SCHOOL ON EXOTIC SUPERCONDUCTIVITY: EXPSUP2022

#1 SCANNED JOSEPHSON TUNNELING MICROSCOPY

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University College Cork – Coláiste na hOllscoille Corcaigh

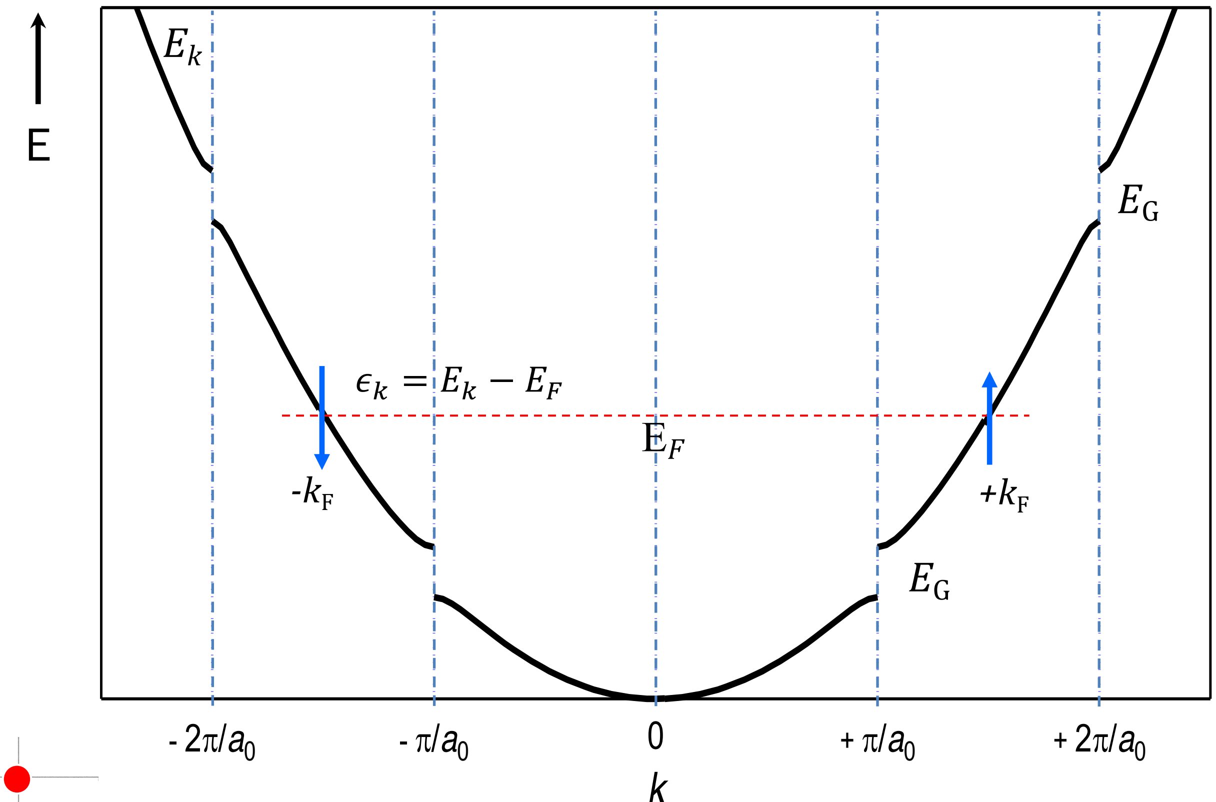
erc European Research Council

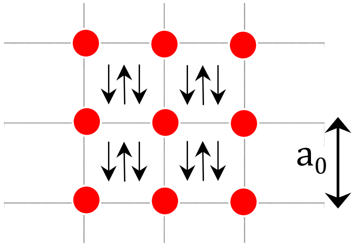
University of Oxford

Gordon and Betty MOORE FOUNDATION

Cornell University Founded A.D. 1865

SINGLE ELECTRON (FERMI) LIQUID





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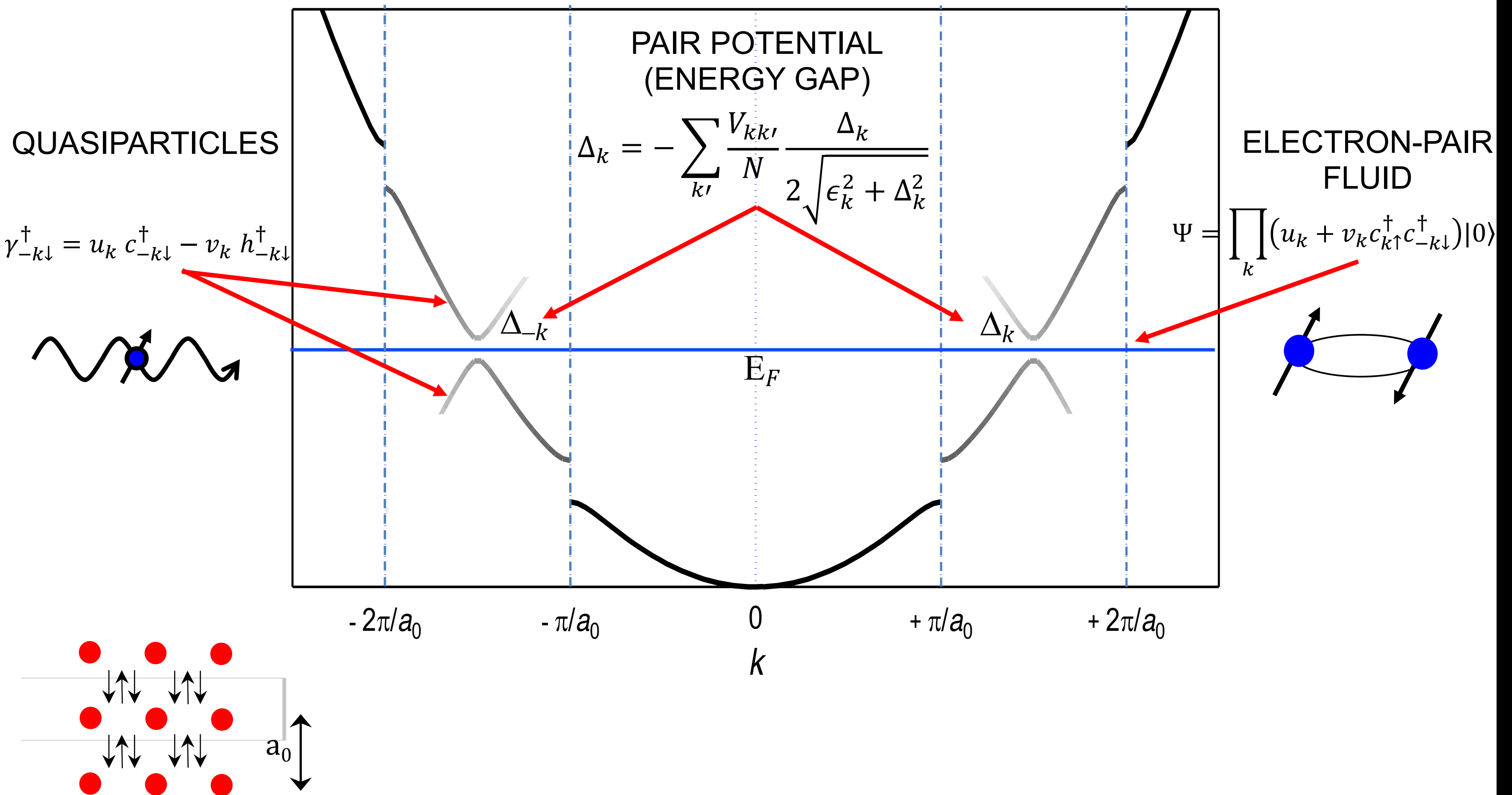
ELECTRON-PAIR CONDENSATE

CONDENSATE ORDER PARAMETER $\psi = \langle c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} \rangle$

$\Psi = \prod_k (u_k + v_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow})|0\rangle$ BCS WAVEFUNCTION



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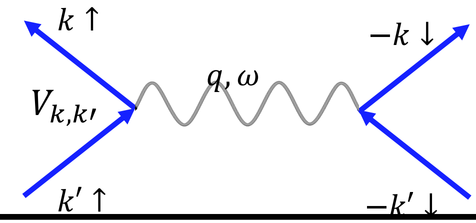
ELECTRON-PAIR CONDENSATE

MODEL

$$H = \sum_{k\sigma} \epsilon_{k\sigma} c^{\dagger}_{k\sigma} c_{k\sigma} + \sum_{kk'} \frac{V_{kk'}}{N} c^{\dagger}_{k\uparrow} c^{\dagger}_{-k\downarrow} c_{-k'\downarrow} c_{k'\uparrow}$$



APPROX.

$$\left\langle c^{\dagger}_{k\uparrow} c^{\dagger}_{-k\downarrow} c_{-k'\downarrow} c_{k'\uparrow} \right\rangle \approx \left\langle c^{\dagger}_{k\uparrow} c^{\dagger}_{-k\downarrow} \right\rangle c_{-k'\downarrow} c_{k'\uparrow} + c^{\dagger}_{k\uparrow} c^{\dagger}_{-k\downarrow} \left\langle c_{-k'\downarrow} c_{k'\uparrow} \right\rangle - \left\langle c^{\dagger}_{k\uparrow} c^{\dagger}_{-k\downarrow} \right\rangle \left\langle c_{-k'\downarrow} c_{k'\uparrow} \right\rangle$$

BILINEAR

$$\Delta_k \equiv -\sum\nolimits_{k'} \frac{V_{kk'}}{N} \left\langle c_{-k'\downarrow} c_{k'\uparrow} \right\rangle$$

$$H = \sum_{k\sigma} \epsilon_{k\sigma} c^{\dagger}_{k\sigma} c_{k\sigma} - \sum_{k} \Delta_k c^{\dagger}_{k\uparrow} c^{\dagger}_{-k\downarrow} + \Delta^{*}_{k} c_{-k\downarrow} c_{k\uparrow} + \sum_{k} \Delta_k \left\langle c^{\dagger}_{k\uparrow} c^{\dagger}_{-k\downarrow} \right\rangle$$

ANSATZ

$$\Psi = \prod_{k} \left(u_k + v_k c^{\dagger}_{k\uparrow} c^{\dagger}_{-k\downarrow} \right) |0\rangle$$

$$\langle \Psi | c^{\dagger}_{k\uparrow} c^{\dagger}_{-k\downarrow} | \Psi \rangle = \langle 0 | \prod\nolimits_p \left(u^{*}_{p} + v^{*}_{p} c_{-p\downarrow} c_{p\uparrow} \right) c^{\dagger}_{k\uparrow} c^{\dagger}_{-k\downarrow} \prod\nolimits_q \left(u_q + v_q c^{\dagger}_{q\uparrow} c^{\dagger}_{-q\downarrow} \right) |0\rangle$$

$$\left\langle c^{\dagger}_{k\uparrow} c^{\dagger}_{-k\downarrow} \right\rangle = v^{*}_{k} u_k \qquad \left\langle c_{k\uparrow} c_{-k\downarrow} \right\rangle = v_k u^{*}_{k}$$

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FERMI-PAIR CONDENSATE

MINIMIZE: $\langle\Psi|H|\Psi\rangle$

$$2\epsilon_k\, u_k v_k - \Delta_k\, u_k^2 + \Delta_k^* v_k^2 = 0 \quad \Longrightarrow \quad \begin{array}{l} |u_k|^2 = \frac{1}{2}\left(1 - \frac{\epsilon_k}{\sqrt{\epsilon_k^2 + \Delta_k^2}}\right) \\ |v_k|^2 = \frac{1}{2}\left(1 + \frac{\epsilon_k}{\sqrt{\epsilon_k^2 + \Delta_k^2}}\right) \end{array} \quad \Longrightarrow \quad v_k u_k^* = \frac{\Delta_k}{2\sqrt{\epsilon_k^2 + \Delta_k^2}}$$

GAP EQUATION

$$\Delta_k \equiv -\sum_{k'} \frac{V_{kk'}}{N} \left\langle c_{-k'\downarrow} c_{k'\uparrow} \right\rangle = -\sum_{k'} \frac{V_{kk'}}{N} v_{k'} u_{k'}^* \quad \Longrightarrow \quad \Delta_k = -\sum_{k'} \frac{V_{kk'}}{N} \frac{\Delta_k}{2\sqrt{\epsilon_k^2 + \Delta_k^2}}$$

ORDER PARAMETER

$$\psi = \left\langle c_{k\uparrow}^\dagger\, c_{-k\downarrow}^\dagger \right\rangle \quad or \quad \psi = \left\langle c_{-k\downarrow} c_{k\uparrow} \right\rangle \qquad not \quad \Delta_k = -\sum_{k'} \frac{V_{kk'}}{N} \frac{\Delta_k}{2\sqrt{\epsilon_k^2 + \Delta_k^2}}$$

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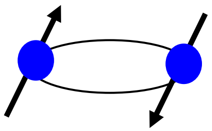
MACROSCOPIC QUANTUM STATE $\Psi(\vec{r},t)$

COOPER PAIR



$$\psi(\boldsymbol{r}_1-\boldsymbol{r}_2)=\phi(\boldsymbol{r}_1-\boldsymbol{r}_2)\left(\frac{\uparrow_1\downarrow_2-\downarrow_1\uparrow_2}{\sqrt{2}}\right)=\phi(\boldsymbol{r}_1-\boldsymbol{r}_2)\chi_{12}=\sum_k g_k e^{ik\cdot(r_1-r_2)}\chi_{12}=\sum_k g_k c^\dagger_{k\uparrow}c^\dagger_{-k\downarrow}|0\rangle$$

MACROSCOPIC QUANTUM STATE

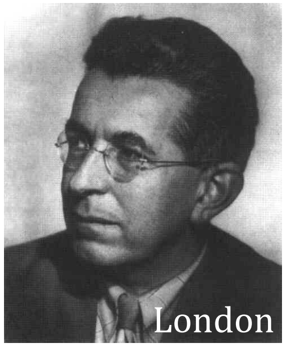
$$\Psi=\prod_k\left(u_k+v_k c^\dagger_{k\uparrow}c^\dagger_{-k\downarrow}\right)|0\rangle$$

ENORMOUS NUMBER IN SAME QUANTUM STATE

$$\Psi(\vec{r},t)=\psi e^{i\phi(r,t)}$$

CONDENSED PAIR DENSITY n_P $\quad \psi=\sqrt{n_P}$

MACROSCOPIC QUANTUM PHASE $\quad \phi(r,t)$



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MACROSCOPIC QUANTUM ELECTRODYNAMICS

CONDENSATE PARTICLES (*q,m,n*)

$$\Psi(\vec{r},t) = \psi e^{i\phi(r,t)}$$

<u>NEWTON</u>

$$\frac{\partial \boldsymbol{p}}{\partial t} = q\boldsymbol{E}$$

$$\boldsymbol{j} \equiv nq\boldsymbol{p}/m$$

<u>FARADAY</u>

$$\nabla\times\boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla\times\boldsymbol{E} = \frac{m}{nq^2}\frac{\partial\nabla\times\boldsymbol{j}}{\partial t}$$

$$\frac{m}{nq^2}\frac{\partial\nabla\times\boldsymbol{j}}{\partial t} + \frac{\partial\boldsymbol{B}}{\partial t} = 0$$

<u>AMPERE</u>

$$\nabla\times\boldsymbol{B} = \mu_0\boldsymbol{j}$$

$$\nabla\times\nabla\times\boldsymbol{B} = -\frac{nq^2}{m}\mu_0\boldsymbol{B}$$

$$= -\nabla^2\boldsymbol{B}$$

$$\frac{\partial\boldsymbol{j}}{\partial t} = \frac{nq^2}{m}\boldsymbol{E}$$

LONDON1

$$\nabla\times\boldsymbol{j} + \frac{nq^2}{m}\boldsymbol{B} = 0$$

LONDON2

$$\nabla^2\boldsymbol{B} = \frac{nq^2\mu_0}{m}\boldsymbol{B}$$

LONDON3

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MACROSCOPIC QUANTUM ELECTRODYNAMICS

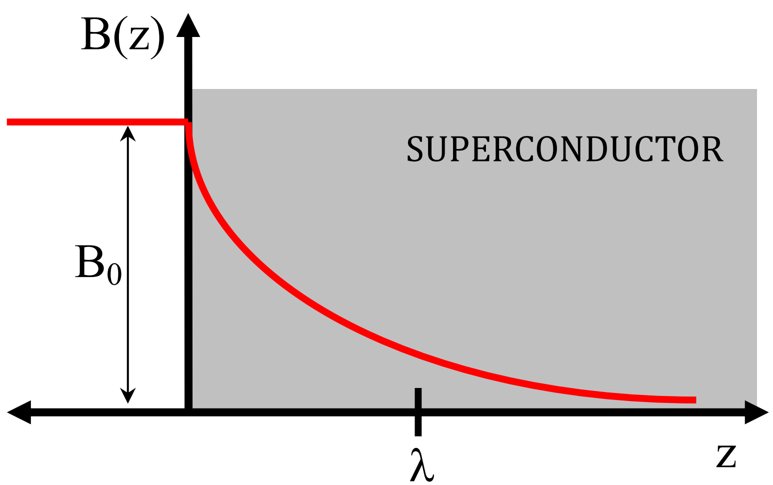
CONDENSATE PARTICLES (*q,m,n*)

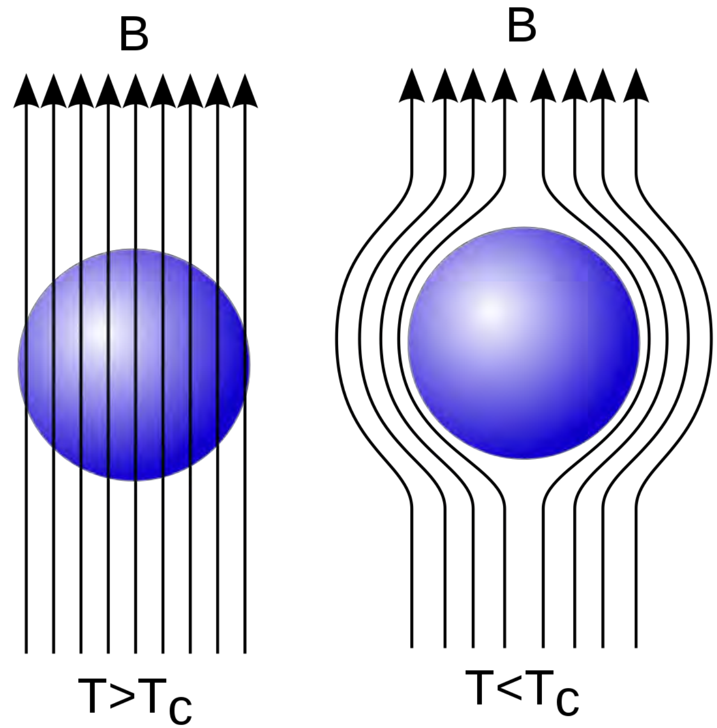
$$\Psi(\vec{r},t) = \psi e^{i\phi(r,t)}$$

$$\nabla^2 \boldsymbol{B} = \frac{nq^2\mu_0}{m}\boldsymbol{B} = \frac{1}{\lambda^2}\boldsymbol{B}$$

$$\lambda = \sqrt{m/nq^2\mu_0}$$

$$B(z) = B_0 e^{-z/\lambda}$$





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MACROSCOPIC QUANTUM ELECTRODYNAMICS

