imaginary time/Matsubara frequency correlations (real frequency: requires analytic continuation)

Is superconductivity enhanced near the QCP?

Yes.





Do any other types of order emerge generically near the QCP?

Generically, apparently not…

Description of the quantum critical regime?

Weak coupling:

Strongly renormalized bosons
weakly renormalized (FL) fermions

Stronger coupling:

Signatures of non-Fermi
Liquid behavior

EB, S. Lederer, Y. Schattner, S. Trebst, Ann. Rev. CMP (2019)

Results

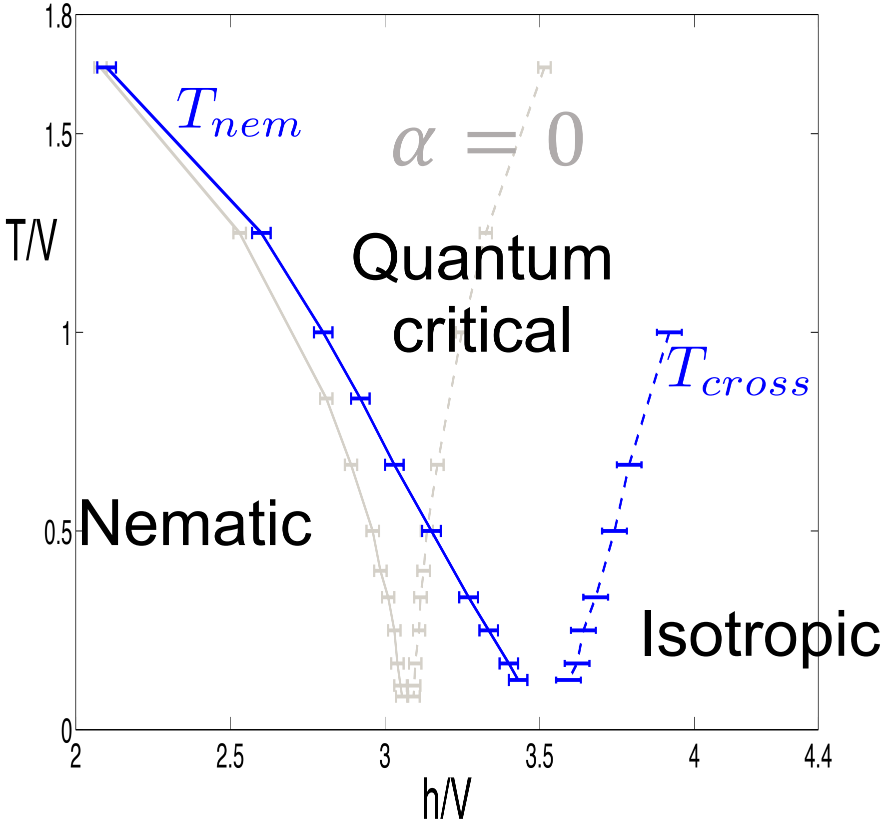
Ising nematic critical point: phase diagram

Ising nematic transition is continuous

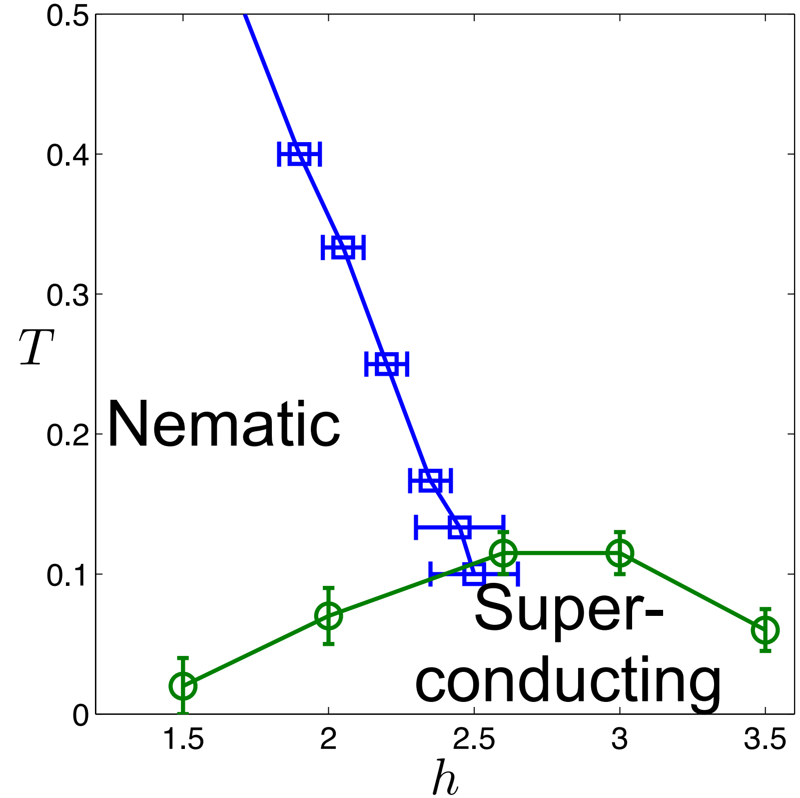
QCP masked by anisotropic s-wave superconducting "dome"

S. Lederer, Y. Schattner, S. Kivelson, EB, PRX (2016); PNAS (2017)

$\alpha = 0.5$



$\alpha = 1.5$



Results

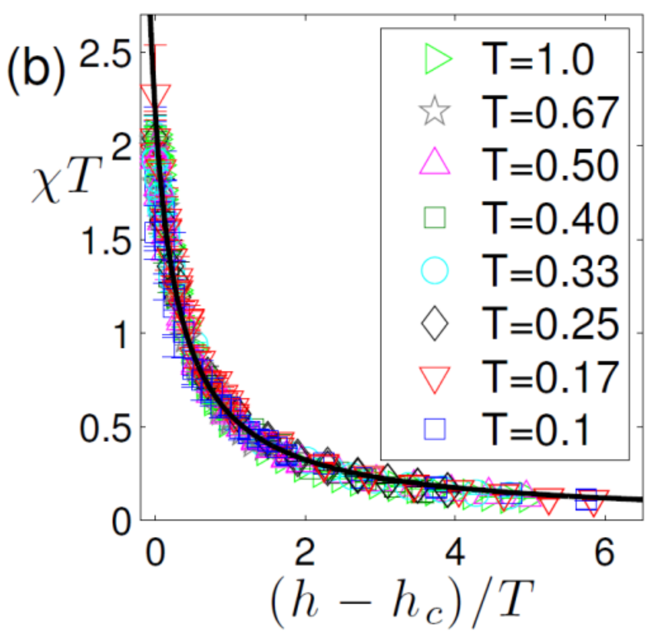
Ising nematic critical point

Divergent nematic susceptibility:

$$\chi \propto \frac{1}{T + A(h - h_c) + Bq^2}$$

ω_n dependence: Landau damped
q dependence of coefficient is complex

Effect of thermal fluctuations:
Klein, Schattner, Chubukov, EB, PRX (2020)

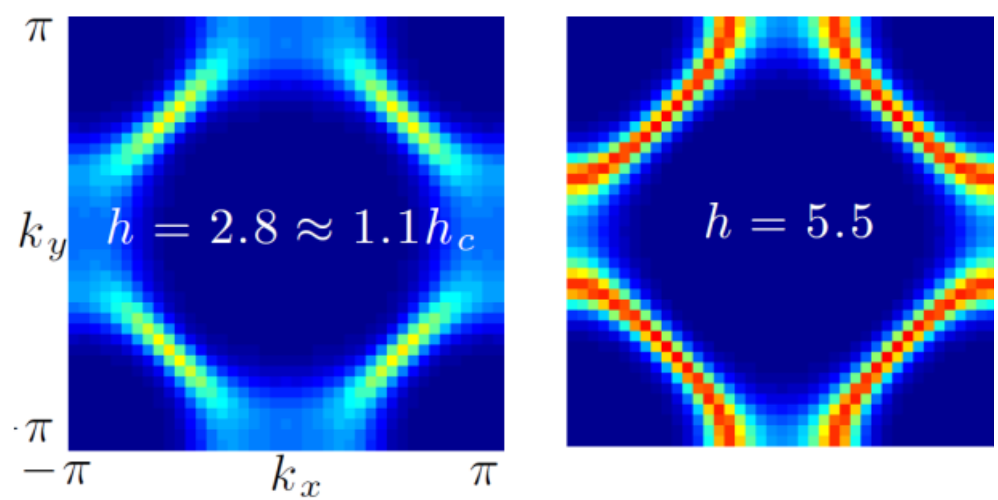


Low energy electronic spectrum:

$$G\left(\tau = \frac{\beta}{2}\right) \approx \int_{-T}^{T} d\omega A(\mathbf{k}, \omega)$$

Non-Fermi liquid behavior
away from "cold spots"

Unexpected behavior:
$\mathrm{Im}\Sigma_{k_F}(i\omega_n, T) \approx const$

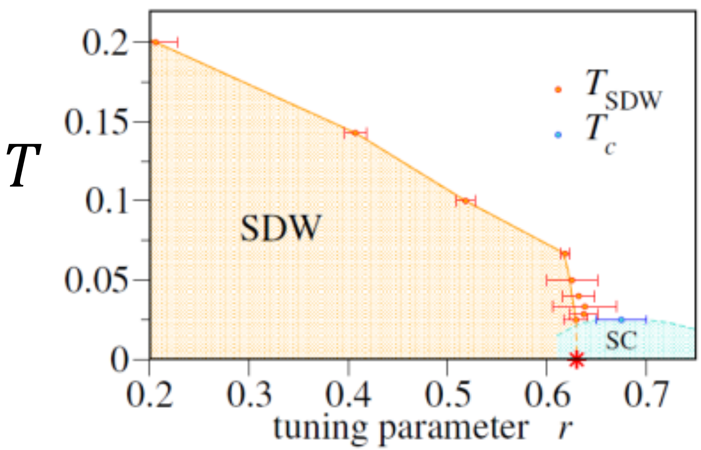


Schattner, Lederer, Kivelson, EB, PRX (2016)