CONDENSATE PARTICLES (q,m,n)

$$\Psi(\vec{r},t) = \psi e^{i\phi(r,t)}$$

$$\boldsymbol{j} \equiv nq\boldsymbol{p}/m \qquad m\langle \boldsymbol{v}\rangle \equiv \langle \boldsymbol{p} - q\boldsymbol{A}\rangle = \left\langle \left(\frac{\hbar}{i}\right)\nabla - q\boldsymbol{A}\right\rangle$$

$$\boldsymbol{j} = \frac{q}{2m}\left\{\Psi^*\left(\frac{\hbar}{i}\nabla - q\boldsymbol{A}\right)\Psi + CC\right\}$$

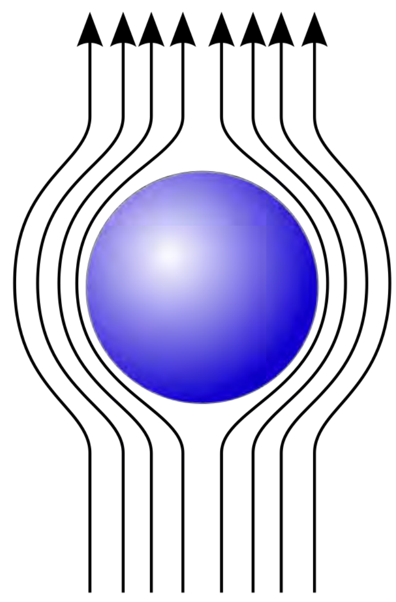
$$\boldsymbol{j} = \frac{1}{i}\frac{q}{2m}\{\hbar\Psi^*\nabla\Psi - \hbar\Psi\nabla\Psi^*\} - \frac{q^2}{m}\boldsymbol{A}\,\Psi^*\Psi$$

$$\boldsymbol{j} = \frac{q}{m}|\psi|^2(\hbar\nabla\phi - q\boldsymbol{A})$$

$$|\psi|^2 \equiv n$$

$$\nabla\times\boldsymbol{j} = \frac{-q^2 n}{m}(\nabla\times\boldsymbol{A}) = \frac{-q^2 n}{m}\boldsymbol{B} \qquad \text{LONDON2}$$

$$\nabla^2\boldsymbol{B} = \frac{1}{\lambda^2}\boldsymbol{B}$$



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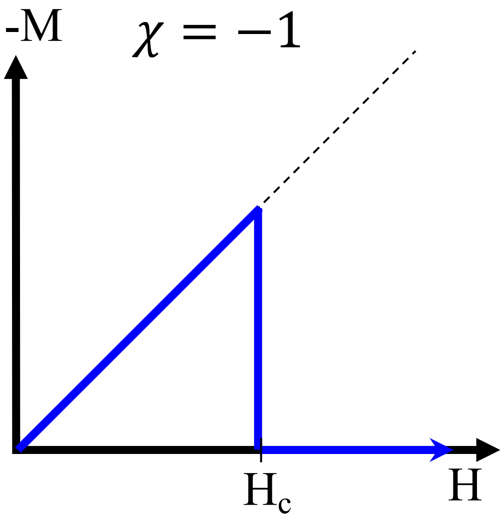
PERFECT DIAMAGNET

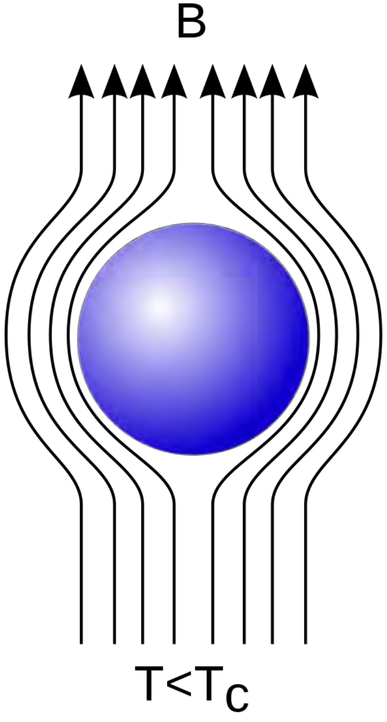
SUPERCONDUCTOR: $q = -2e$

$\Psi(\vec{r}, t) = \psi e^{i\phi(r,t)}$

W. Meißner and R. Ochsenfeld, Naturwissenschaften 21, 787 (1933).



$$\nabla^2 \boldsymbol{B} = \frac{1}{\lambda^2} \boldsymbol{B}$$



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MAGNETIC FLUX QUANTUM

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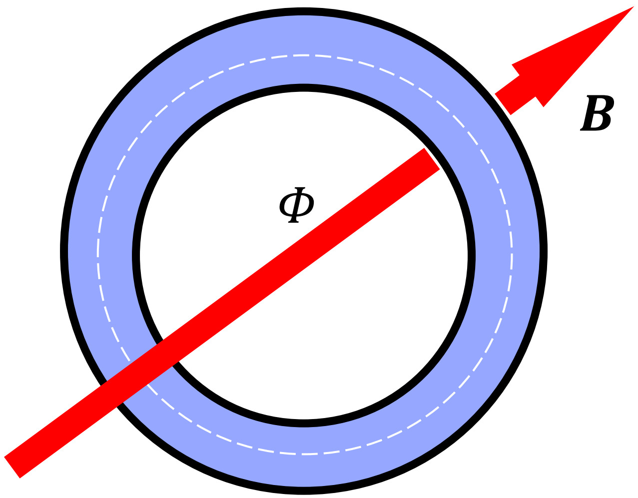
MAGNETIC FLUX QUANTUM

SUPERCONDUCTOR: $q = -2e$

$\Psi(\vec{r},t) = \psi e^{i\phi(r,t)}$



MEISSNER EFFECT:

$$\boldsymbol{j} = \frac{q}{m}|\psi|^2(\hbar\nabla\phi - q\boldsymbol{A}) = \boldsymbol{0}$$

$$\hbar\nabla\phi = q\boldsymbol{A}$$

UNIQUENESS:

$$\oint \nabla\phi \cdot dl = q/\hbar \oint \boldsymbol{A} \cdot dl$$

$$= N2\pi$$

$$\oint \boldsymbol{A} \cdot dl = \int \boldsymbol{\nabla}\times\boldsymbol{A} \cdot da$$

$$= \int \boldsymbol{B} \cdot da \equiv \Phi$$

FLUX QUANTUM:

$$\Phi = \mathrm{N}(h/2e) \qquad \Phi_0 = \frac{h}{2e} = 2\times10^{-15}\,Tm^2$$

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MAGNETIC FLUX QUANTUM

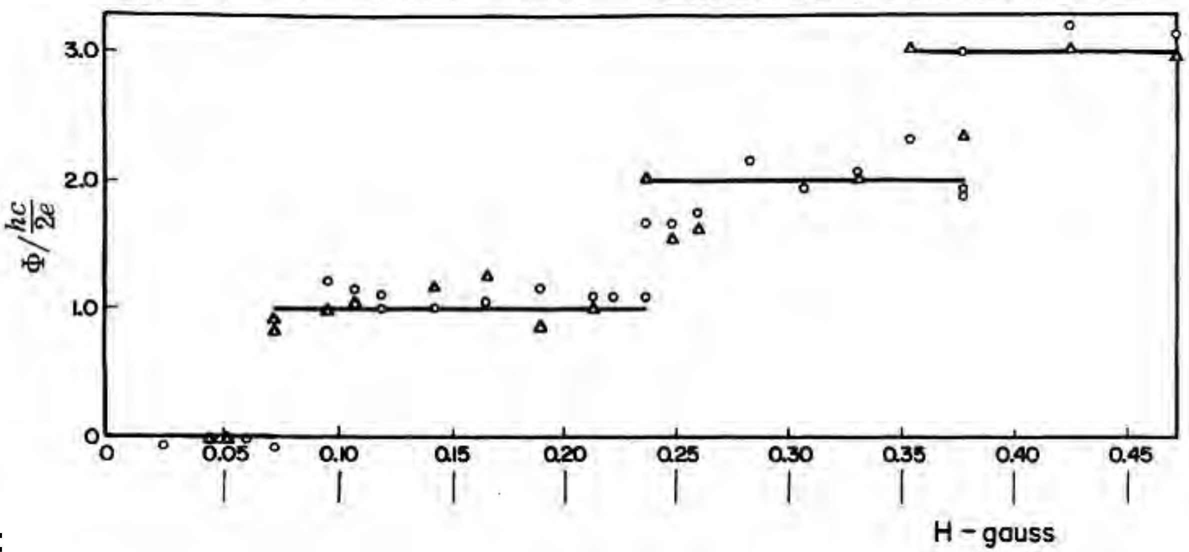
SUPERCONDUCTOR: $q = -2e$

$$\Psi(\vec{r}, t) = \psi e^{i\phi(r,t)}$$

Volume 7, Number 2 PHYSICAL REVIEW LETTERS July 15, 1961

EXPERIMENTAL EVIDENCE FOR QUANTIZED FLUX IN SUPERCONDUCTING CYLINDERS*

Bascom S. Deaver, Jr., and William M. Fairbank
Department of Physics, Stanford University, Stanford, California
(Received June 16, 1961)



FLUX QUANTUM:

$$\Phi = \mathrm{N}(h/2e) \qquad \Phi_0 = \frac{h}{2e} = 2\times10^{-15}\ Tm^2$$

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PERSISTENT ELECTRICAL CURRENTS

SUPERCONDUCTOR: $q = -2e$

$$\Psi(\vec{r}, t) = \psi e^{i\phi(r,t)}$$

MEISSNER EFFECT:

$$\boldsymbol{j} = \frac{q}{m}|\psi|^2(\hbar\nabla\phi - q\boldsymbol{A}) = \mathbf{0}$$

$$\hbar\nabla\phi = q\boldsymbol{A}$$

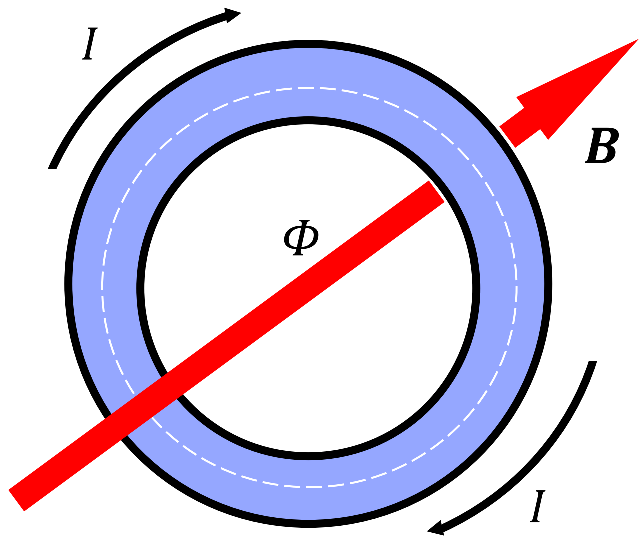
UNIQUENESS:

$$\oint \nabla\phi \cdot dl = q/\hbar \oint \boldsymbol{A} \cdot dl$$

$$= N2\pi$$

FLUX QUANTUM:

$$\Phi = \mathrm{N}(\Phi_0)$$



PERSISTENT

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PERSISTENT ELECTRICAL CURRENTS

SUPERCONDUCTOR: $q = -2e$

$$\Psi(\vec{r}, t) = \psi e^{i\phi(r,t)}$$

MEISSNER EFFECT:

$$\boldsymbol{j} = \frac{q}{m}|\psi|^2(\hbar\nabla\phi - q\boldsymbol{A}) = \mathbf{0}$$

$$\hbar\nabla\phi = q\boldsymbol{A}$$

UNIQUENESS:

$$\oint \nabla\phi \cdot dl = q/\hbar \oint \boldsymbol{A} \cdot dl$$

$$= N2\pi$$

FLUX QUANTUM:

$$\Phi = \mathrm{N}(\Phi_0)$$

CMS SUPERCONDUCTING SOLENOID - CERN

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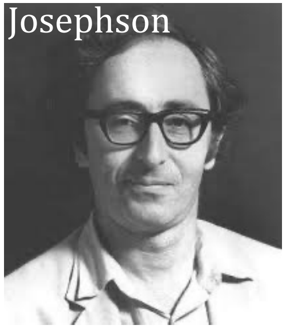
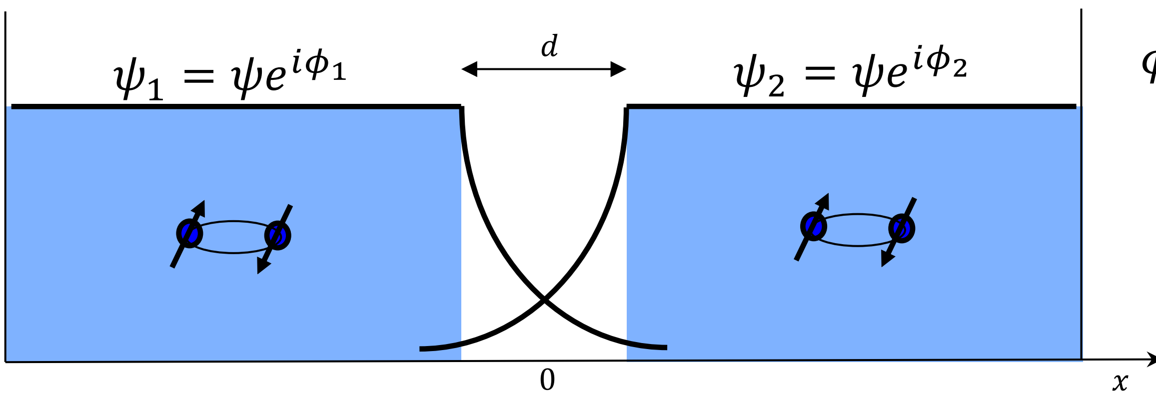
JOSEPHSON EFFECT(S)

SUPERCONDUCTING JOSEPHSON EFFECT



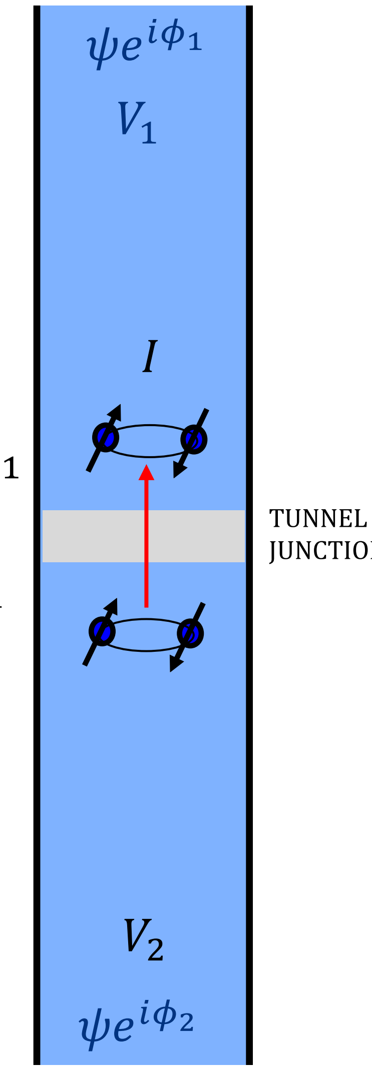
$$I = I_c sin\phi$$

JOSEPHSON TUNNELING:



$$\phi = \phi_2 - \phi_1$$

$$V = V_2 - V_1$$



$$\psi_J = \psi e^{i(\phi_1 - \gamma(x + \frac{d}{2}))} + \psi e^{i(\phi_2 + \gamma(x - \frac{d}{2}))}$$

$$I = 2e\frac{i\hbar}{2m}\{\psi_J^* \nabla\psi_J - \psi_J \nabla\psi_J^*\}$$

$$\Rightarrow \quad I = I_c \sin(\phi_2 - \phi_1)$$

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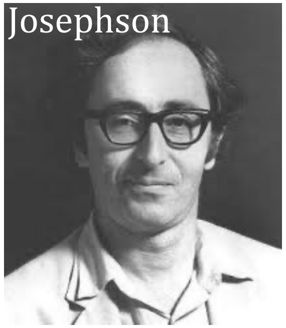
SUPERCONDUCTING JOSEPHSON EFFECT



$$I = I_c sin\phi$$

$$\hbar\frac{\partial\phi}{\partial t} = 2eV$$

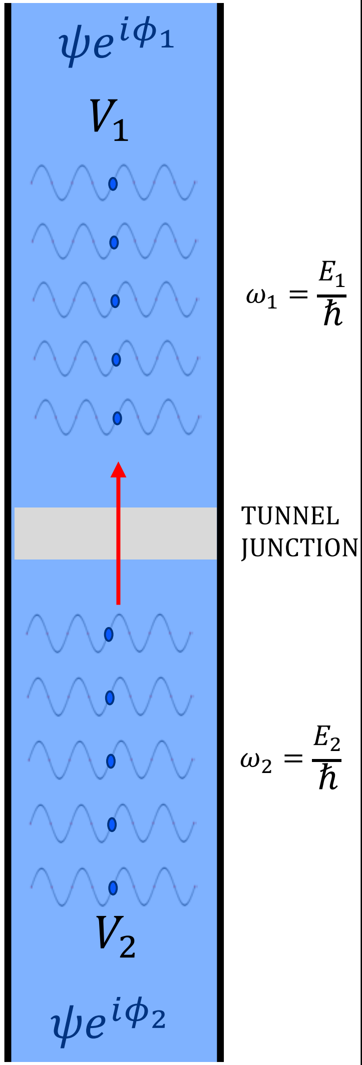
JOSEPHSON FREQUENCY:

$$\frac{\partial\phi}{\partial t} = \omega_2 - \omega_1 = \frac{E_2}{\hbar} - \frac{E_1}{\hbar} = \frac{2e}{\hbar}(V_2 - V_1)$$

$$\Rightarrow \hbar\frac{\partial\phi}{\partial t} = 2eV$$

$$\phi = \phi_2 - \phi_1$$

$$V = V_2 - V_1$$



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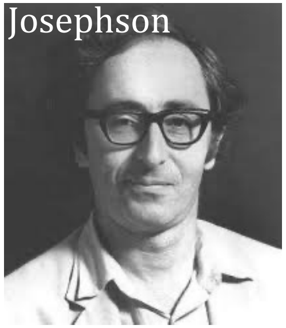
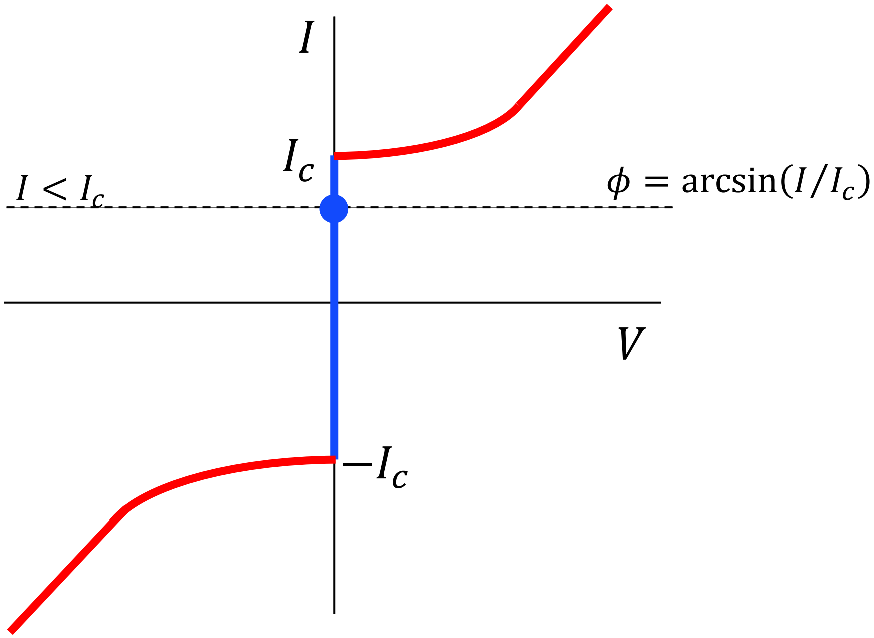
JOSEPHSON JUNCTION