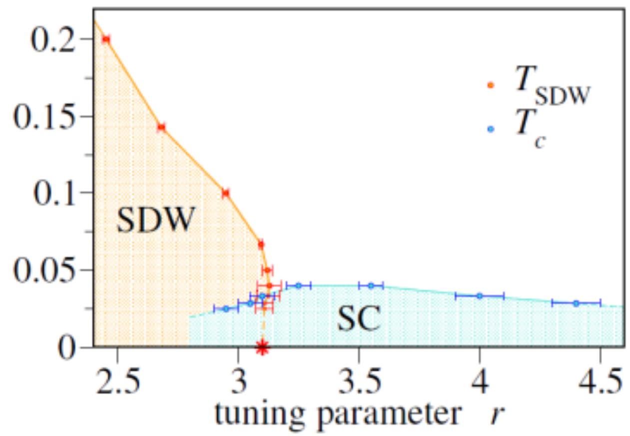
Results

Easy-plane O(2) AFM critical point: phase diagram

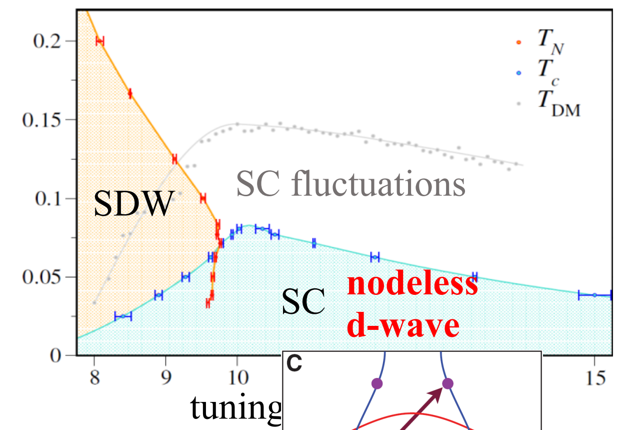
$\lambda = 1.5$

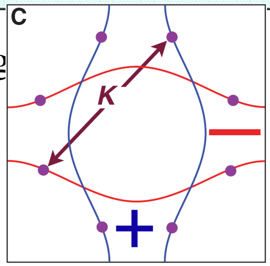


$\lambda = 2$



$\lambda = 3$

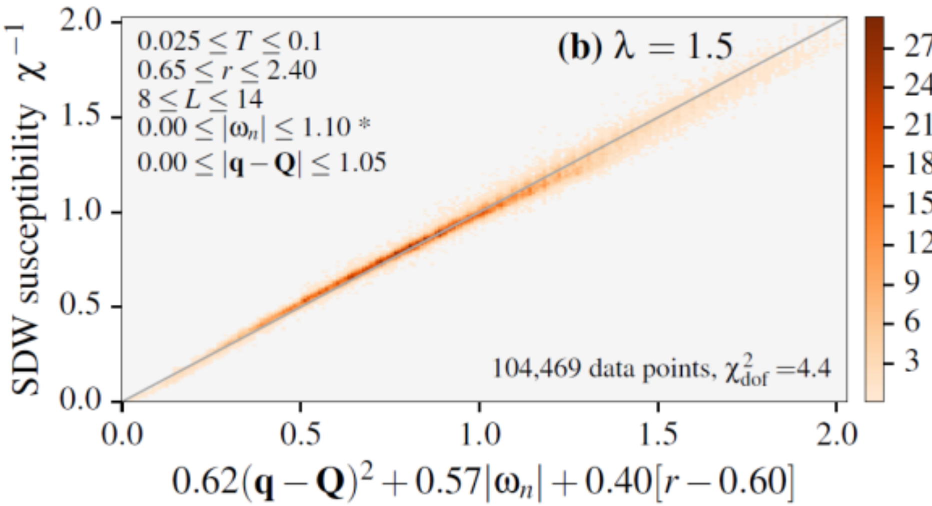




QCP covered by nodeless d-wave SC dome

Magnetic susceptibility above T_c:

$$\chi \propto \frac{1}{|\omega_n| + Aq^2 + B(r - r_c) + C(T)}$$

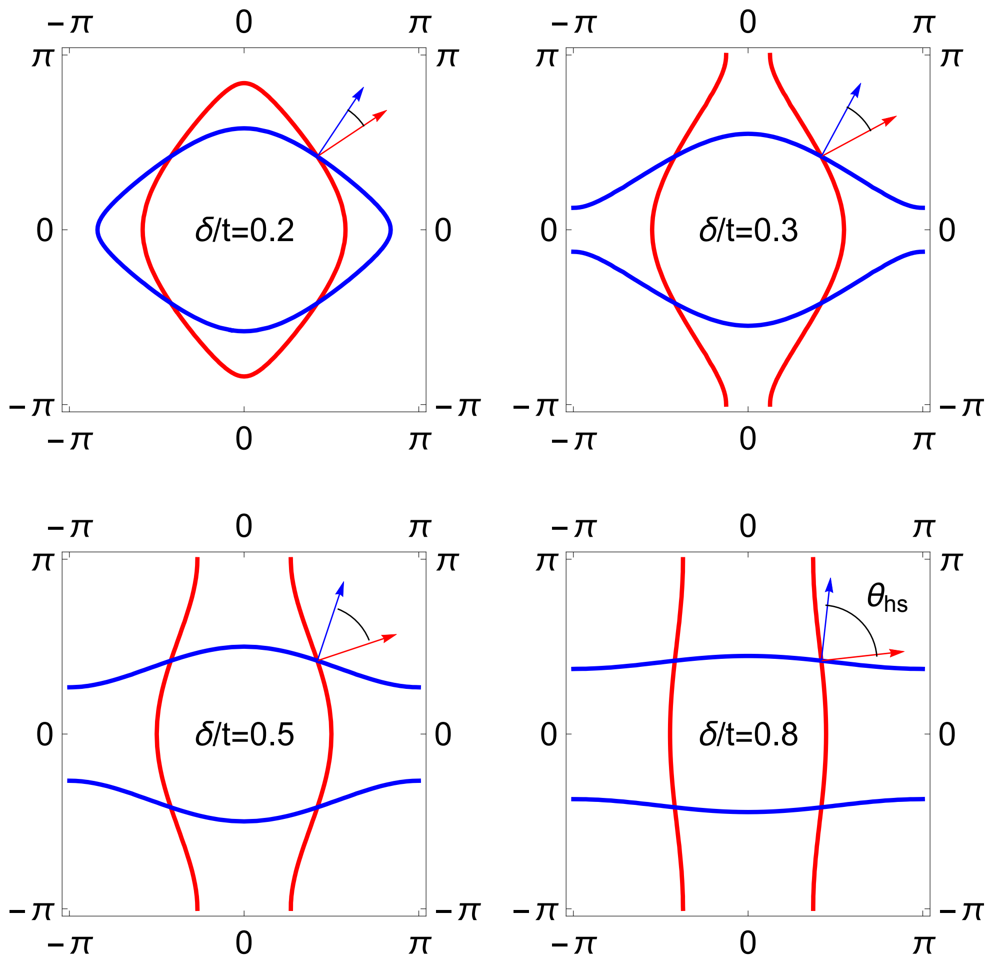


O(3) AFM transition: similar SC T_c,
χ has similar form
Bauer, Schattner, Trebst, EB, PRR (2020)

Schattner, Gerlach, Trebst, EB, PRL (2016); PRB (2017)

What controls T_c near the QCP?