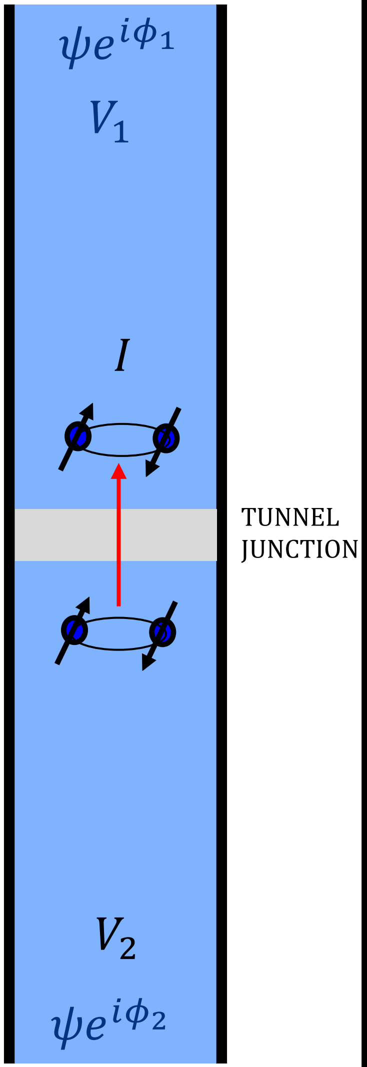


$$I = I_c sin\phi$$

$$\hbar\frac{\partial\phi}{\partial t} = 2eV$$

JOSEPHSON JUNCTION:





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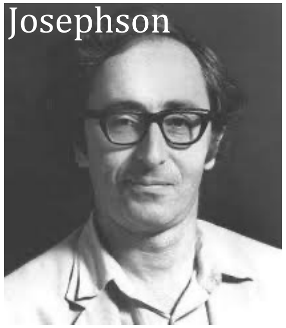
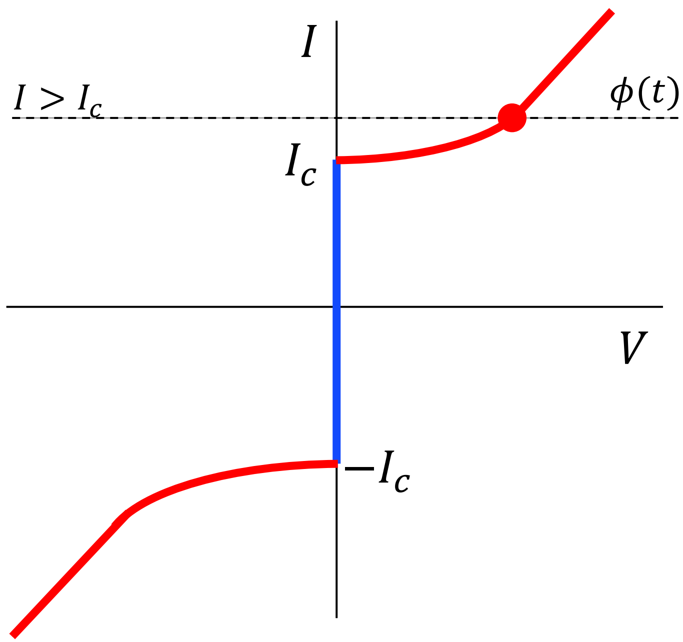
JOSEPHSON JUNCTION



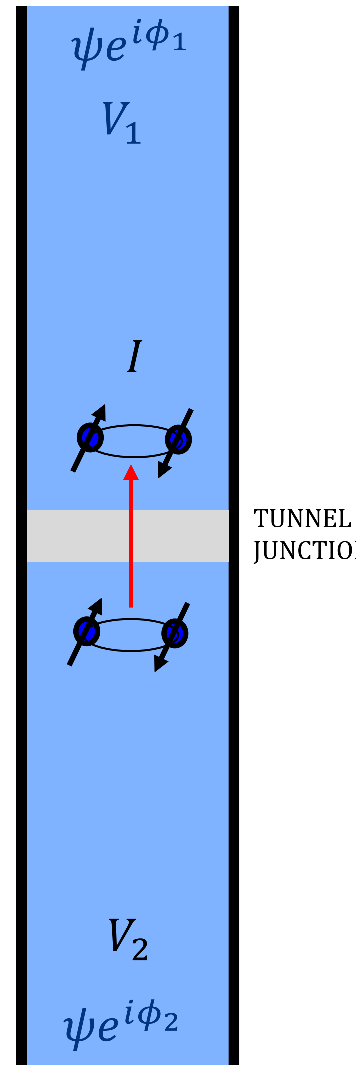
$$I = I_c sin\phi$$

$$\hbar \frac{\partial \phi}{\partial t} = 2eV$$

JOSEPHSON JUNCTION:



$$\phi(t) = \int (2eV/\hbar)\mathrm{d}t$$



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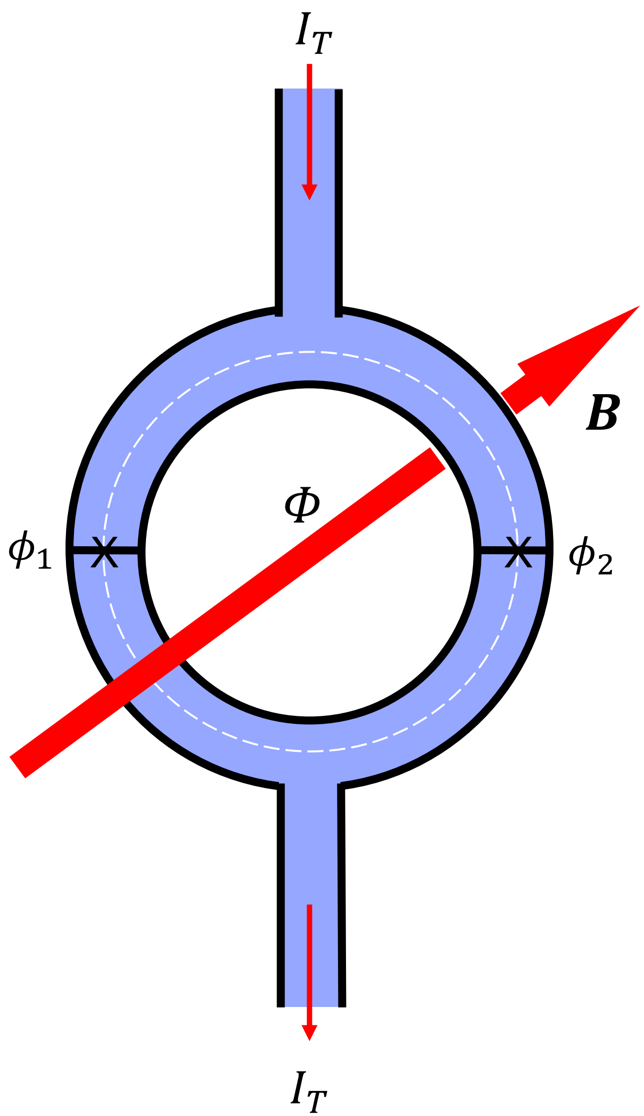
SUPERCONDUCTING QUANTUM INTERFERENCE



$$I_T = I_C Sin\phi_1 + I_C Sin\phi_2$$

$$I_T = 2I_C\ Sin\left(\frac{\phi_1 + \phi_2}{2}\right) Cos\left(\frac{\phi_1 - \phi_2}{2}\right)$$

$$\boldsymbol{j} = \frac{q}{m}|\psi|^2(\hbar\nabla\phi - q\boldsymbol{A}) = \mathbf{0} \qquad \hbar\nabla\phi = q\boldsymbol{A}$$

$$\oint \nabla\phi \cdot dl = N2\pi = \phi_1 - \phi_2 - 2e/\hbar \oint \boldsymbol{A} \cdot dl$$

$$\phi_1 - \phi_1 = \left(\frac{2e}{\hbar}\right)\Phi = 2\pi\frac{\Phi}{\Phi_0} \qquad \Phi_0 = \frac{h}{2e}$$

$$|I_{MAX}| = 2I_C\ \left|Cos\left(\pi\frac{\Phi}{\Phi_0}\right)\right|$$

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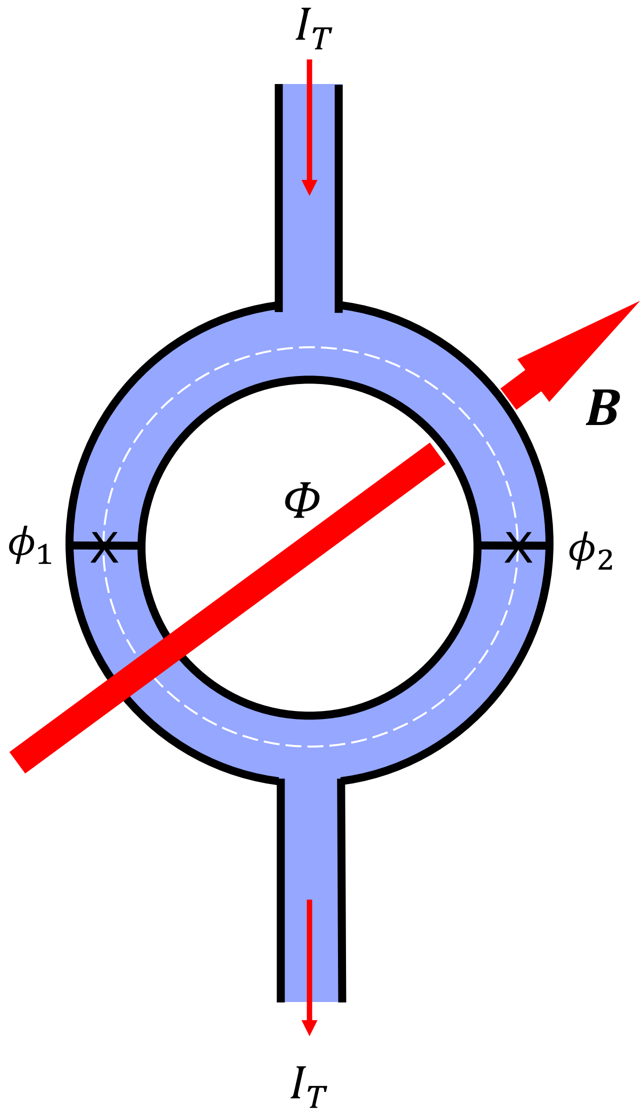
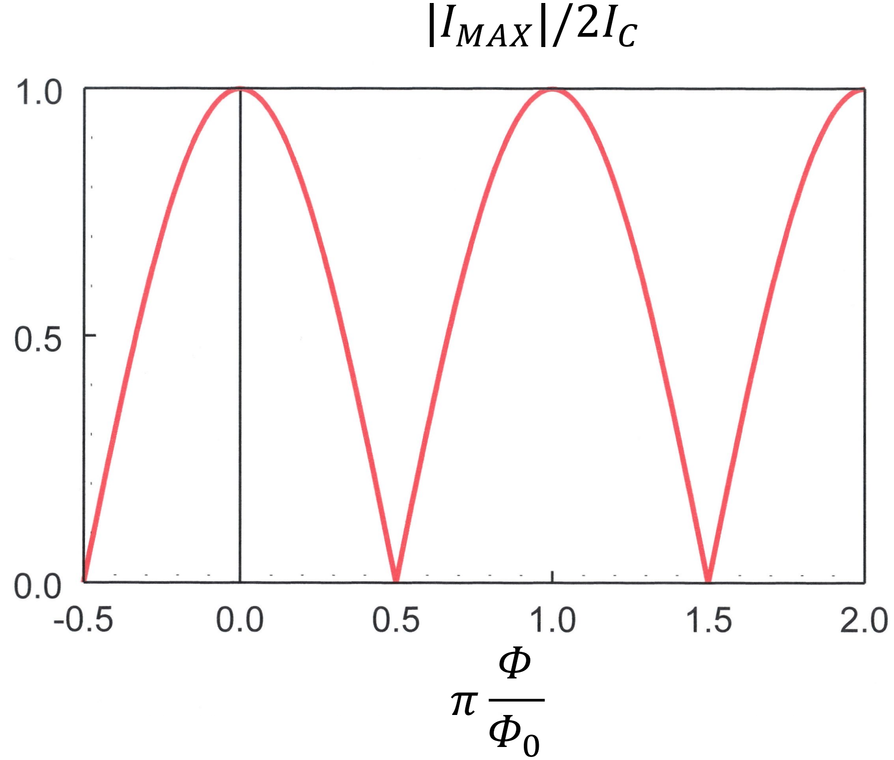
SUPERCONDUCTING QUANTUM INTERFERENCE





$$|I_{MAX}| = 2I_C \left|Cos\left(\pi \frac{\Phi}{\Phi_0}\right)\right|$$

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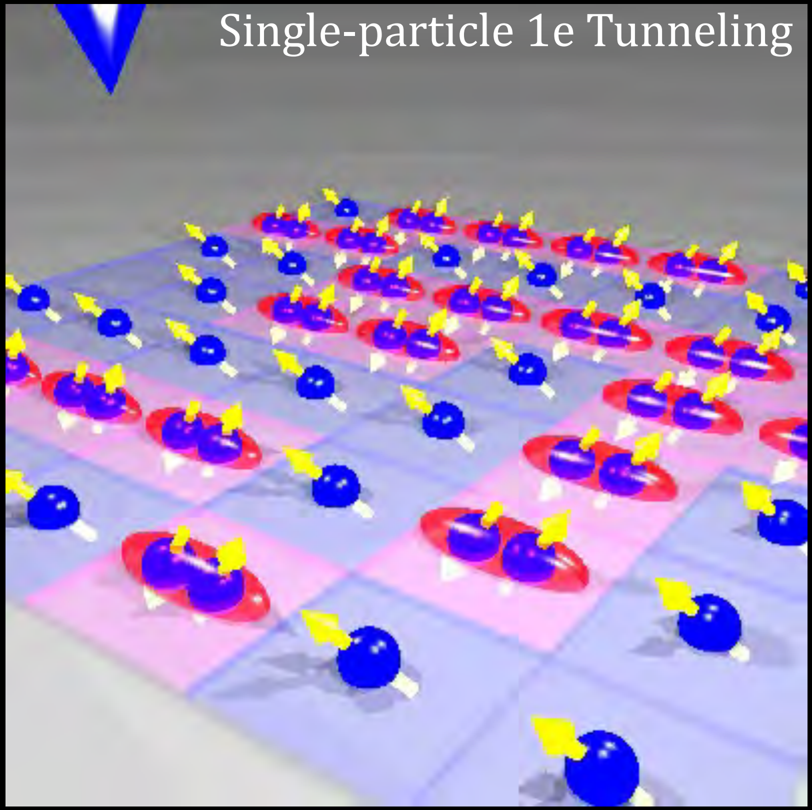
SISTM & SJTM

SISTM

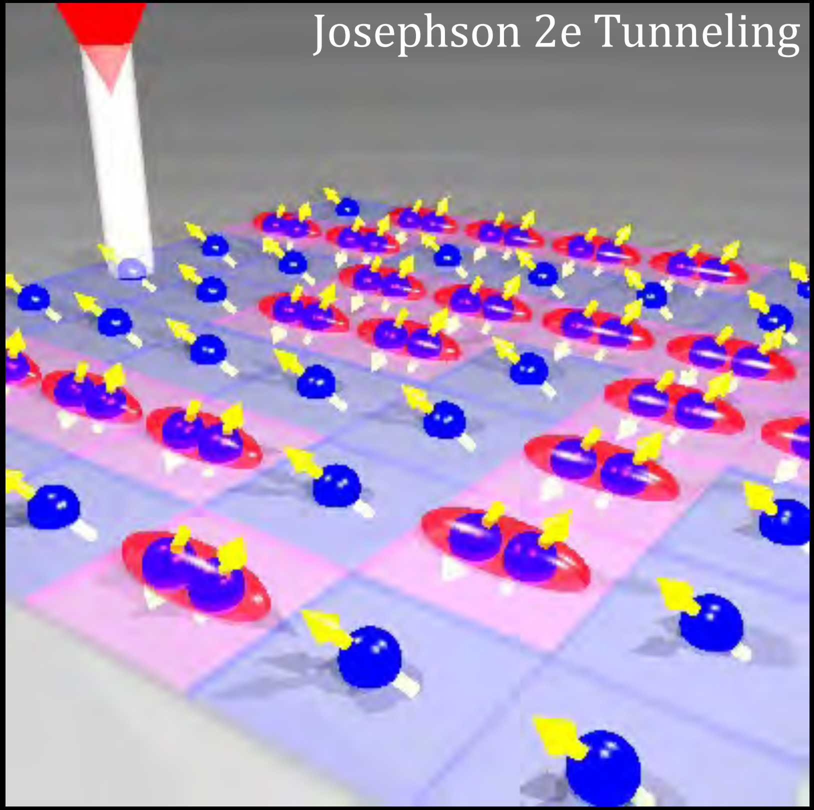
NORMAL METAL TIP



VISUALIZE QUASIPARTICLES

SJTM

SUPERCONDUCTING TIP



VISUALIZE ELECTRON-PAIRS

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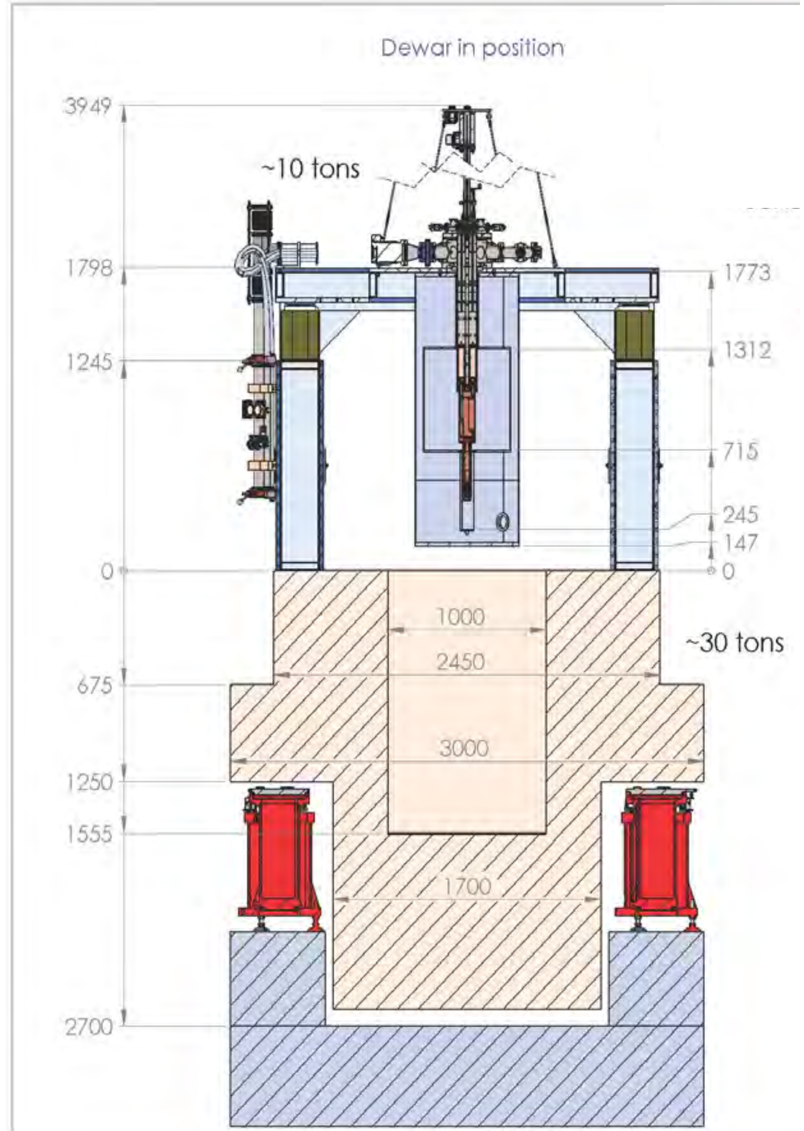
ULTRA LOW VIBRATION LABS & CRYOSTATS