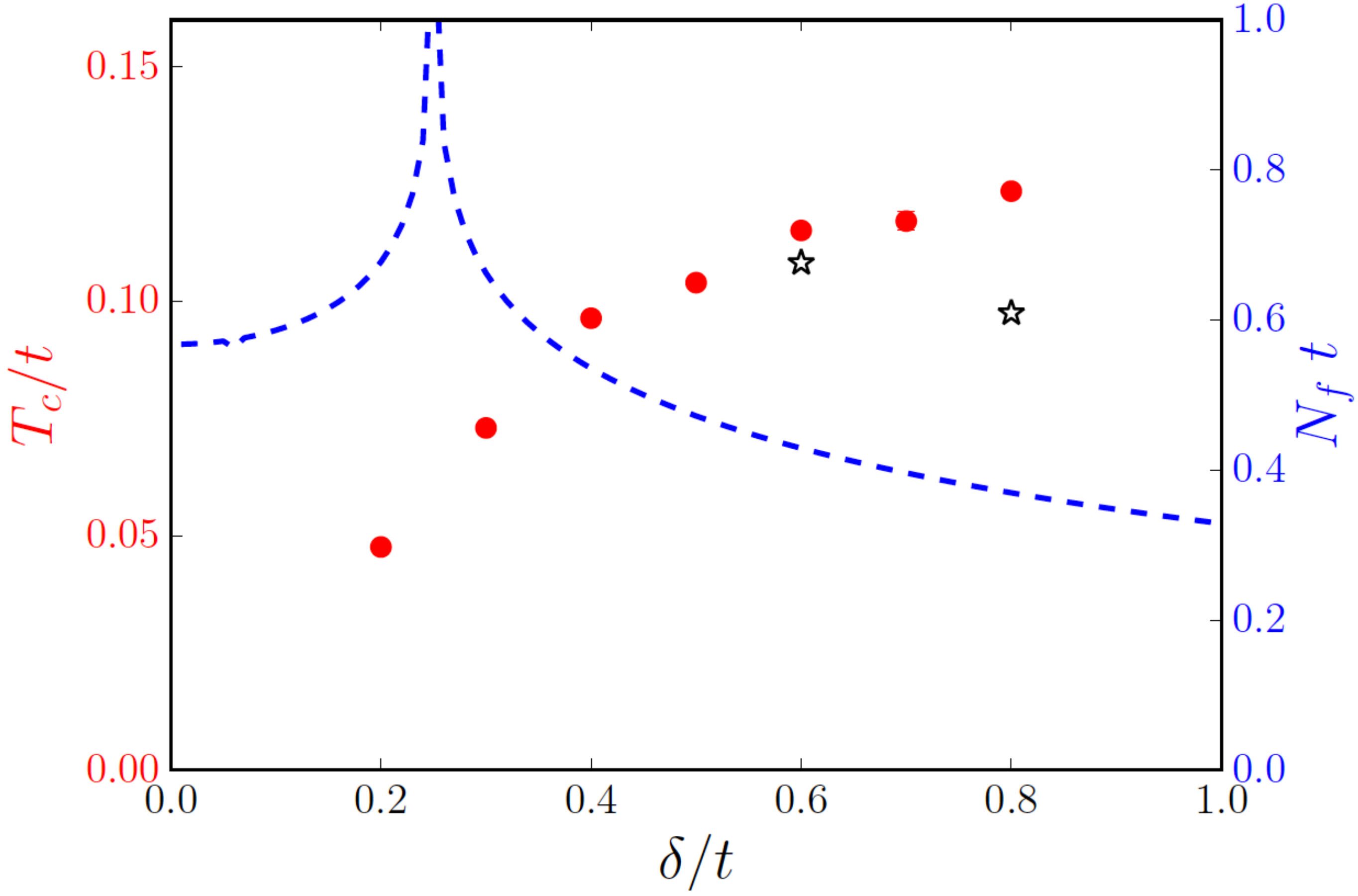
Vary angle between Fermi surfaces at hot spots:



Wang, Schattner, Berg, Fernandes, PRB (2017)

What controls T_c near the QCP?

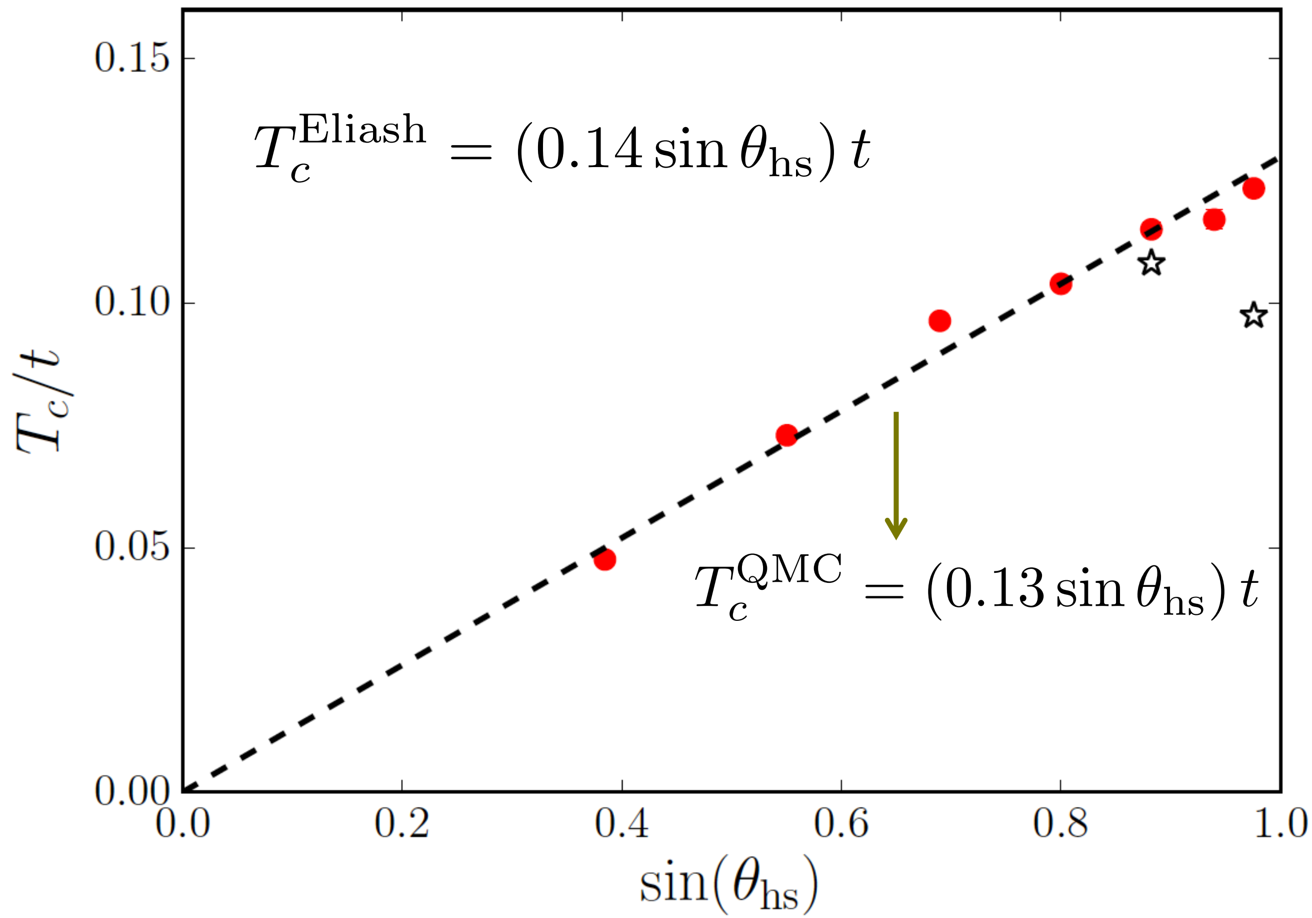
T_c variation is not due to density of states effects



Wang, Schattner, Berg, Fernandes, PRB (2017)

What controls T_c near the QCP?

T_c near antiferromagnetic QCP vs. $\delta/t \sim \sin\theta_{\rm hs}$:



New, non-SC QCP with $\theta_{\rm hs} \to 0$?

Schlief, Lunts, S-S. Lee (PRX, 2017); Lunts, Albergo, Lindsey (arXiv 22')

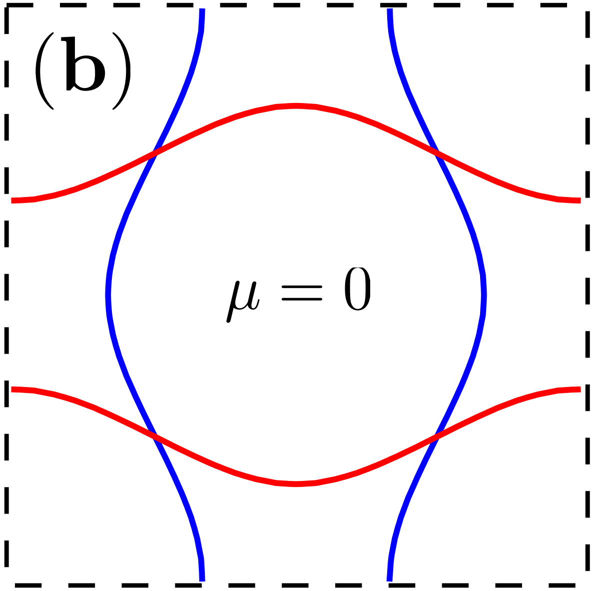
Wang, Schattner, Berg, Fernandes, PRB (2017)

CDW near antiferromagnetic QCP?

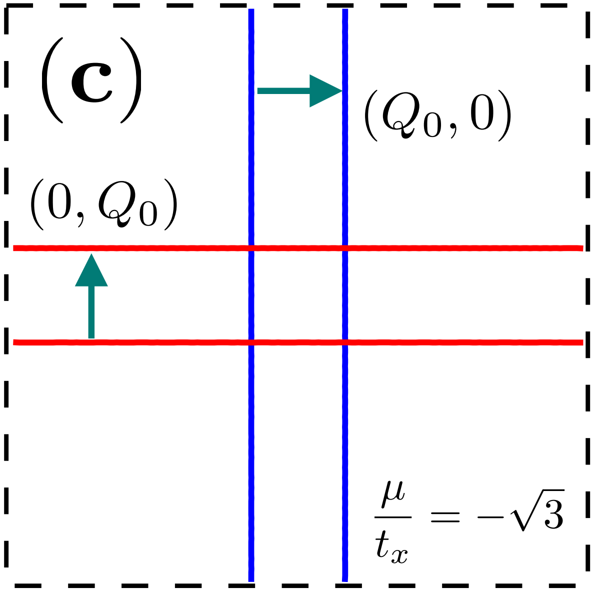
AFM is attractive for both CDW and superconductivity

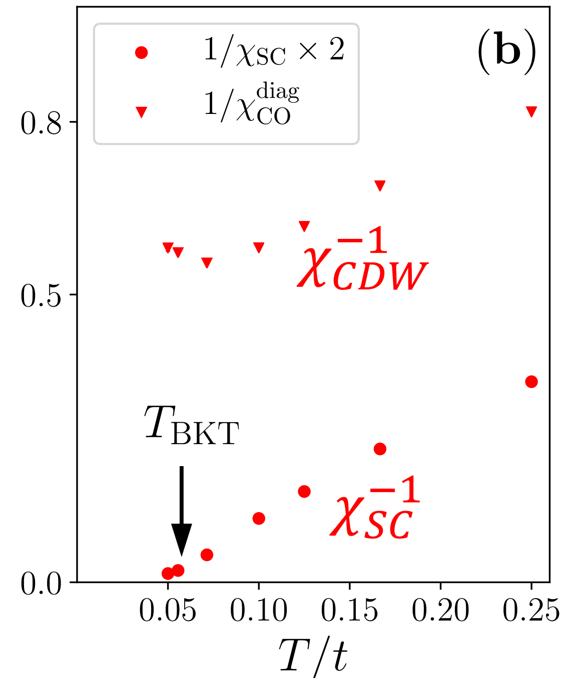
Metlitski, Sachdev (PRB, 2011); Efetov, Meier, Pepin (Nat. Phys. 2013); Wang, Chubukov (PRB, 2014)

$\mu = 0$
(P-H symmetric)