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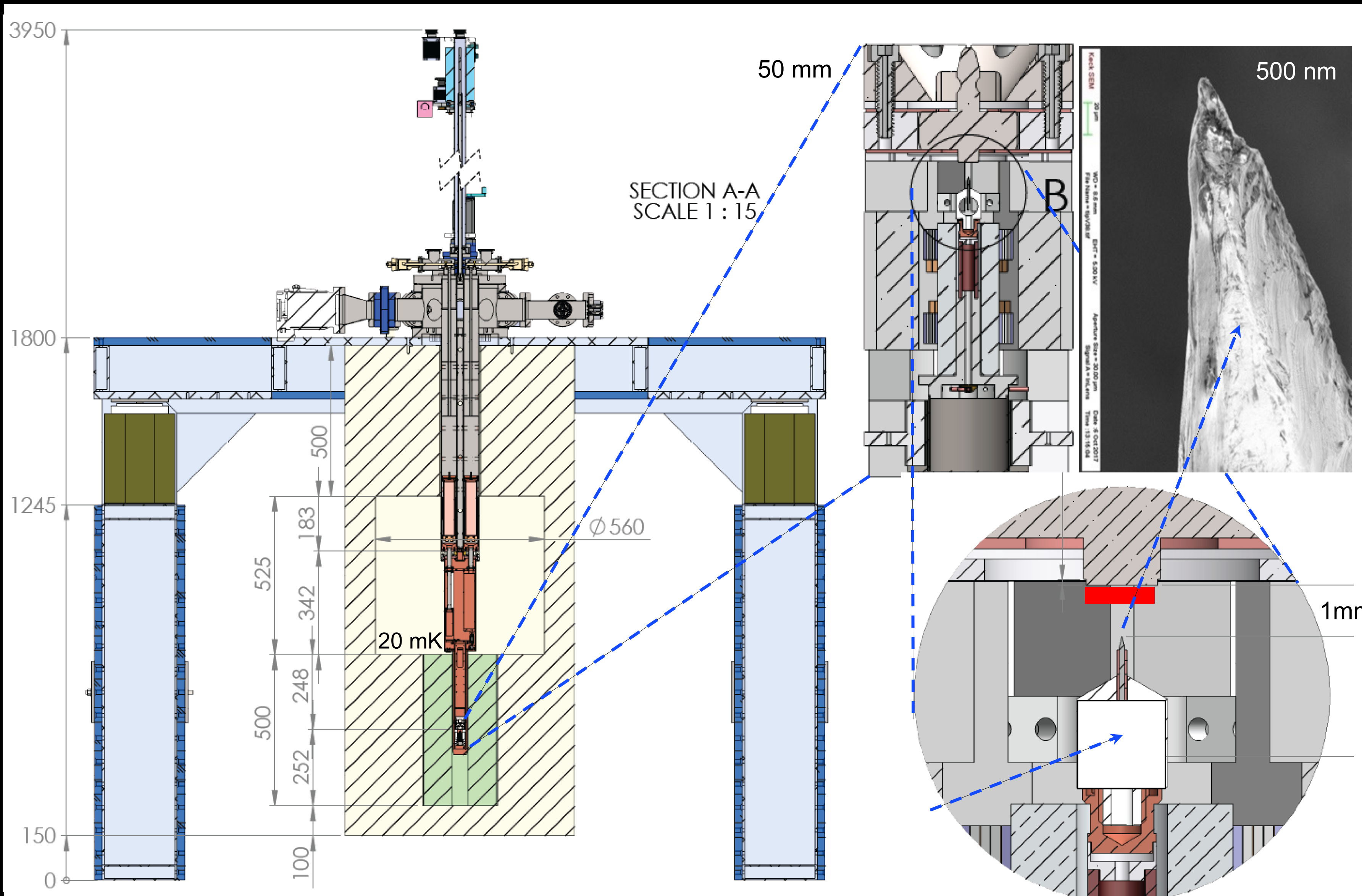
MILLIKELVIN ULV SISTM/SJTM MICROSCOPE



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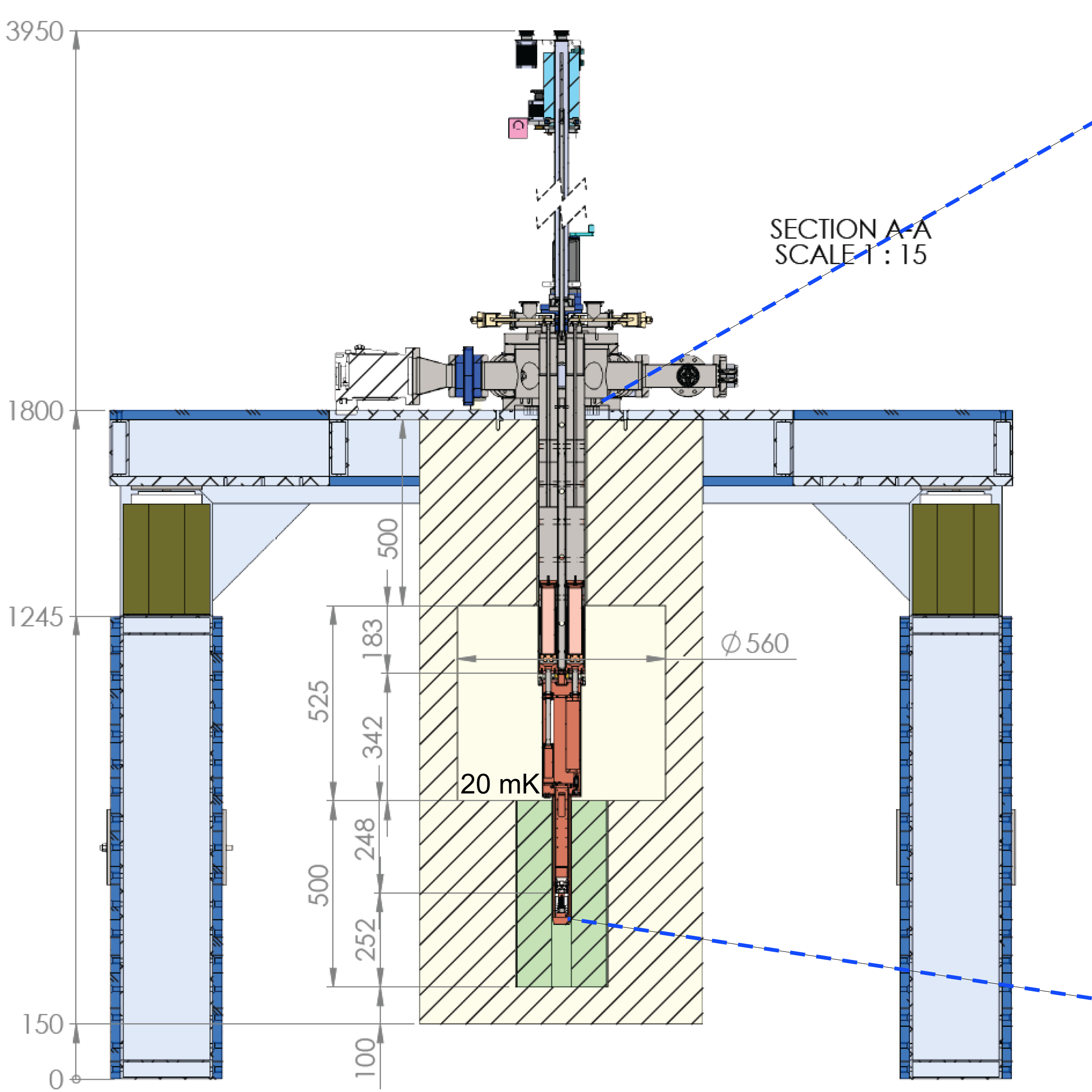
MILLIKELVIN ULV SISTM/SJTM MICROSCOPE



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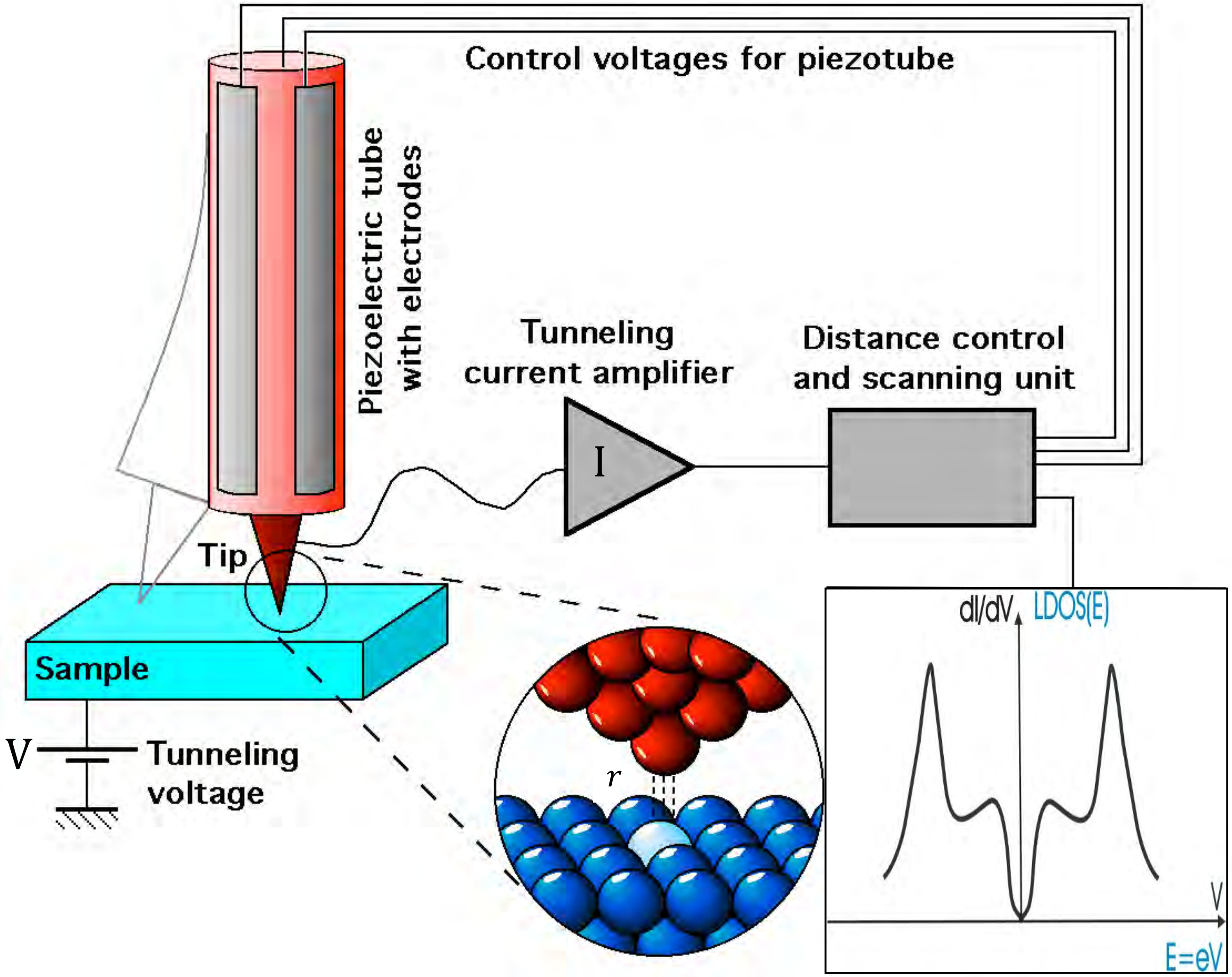
SPECTROSCOPIC IMAGING STM



$$g((r,V)) \equiv dI/dV(r,V) \propto N(r,E=eV)$$

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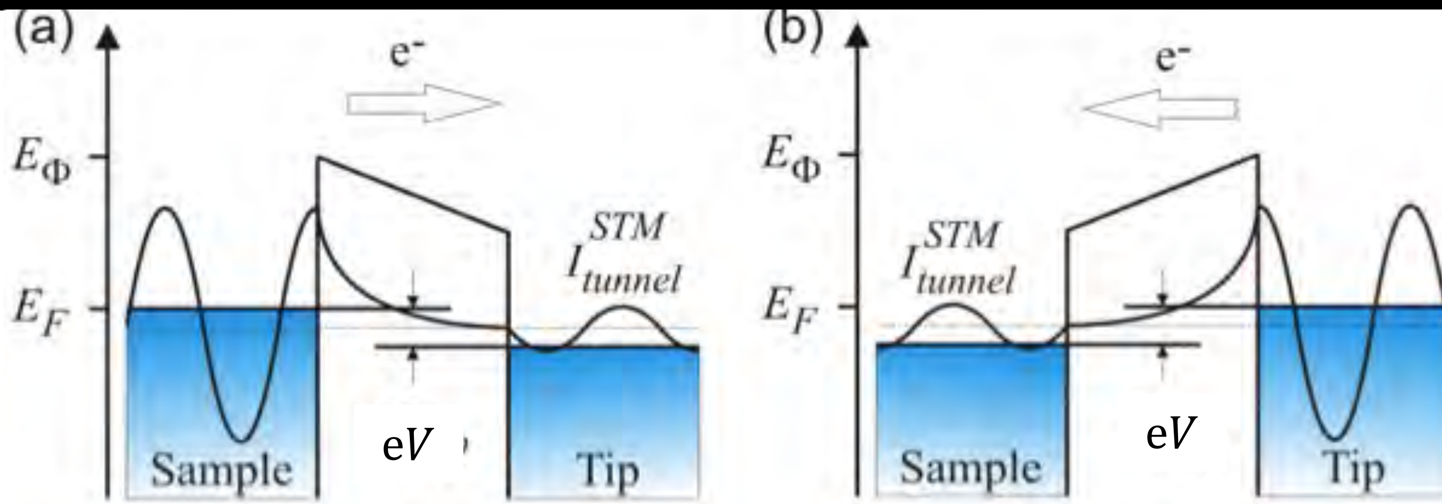
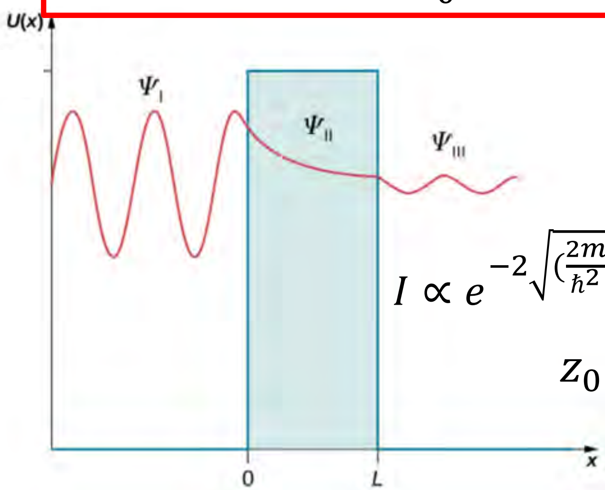
SPECTROSCOPIC IMAGING STM

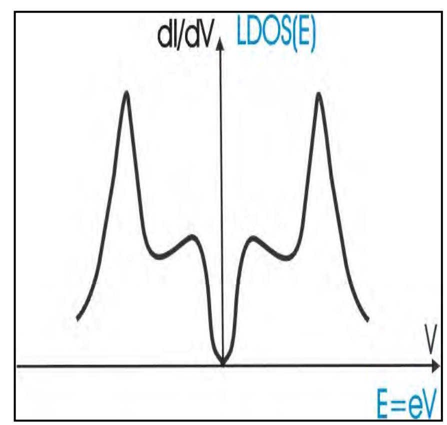


$$I(r,V) = Ce^{\frac{-z(r)}{z_0}} \int_0^{eV} N(r,\varepsilon)d\varepsilon \qquad dI/dV(r,V) = Ce^{\frac{-z(r)}{z_0}} N(r,E)$$



$$I \propto e^{-2\sqrt{(\frac{2m}{\hbar^2})(E_\Phi - E_F)z}} = e^{\frac{-z(r)}{z_0}}$$

$$z_0 \approx 1\text{Å}$$



$$g((r,V)) \equiv dI/dV(r,V) \propto N(r,E=eV)$$

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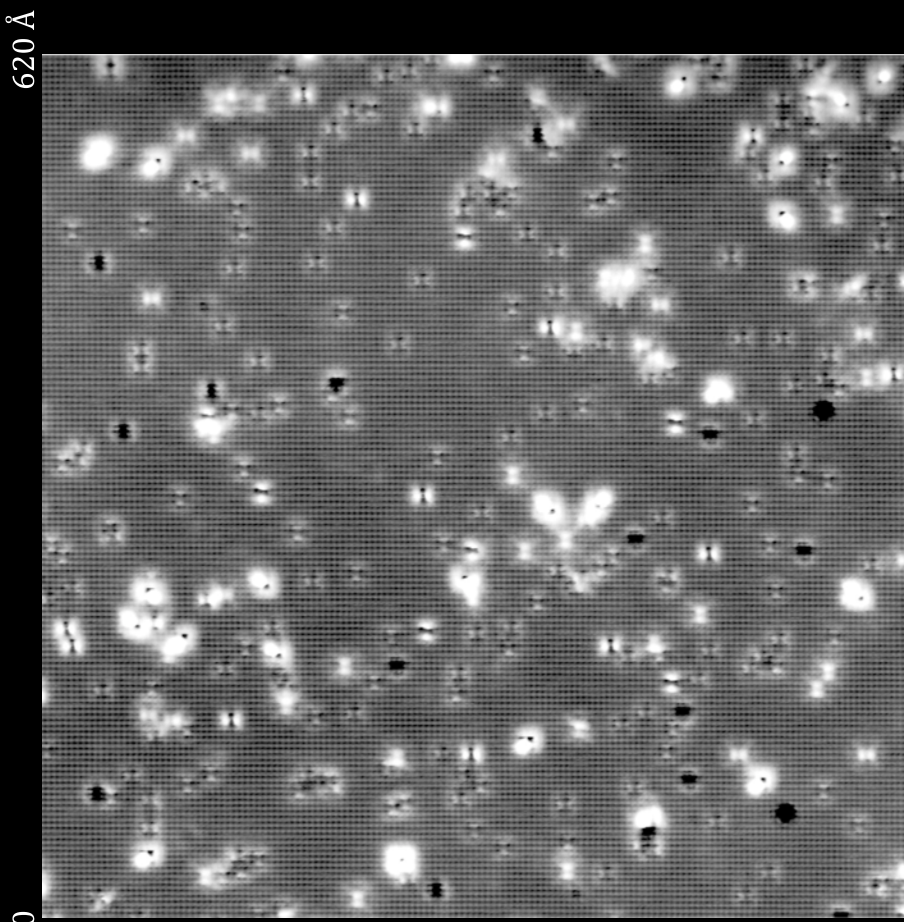
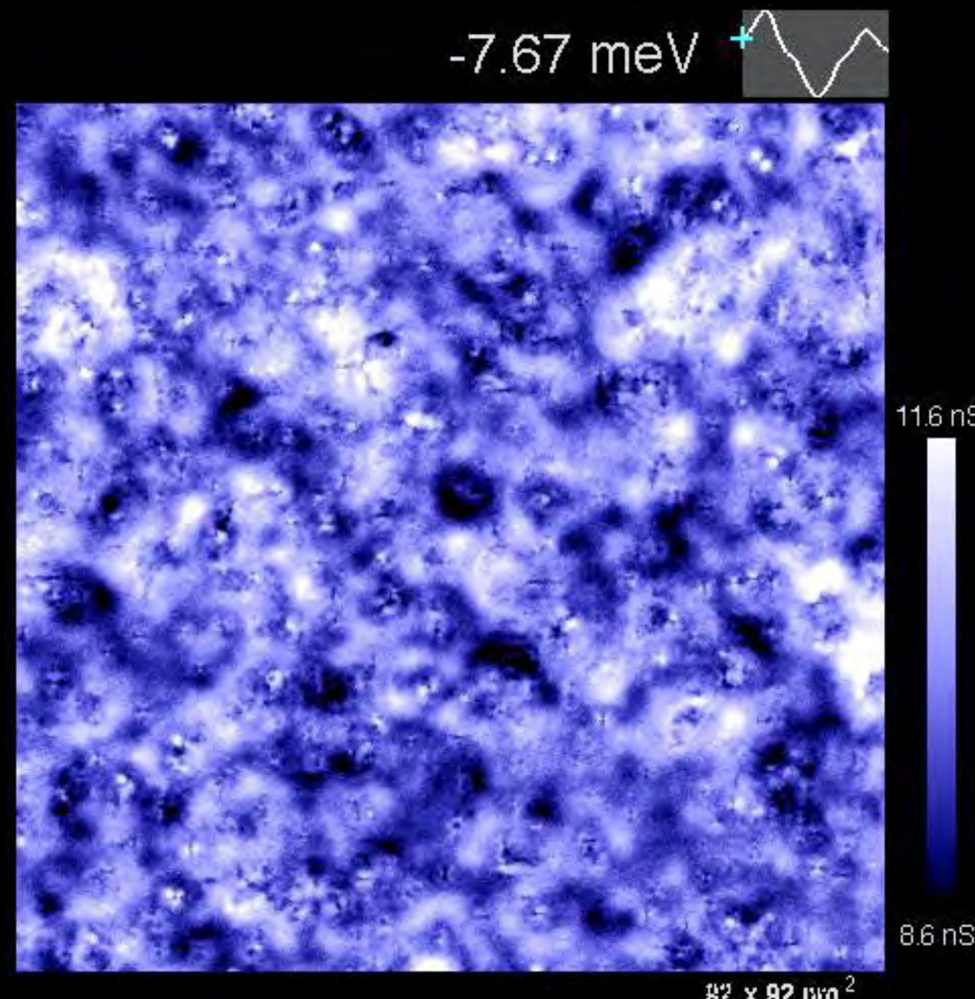
DENSITY OF STATES $N(r, E)$

dI/dV spectrum at every atom $\longrightarrow$ Atomically & Energy-Resolved $N(r, E)$



T(r)



$dI/dV(r, V) \propto N(r, E)$

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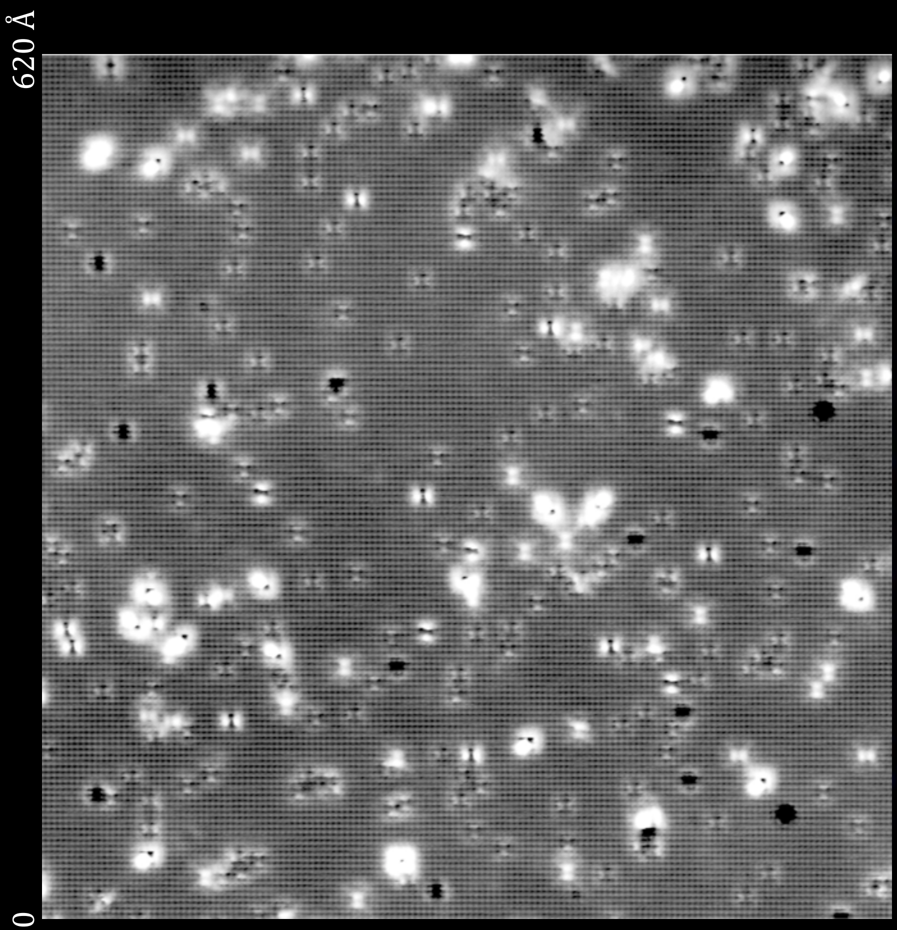
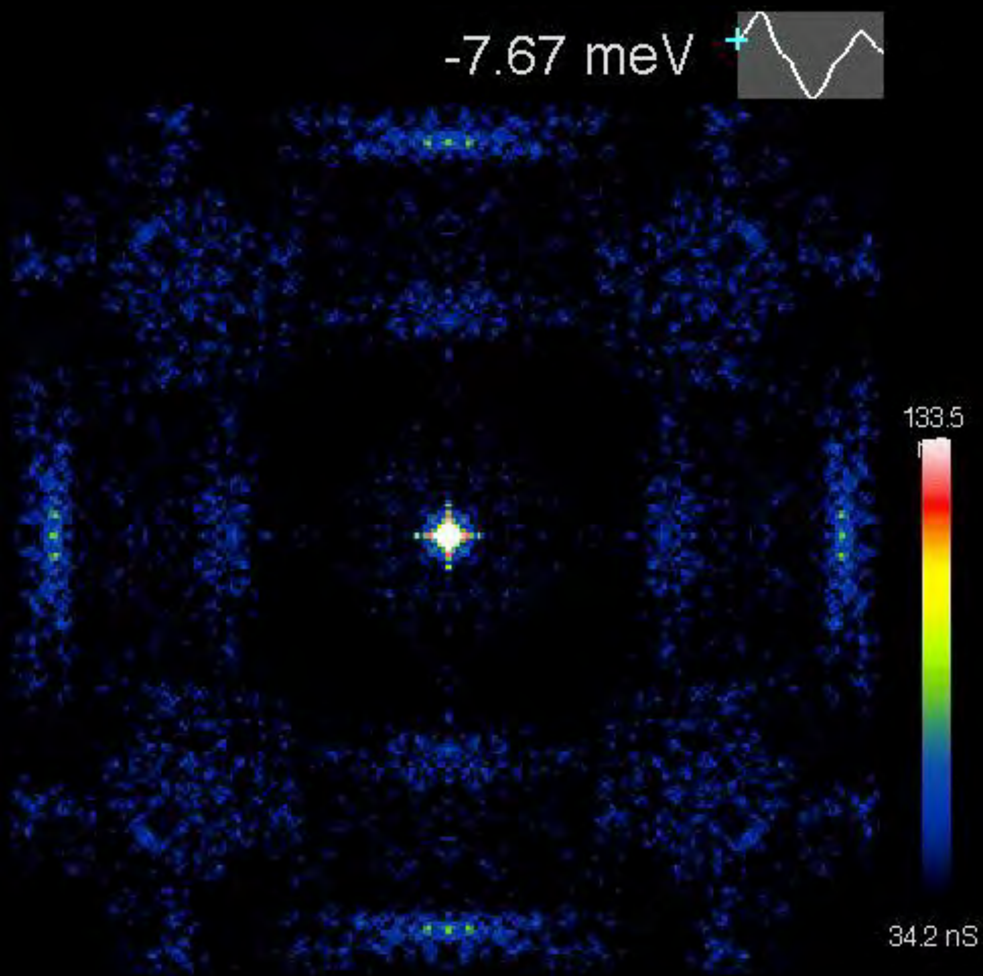
QUASIPARTICLE INTERFERENCE $N(q, E)$

dI/dV spectrum at every atom → Energy-Resolved $N(q, E)$



T(r)



$N(q, E) \propto ImG(k, E)TG(k - q, E)$

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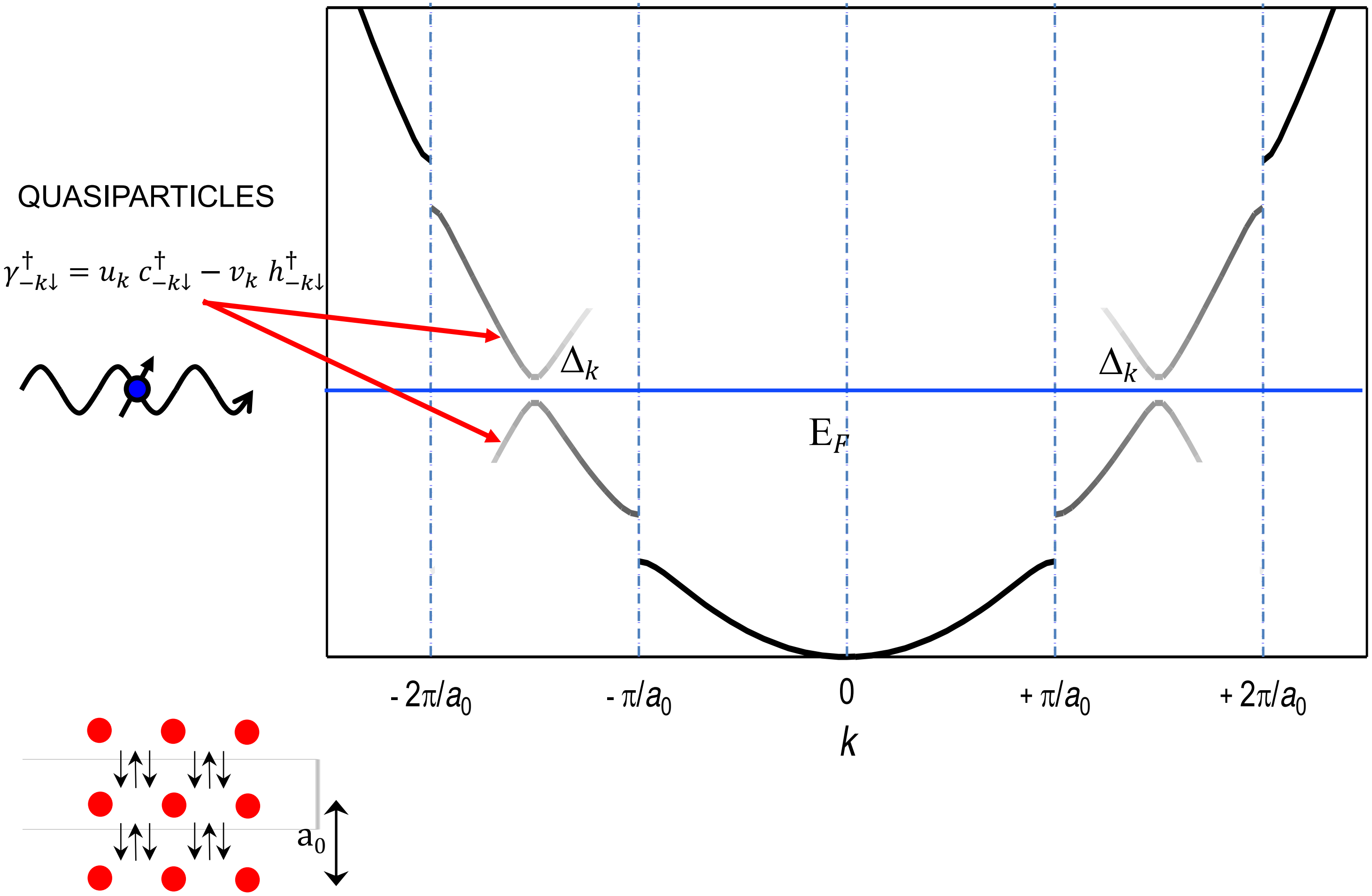
QUASIPARTICLE VISUALIZATION $N(r,E) : N(q,E)$

$$\psi = \langle c^\dagger_{k\uparrow}\, c^\dagger_{-k\downarrow} \rangle \qquad \Psi = \prod_k (u_k + v_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow})|0>$$

QUASIPARTICLES

$$\gamma^\dagger_{-k\downarrow} = u_k\, c^\dagger_{-k\downarrow} - v_k\, h^\dagger_{-k\downarrow}$$



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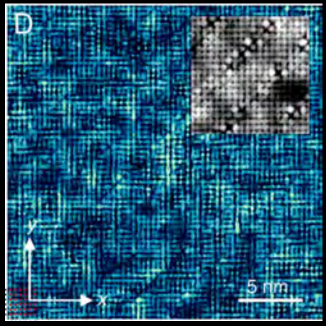
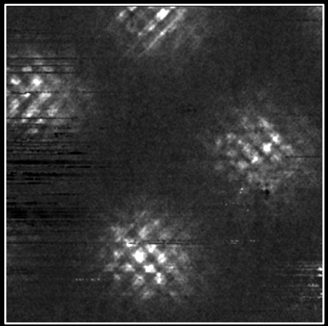
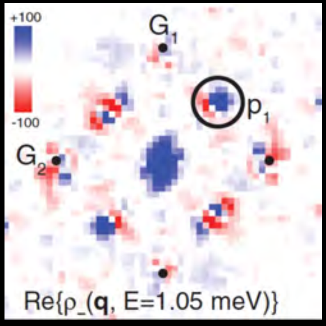
QUASIPARTICLE VISUALIZATION $N(r,E) : N(q,E)$



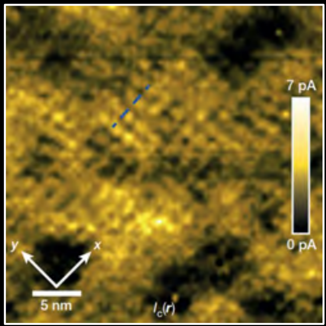
Nature 570, 484 (2019)



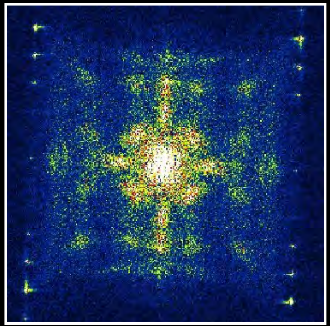
Science 364, 976 (2019)



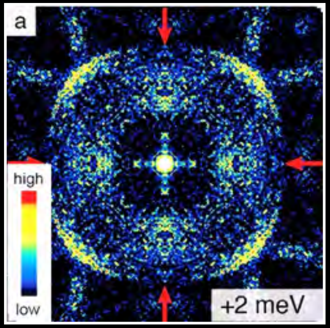
Science 357, 75 (2017)



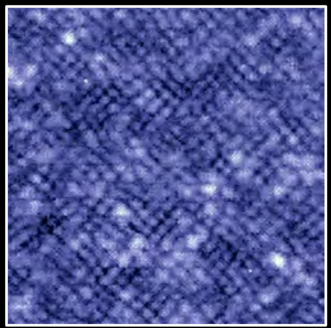
Nature 532, 343 (2016)



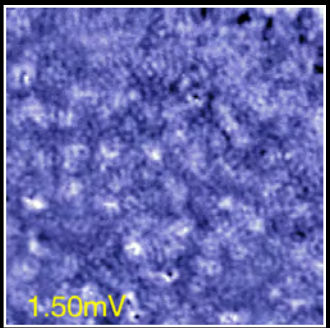
Science 344, 612 (2014)



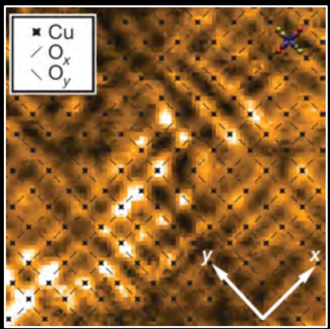
Science 336, 563 (2012)



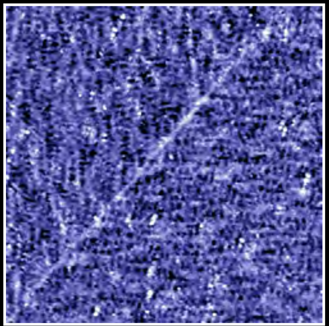
Science 333, 426 (2011)



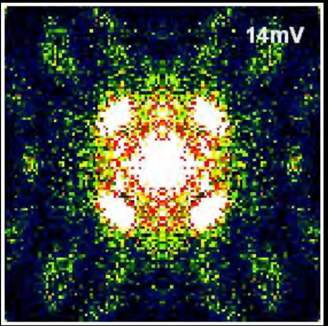
Nature 466, 374 (2010)



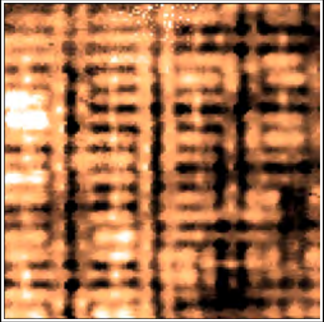
Nature 465, 570 (2010)



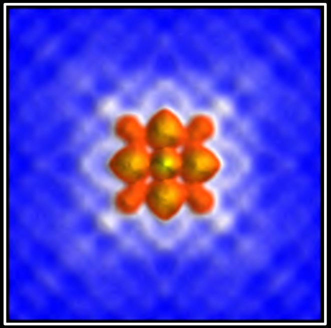
Science 327, 181 (2010)



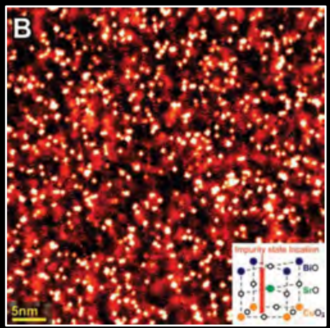
Nature 454, 1072 (2008)