$\mu \neq 0$
Perfect nesting





X. Wang, Y. Wang, Schattner, EB, R. Fernandes (PRL, 2018)

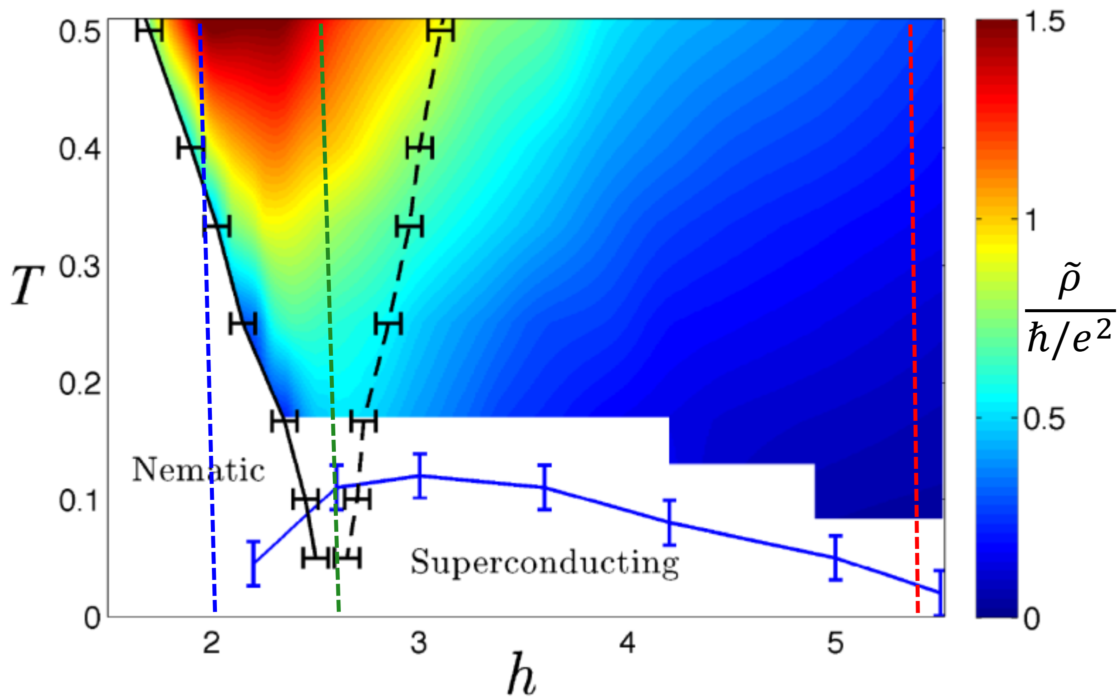
Transport

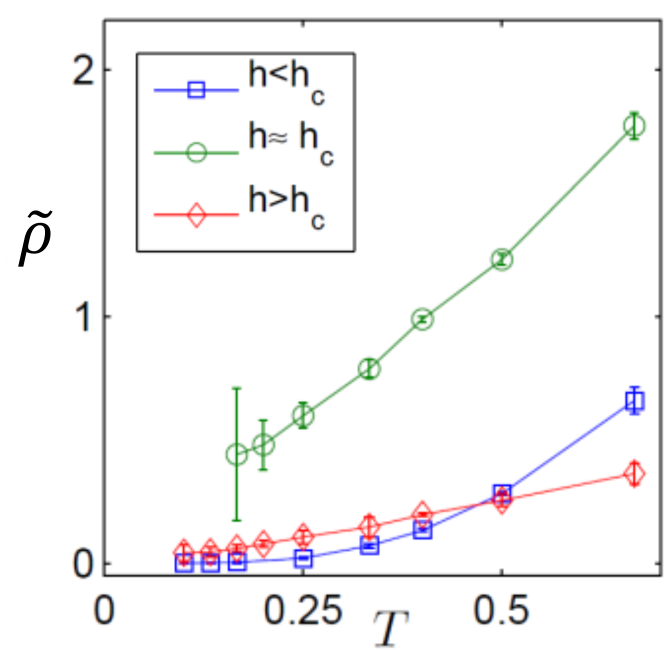
Ising nematic critical point

"Resistivity proxy": $\tilde{\rho} \equiv \frac{\partial_\tau^2 \Lambda(\beta/2)}{2\pi\Lambda^2(\beta/2)} \approx \frac{\int_0^T d\omega\, \omega^2 \sigma(\omega)}{T\left[\int_0^T d\omega \sigma(\omega)\right]^2}$

If $\sigma(\omega)$ is a Lorentzian: $\tilde{\rho} = \rho_{dc}$

Qualitatively similar results for AFM QCP

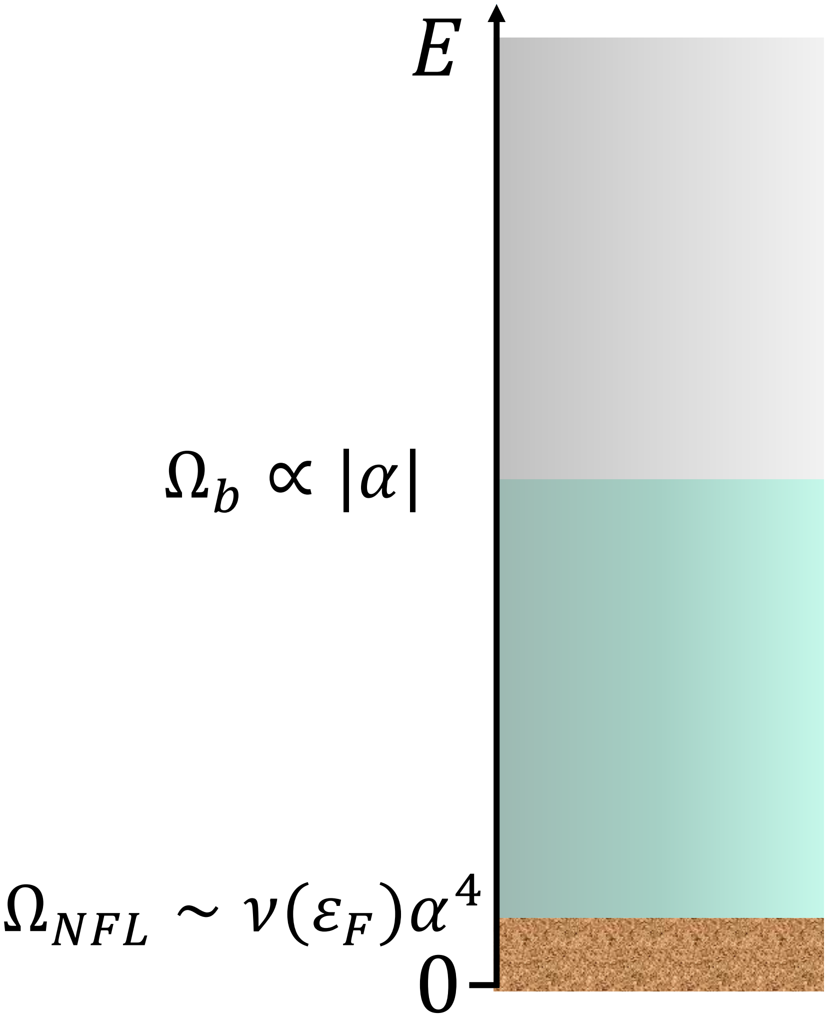




S. Lederer, Y. Schattner, EB, S. Kivelson, PNAS (2017)

Weak coupling, d=2 (Ising nematic)

$\nu(\varepsilon_F)\alpha^2 \ll 1$



Wilson-Fisher
$z = 1$, FL

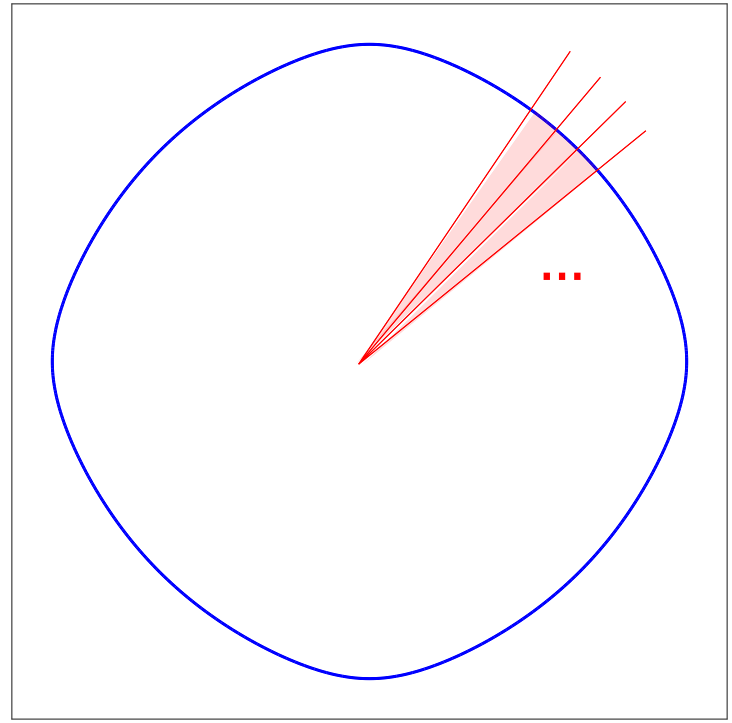
Landau damped bosons,
coherent electrons
$z = 3$

NFL? $(\Sigma(\omega = \Omega_{NFL}) \sim \omega)$
Superconductor? $(T_c \sim \Omega_{NFL})$

Method

Memory matrix method:
identify "slow variables"

Review: S. Hartnoll, A. Lucas, S. Sachdev (2016)



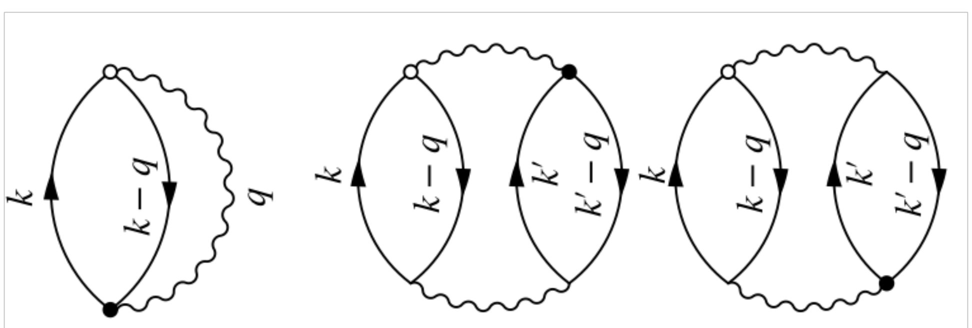
In our case:

Mahajan, Barkeshli, Hartnoll (2013)

$$n_{\hat{k}} = \int dk_\perp c_{\boldsymbol{k}}^\dagger c_{\boldsymbol{k}}$$

Memory matrix:

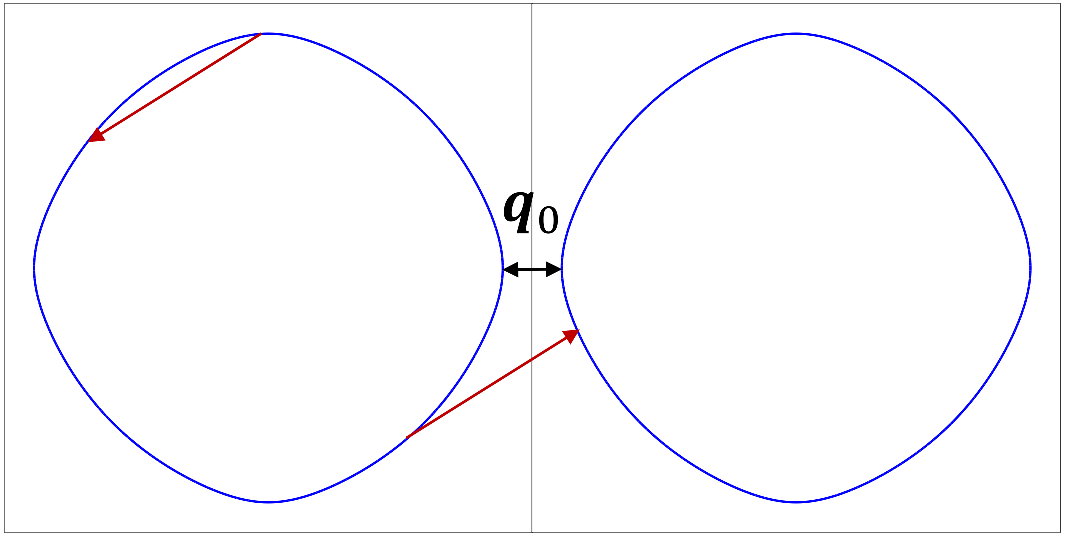
$$M_{\hat{k},\hat{k}'}(\Omega) = \frac{\mathrm{Im}\langle \dot{n}_{\hat{k}} | \dot{n}_{\hat{k}'} \rangle}{\Omega}$$ where $\dot{n}_{\hat{k}} = i[H, n_{\hat{k}}]$



Kinetic equation including multi-particle scattering processes

Xioayu Wang and EB, PRB (2019)