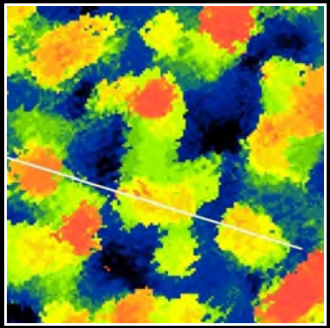
Science 315, 1380(2007)



Nature 442, 546 (2006)



Science 309, 1048 (2005)



Science 297, 1148 (2002)

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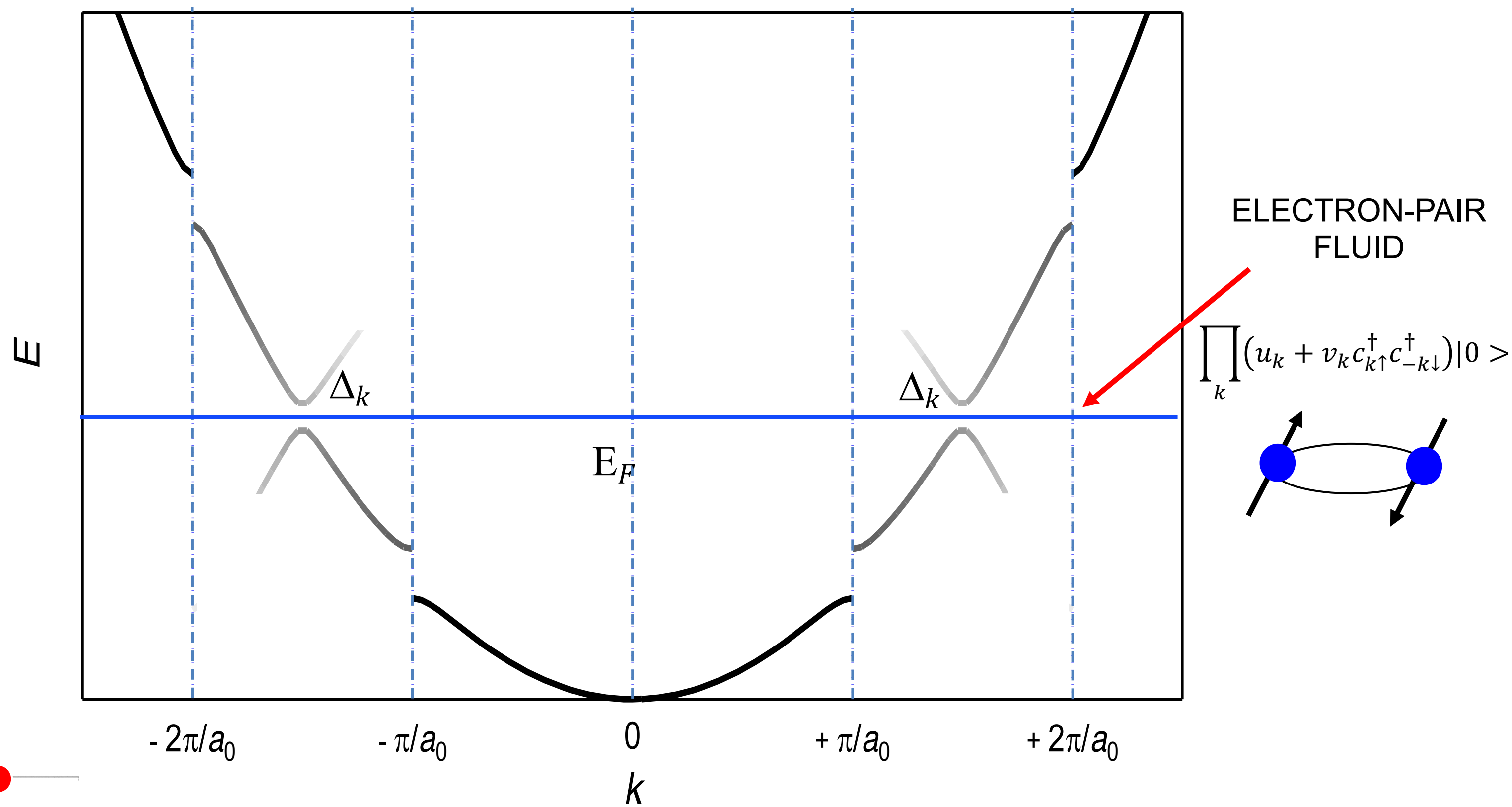
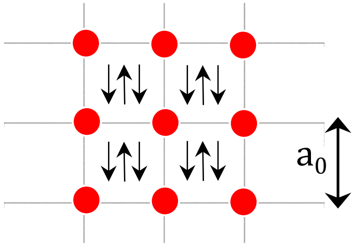
ELECTRON-PAIR VISUALIZATION $n_P(r)$

$$\psi = \langle c^{\dagger}_{k\uparrow}\, c^{\dagger}_{-k\downarrow} \rangle \qquad \Psi = \prod_k (u_k + v_k c^{\dagger}_{k\uparrow} c^{\dagger}_{-k\downarrow})|0>$$





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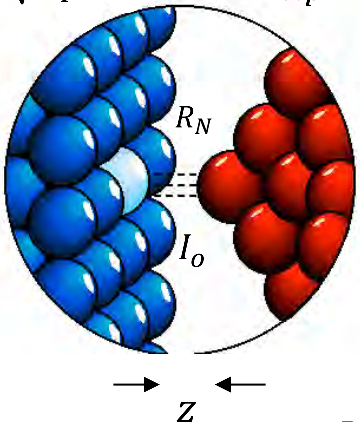
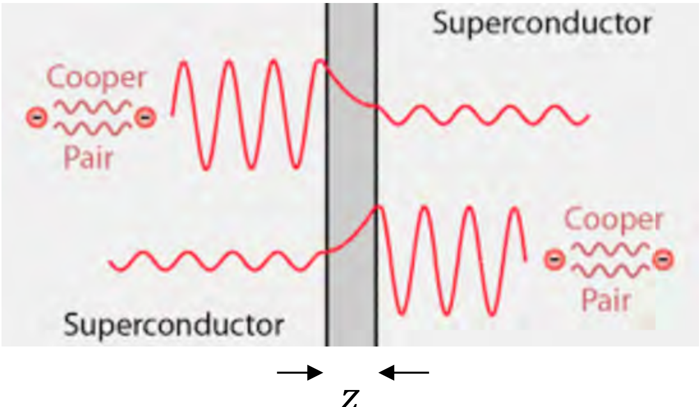
SJTM VISUALIZATION $n_P(r)$ – IMPOSSIBLE!

$\Psi_{sample} = \sqrt{n_P} e^{i\theta_s}$ $\Psi_{tip} = \sqrt{n_t} e^{i\theta_t}$



$$J = J_0 Sin(\theta_t - \theta_s)$$

$\Psi_{sample} = \sqrt{n_P} e^{i\theta_s}$ $\Psi_{tip} = \sqrt{n_t} e^{i\theta_t}$



$$J_0 = \frac{e\hbar}{m}\left\{\frac{2}{\lambda} e^{-2z/\lambda}\right\}\sqrt{n_P}\sqrt{n_t}$$

ELECTRON-PAIR CURRENT	$I_0 R_N \propto \sqrt{n_P}\sqrt{n_t} \implies I_0^2(r) R_N^2(r) \propto n_P(r)$	VISUALIZE ELECTRON-PAIR DENSITY
AMBEGAOKAR-BARATOFF	$I_0 R_N \approx \frac{\pi\Delta}{2e} \approx 1.5\, mV\ @\ \Delta = 1 meV$	$(T \to 0; \sqrt{n_P} = \sqrt{n_t})$
STM TUNNEL JUNCTION	$I_0 \approx 1.5\, pA \quad @\ R_N = 1\, G\Omega$	$\implies E_J = \frac{\hbar I_0}{2e} \approx 5\, neV \to 30\, \mu K$

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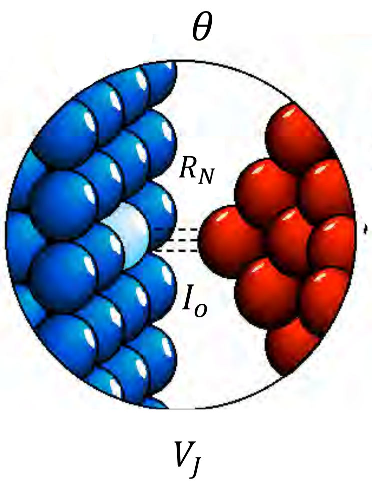
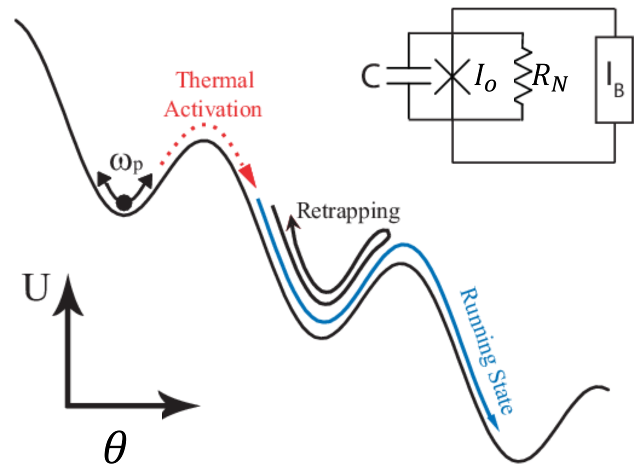
SJTM VISUALIZATION $n_P(r)$



$$\frac{\hbar C}{2e}\ddot{\theta} + \frac{\hbar}{2eR_N}\dot{\theta} + I_o Sin\theta = I_B$$

$$I = I_o \int_{-\infty}^{\infty} d\theta \, Sin\theta \, \mathrm{W}(\theta_t, \theta_o)$$



Ivanchenko & Zilberman, ***JETP 28***, 1272 (1969)

$kT >> E_J$ and $C \rightarrow 0$

$U(\theta) = E_J(1 - Cos\theta - I_B\theta/I_o)$

ELECTRON-PAIR CURRENT

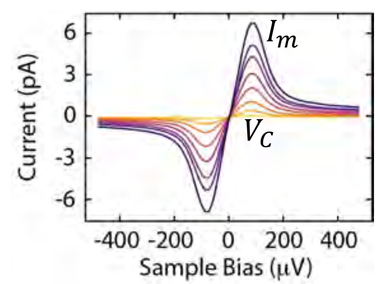
$$I(V_J) = \frac{1}{2} I_0^2 Z \frac{V_J}{V_J^2 + V_C^2}$$

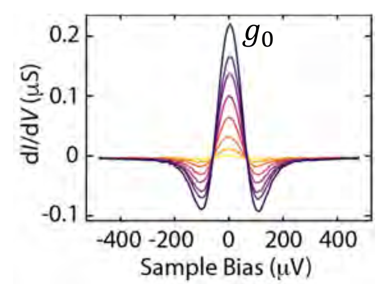
ELECTRON-PAIR CURRENT MAX.

$$I_m = \frac{\hbar}{8ek_BT^*} I_0^2$$

ELECTRON-PAIR V=0 CONDUCTANCE

$$g_0 \equiv \frac{dI}{dV_J}\bigg|_{V=0} = \frac{\hbar I_m}{ek_BT^*Z}$$





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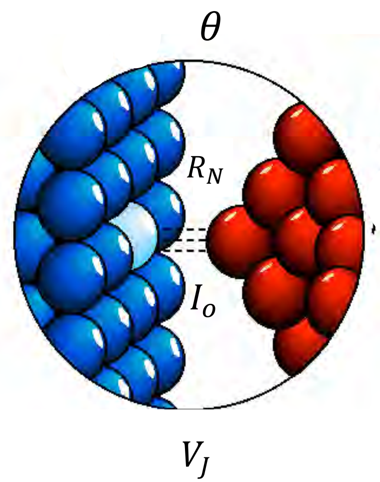
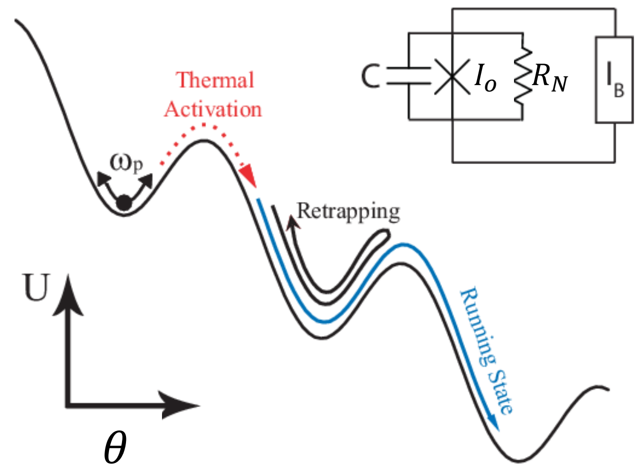
SJTM VISUALIZATION $n_P(r)$ – CHALLENGING!



$$\frac{\hbar C}{2e}\ddot{\theta} + \frac{\hbar}{2eR_N}\dot{\theta} + I_o Sin\theta = I_B$$

$$I = I_o \int_{-\infty}^{\infty} d\theta\, Sin\theta\, \mathrm{W}(\theta_t, \theta_o)$$



Ivanchenko & Zilberman, ***JETP 28***, 1272 (1969)

$kT >> E_J$ *and* $C \to 0$

$U(\theta) = E_J(1 - Cos\theta - I_B\theta/I_o)$

ELECTRON-PAIR CURRENT

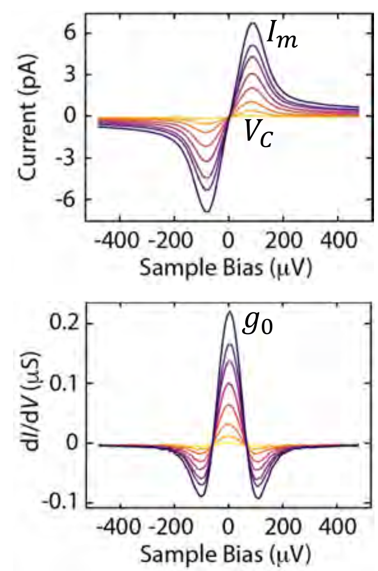
$$I(V_J) = \frac{1}{2} I_0^2 Z \frac{V_J}{V_J^2 + V_C^2}$$

ELECTRON-PAIR CURRENT MAX.

$$I_m = \frac{\hbar}{8ek_B T^*} I_0^2$$

ELECTRON-PAIR V=0 CONDUCTANCE

$$g_0 \equiv \left.\frac{dI}{dV_J}\right|_{V=0} = \frac{\hbar I_m}{ek_B T^* Z}$$



VISUALIZE ELECTRON-PAIR DENSITY

$$n_P(r) \propto I_m(r) {R_N}^2(r)$$

$$n_P(r) \propto g_0(r) {R_N}^2(r)$$

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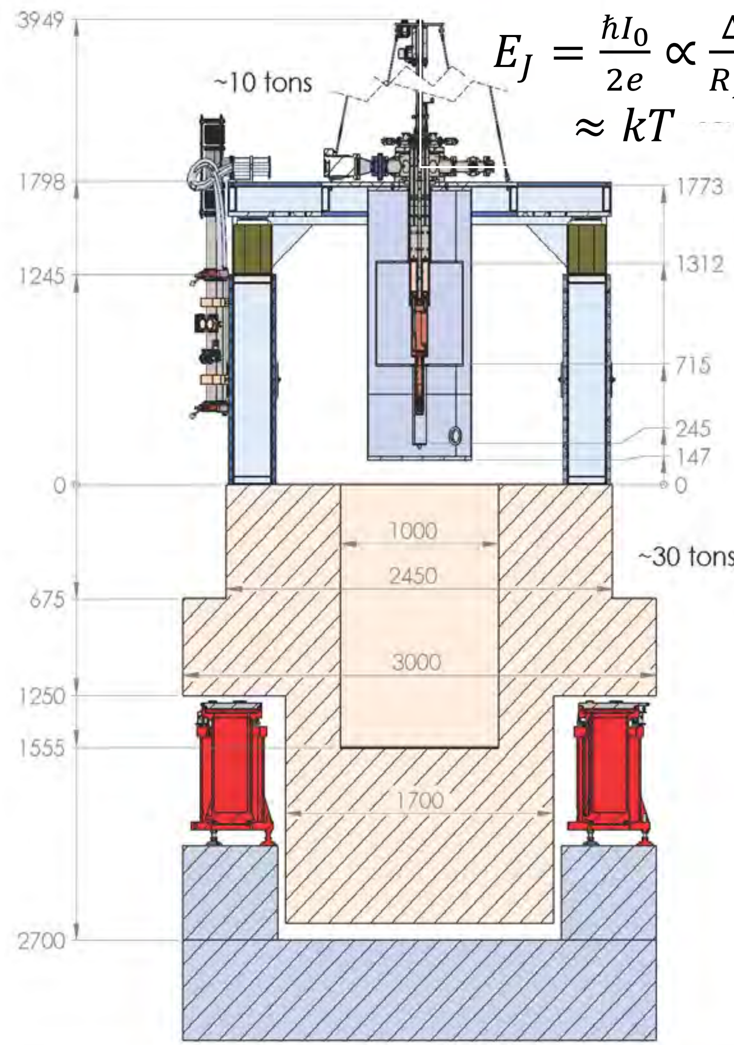
ULTRA-LOW VIBRATION & TEMPERATURE

ULTRA LOW VIBRATION LAB

ULTRA LOW VIBRATION CRYOSTAT

$$E_J = \frac{\hbar I_0}{2e} \propto \frac{\Delta}{R_N} \approx kT$$



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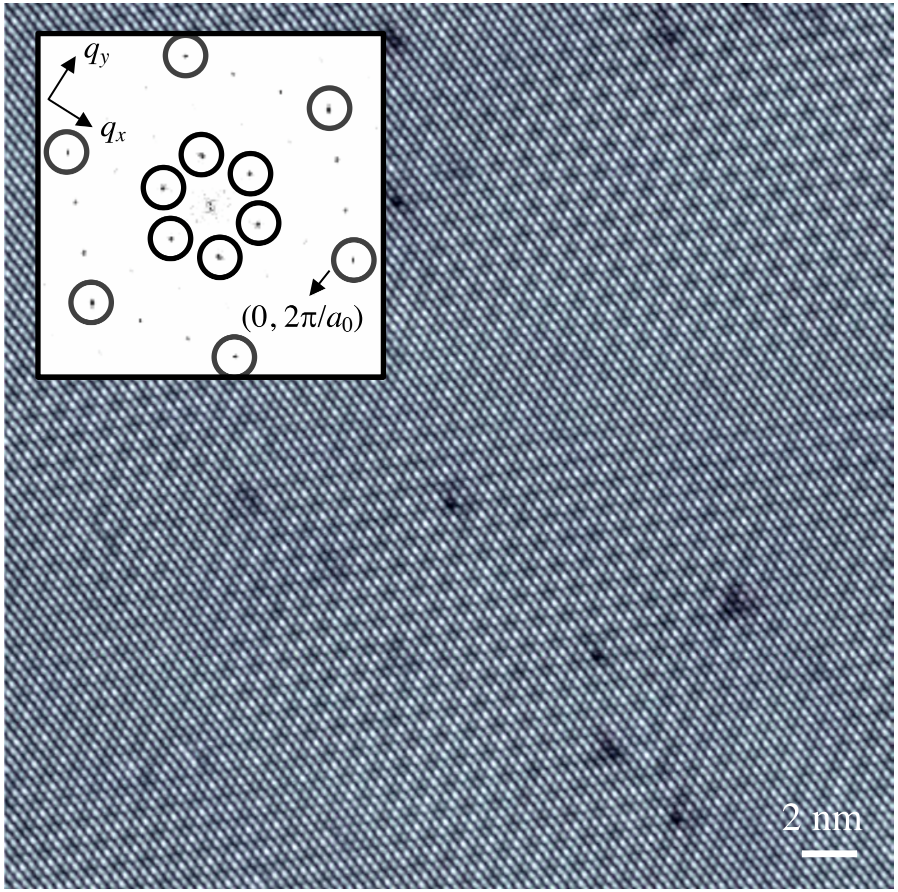
TMD PAIR DENSITY WAVE STATE

DO PDW STATES EXIST IN TMD?