Umklapp processes*



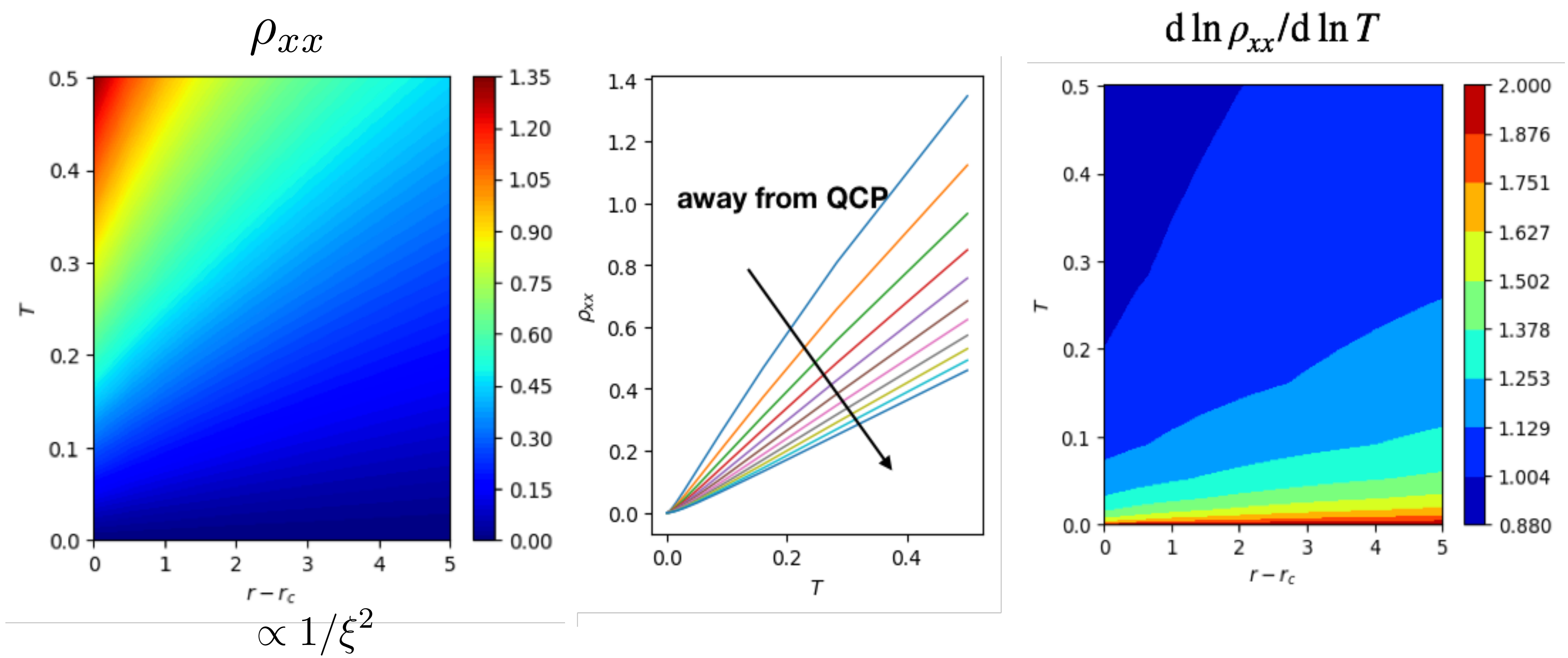
At sufficiently low T, $\rho \sim T^2$ (even at QCP)
Maslov, Yudson, Chubukov (2011)

Behavior at intermediate temperatures? $T > T_0$
(Expect $T_0 \sim |\boldsymbol{q}_0|^z$)

* Compensated metal (like $\mathrm{BaFe_2(As_{1-x}P_x)_2}$): $T_0 = 0$
(*no Umklapp necessary*!)

Xioayu Wang and EB, PRB (2019)

Analytical transport calculation: coherent electron regime



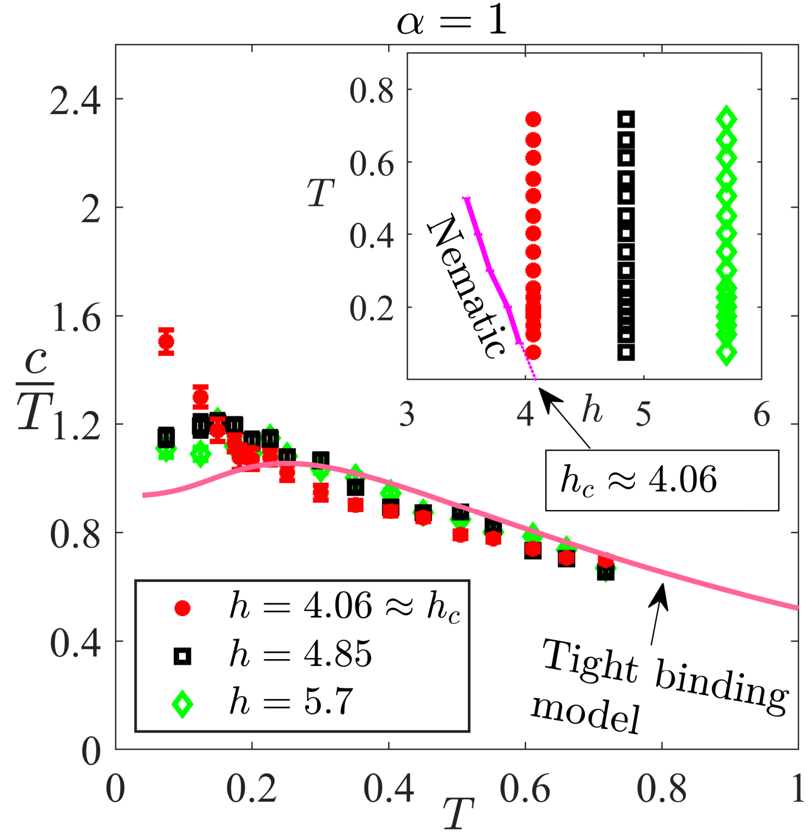
- Non-zero resistivity due to Umklapp proceeses
- Quasi-linear resistivity for $T > T_0$ ($T_0 \sim |\boldsymbol{q}_0|^z$)

Xioayu Wang and EB, PRB (2019)

Specific heat: Ising nematic QCP

Broad “coherent electron regime” above $T_{NFL} \sim T_c$

- $m \sim m^*$, q.p. weight $Z \sim 1$
- Non-Fermi liquid scattering rate



Grossman, Hofmann, Holder, EB, PRL (2021)

Summary

Metallic quantum criticality is accessible via sign problem-free Quantum Monte Carlo simulations.

- **Generic properties:**
  - ***QCP "preempted" by high-T_c superconductor!***
  - ***Quantum critical regime above T_c:***
    - ***Rapid growth of correlations***
    - ***Breakdown of Fermi liquid behavior***
    - ***Anomalous transport***
- **What's missing…**
  - ***No "competing orders" other than SC***
  - ***No "Pseudogap"***

Thank you.