CHARGE DENSITY WAVE + SUPERCONDUCTOR



T(r, -20 mV)

Science 372, 1447 (2021)

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SJTM EXAMPLE : SC+CDW @ $NbSe_2$

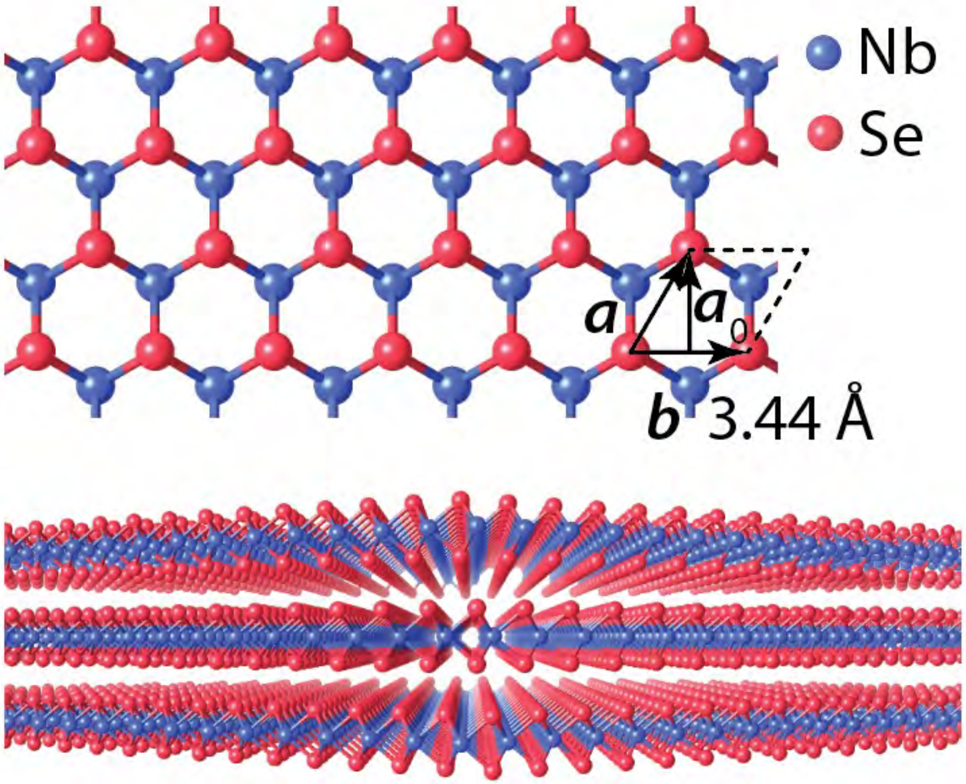
SUPERCONDUCTOR

CHARGE DENSITY WAVE

$T_C \approx 7.2$ K

$T_{CDW} \approx 33$ K

$\Delta_0 \approx 1.2$ meV



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SJTM EXAMPLE : SC+CDW @ $NbSe_2$

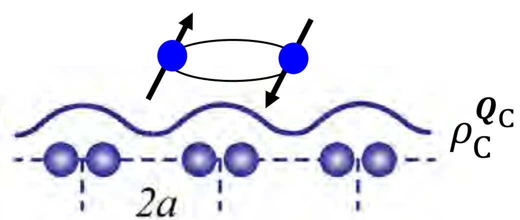
SUPERCONDUCTOR

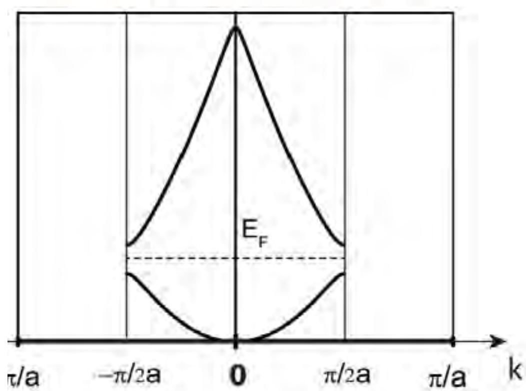
$$\Delta_S = \Delta_0 e^{i\phi}$$

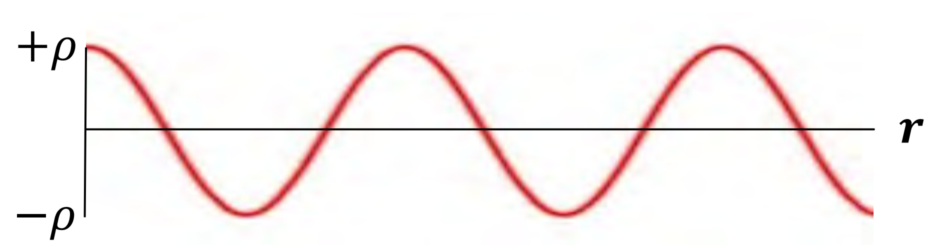
CHARGE DENSITY WAVE

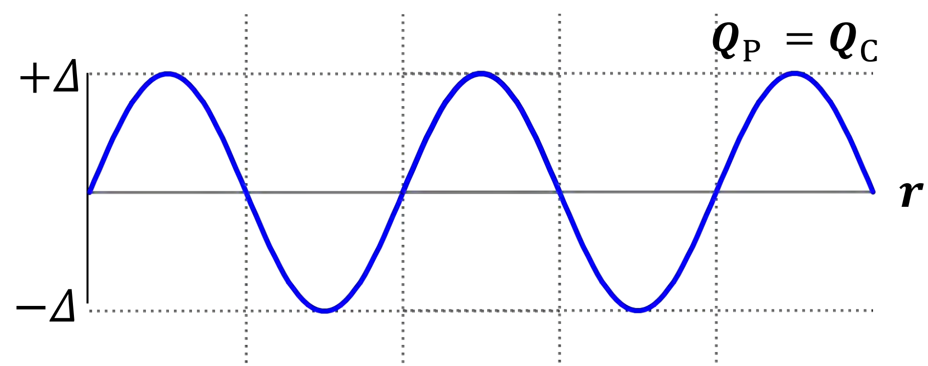
$$\rho_C^{\boldsymbol{Q}_C}(\boldsymbol{r}) = \rho e^{i\boldsymbol{Q}_C\cdot\boldsymbol{r}} + \rho^* e^{-i\boldsymbol{Q}_C\cdot\boldsymbol{r}}$$

$$\mathcal{F} = \mathcal{F}_S + \mathcal{F}_C + \mathcal{F}_P - \lambda\rho_C^{Q}\Delta_S^*\Delta_P^{-Q}$$









$$\Delta_P^{\boldsymbol{Q}_P}(\boldsymbol{r}) = \left[\Delta e^{i\boldsymbol{Q}_P\cdot\boldsymbol{r}} + \Delta^* e^{-i\boldsymbol{Q}_P\cdot\boldsymbol{r}}\right] e^{i\phi}$$

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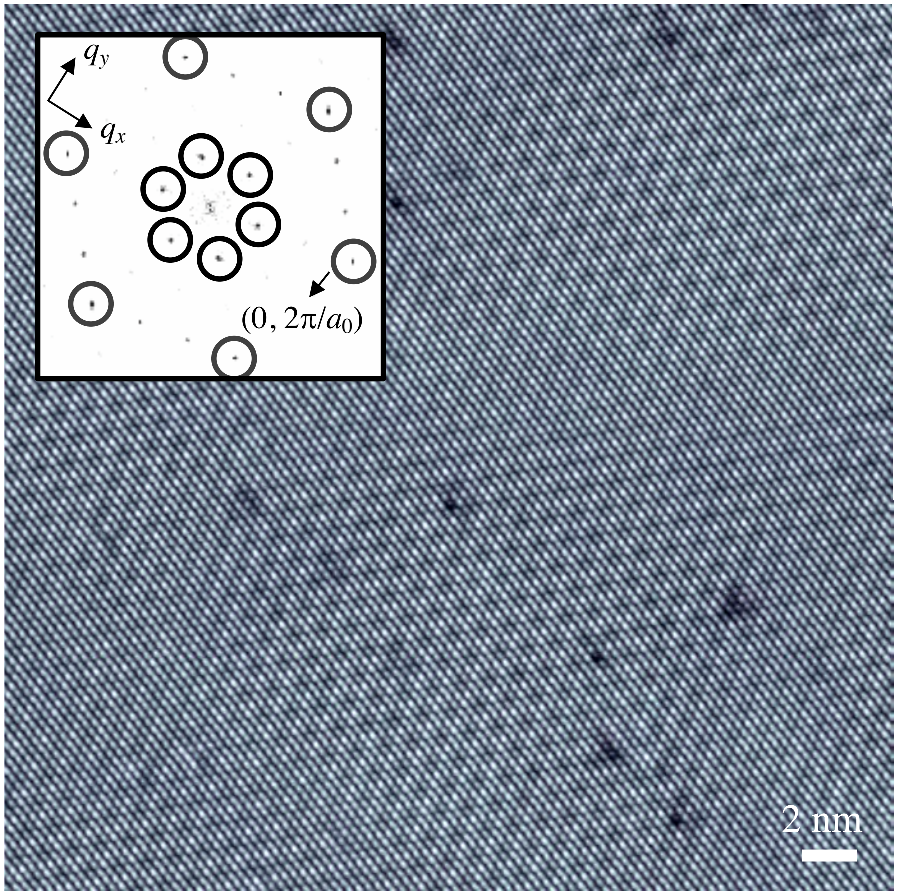
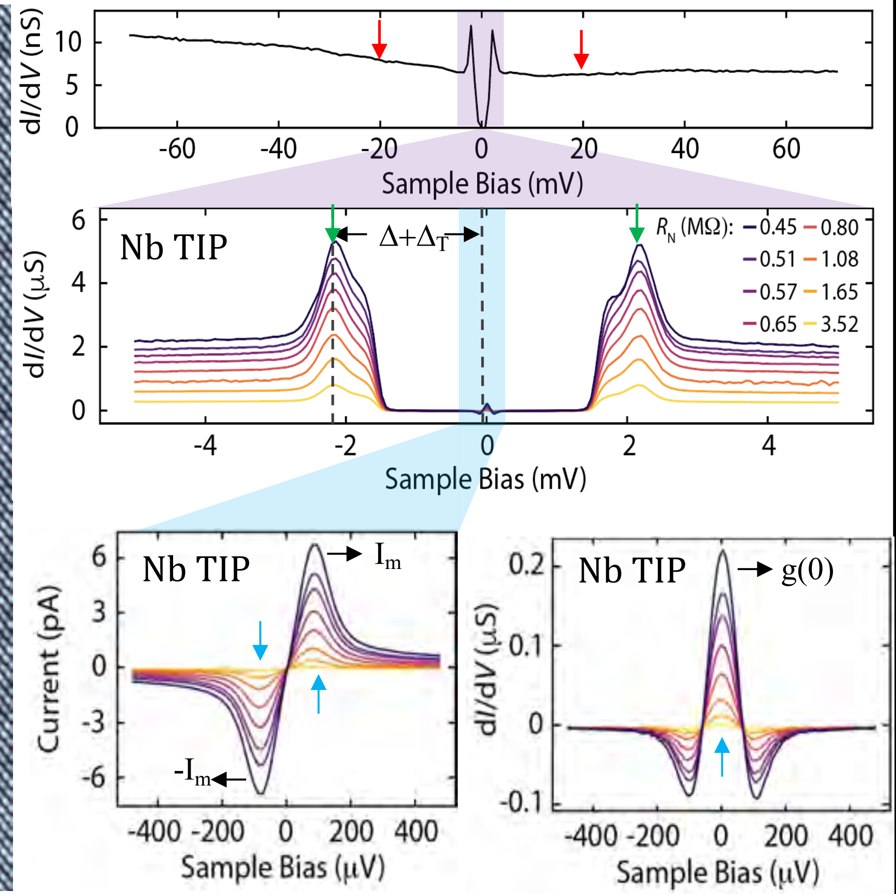
SJTM of $NbSe_2$

CHARGE DENSITY WAVE + SUPERCONDUCTOR



T(r, -20 mV)

CONDUCTANCE IMAGE DYNAMIC RANGE >10^5



T = 280mK

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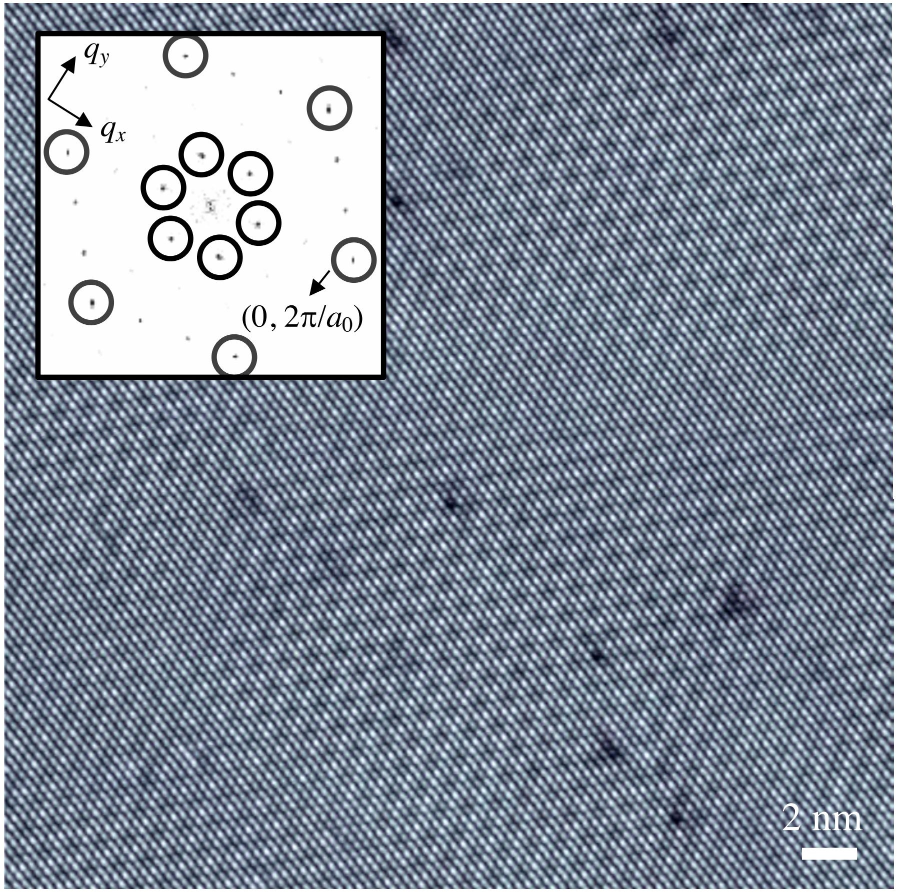
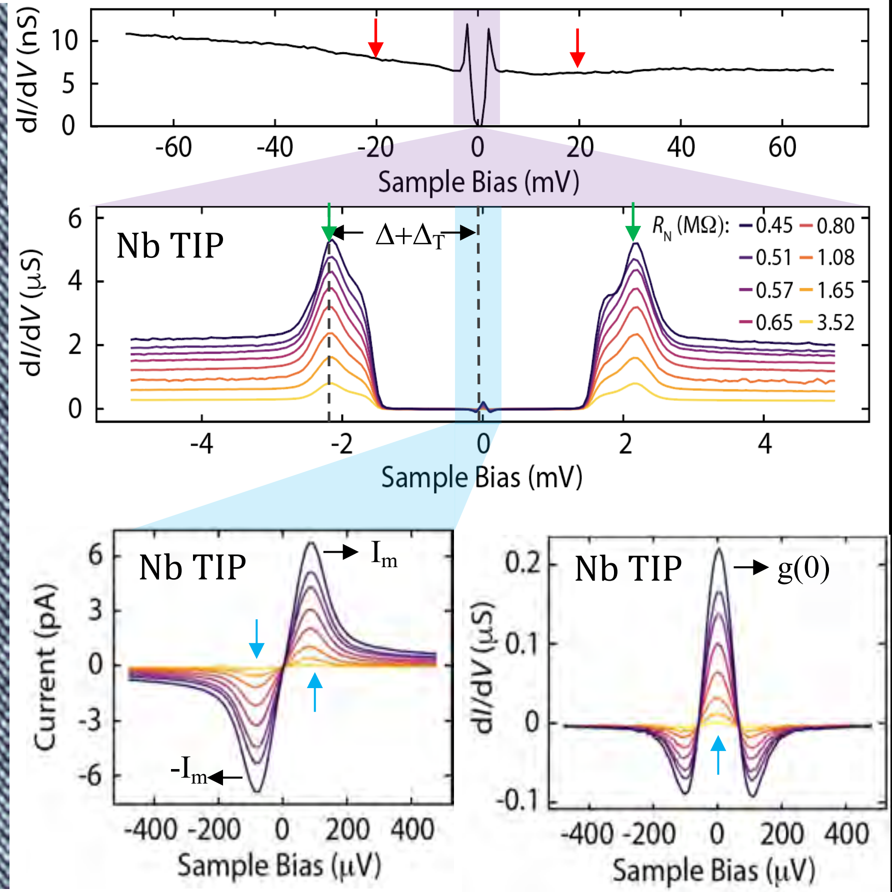
SJTM of $NbSe_2$

CHARGE DENSITY WAVE + SUPERCONDUCTOR

CONDUCTANCE IMAGE DYNAMIC RANGE >10^5



T(r, -20 mV)



T = 280mK

Science 372, 1447 (2021)

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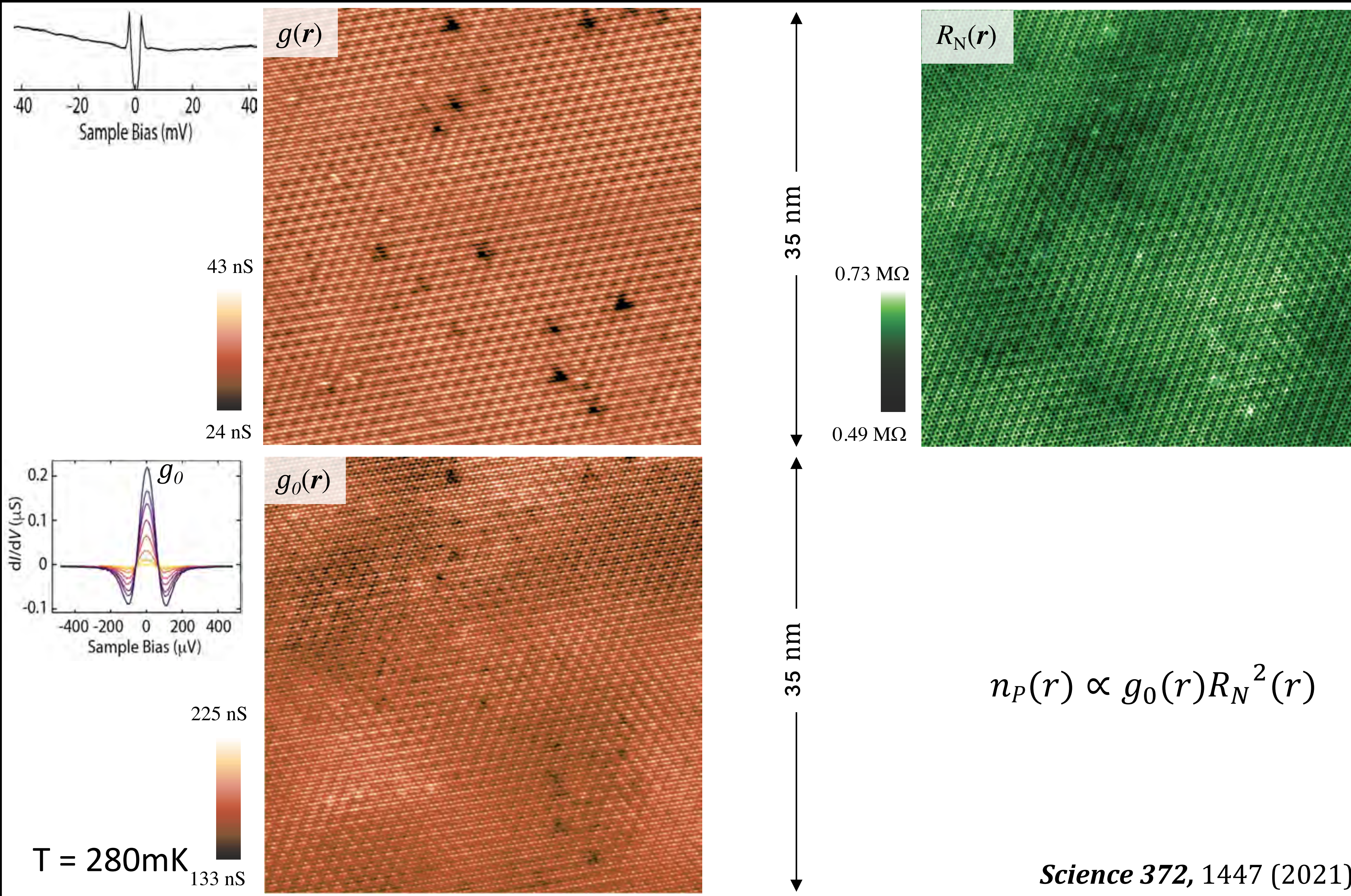
VISUALIZE ELECTRON-PAIR DENSITY $n_P(\boldsymbol{r})$



$$n_P(r) \propto g_0(r) R_N{}^2(r)$$

Science 372, 1447 (2021)

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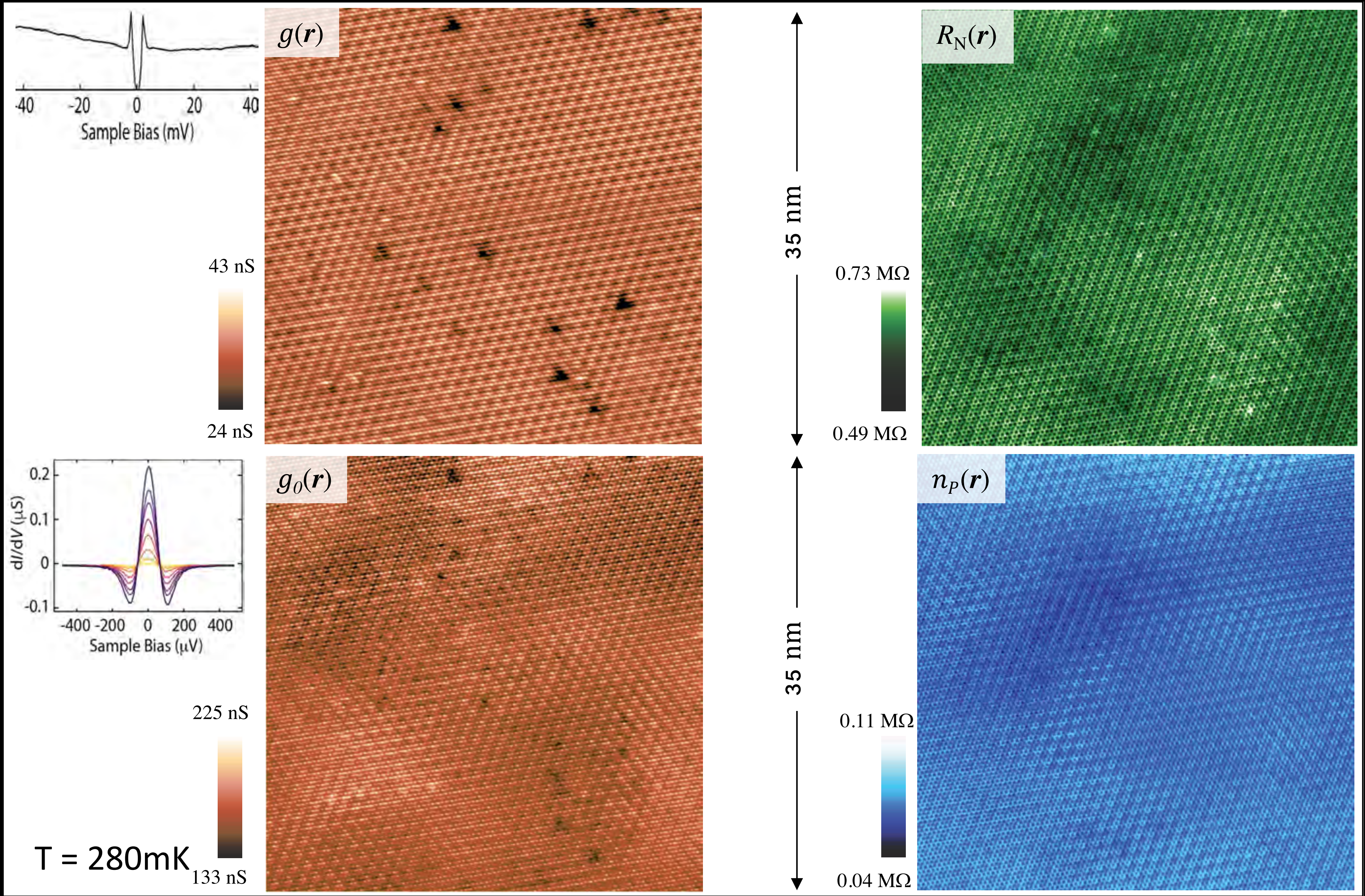
VISUALIZE ELECTRON-PAIR DENSITY $n_P(\boldsymbol{r})$



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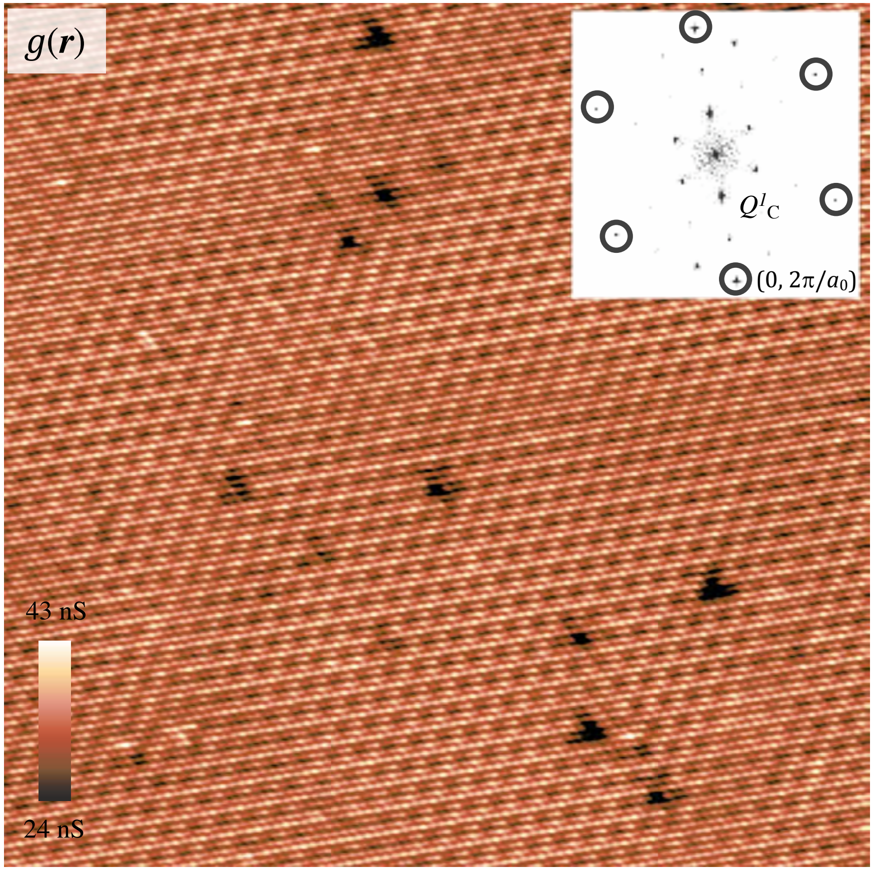
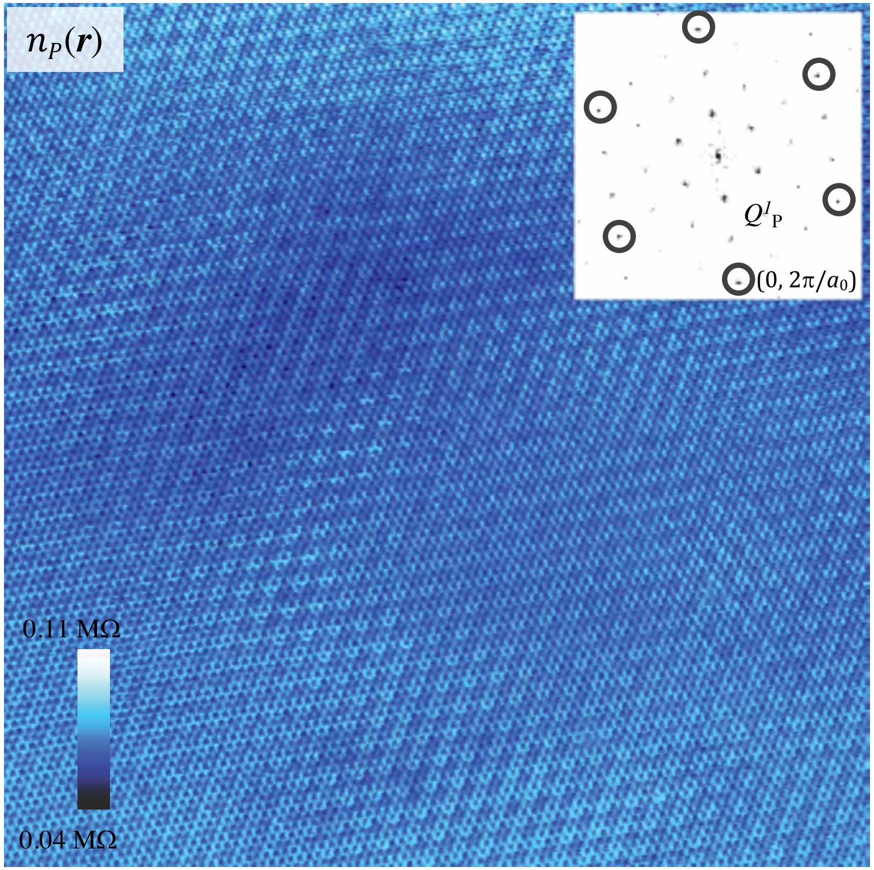
CHARGE DENSITY & PAIR DENSITY MODULATIONS

CRYSTAL LATTICE

CRYSTAL LATTICE





T = 280mK

Science 372, 1447 (2021)

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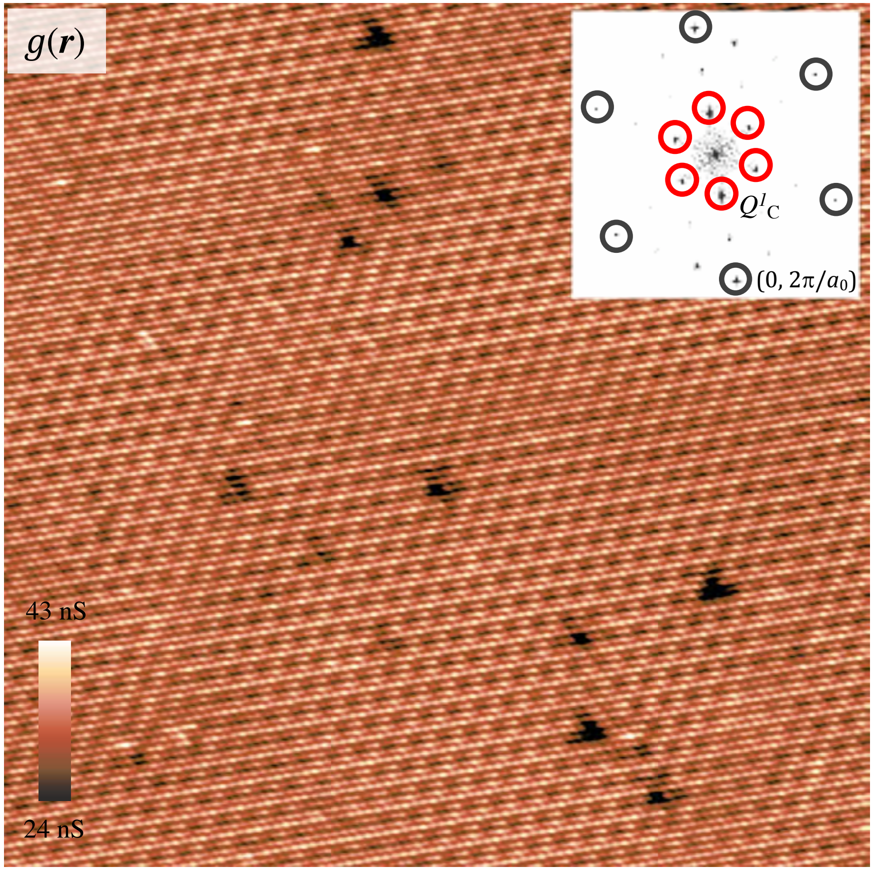
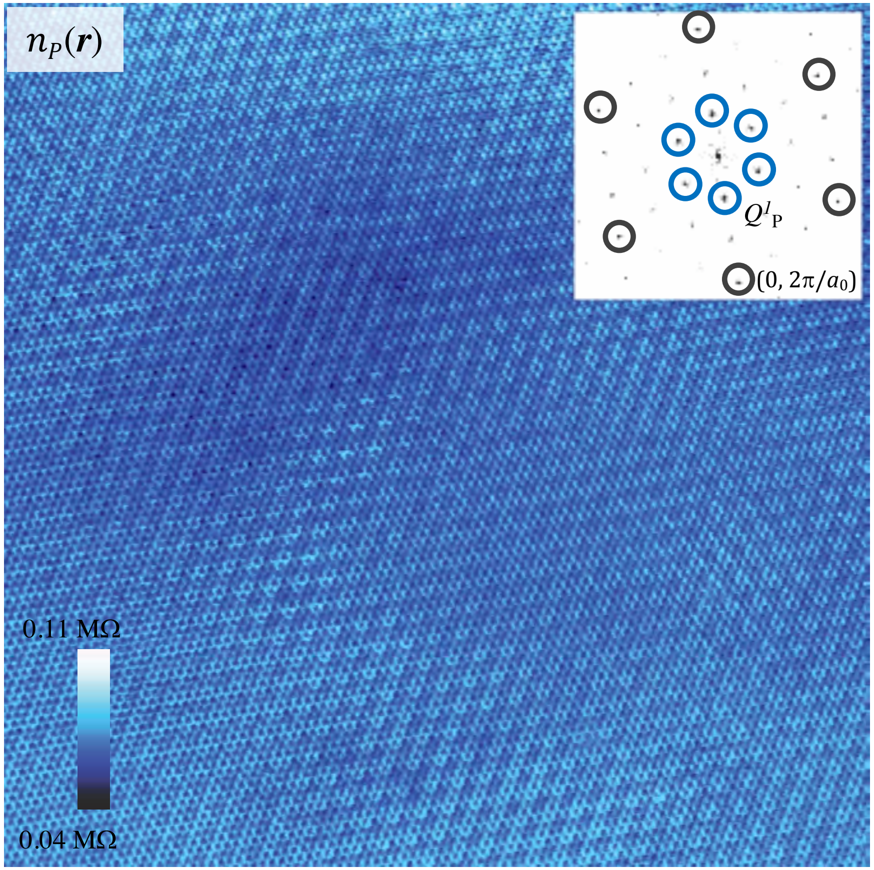
CHARGE DENSITY & PAIR DENSITY MODULATIONS

CHARGE DENSITY WAVE + CRYSTAL LATTICE



PAIR DENSITY WAVE + CRYSTAL LATTICE



T = 280mK

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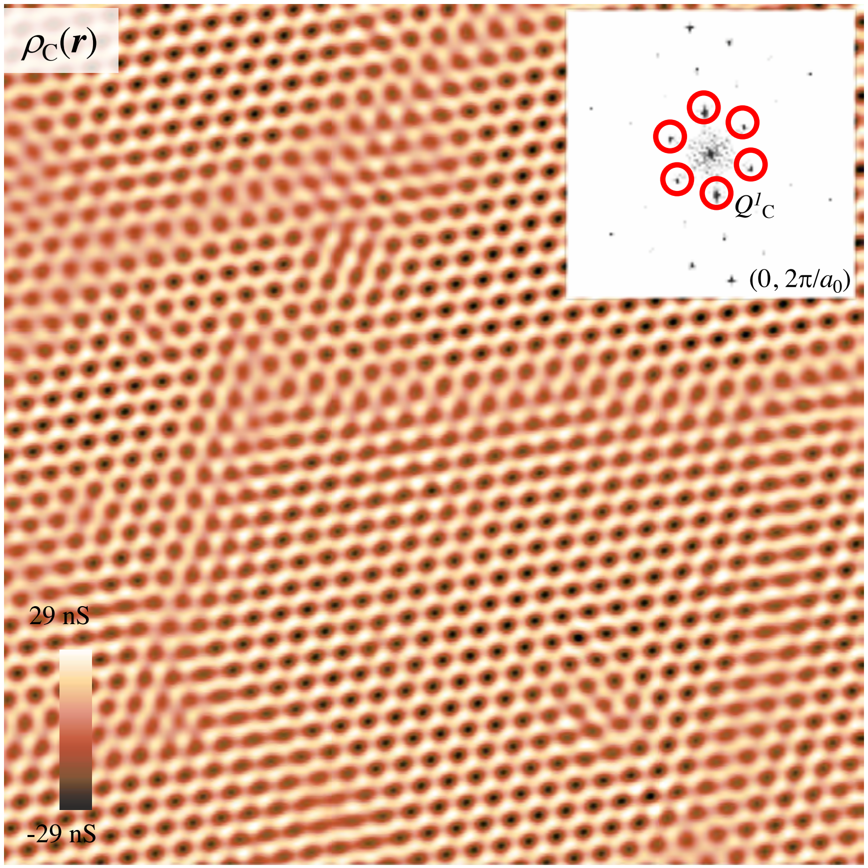
SIMULTANEOUS VISUALIZATION CDW AND PDW STATES

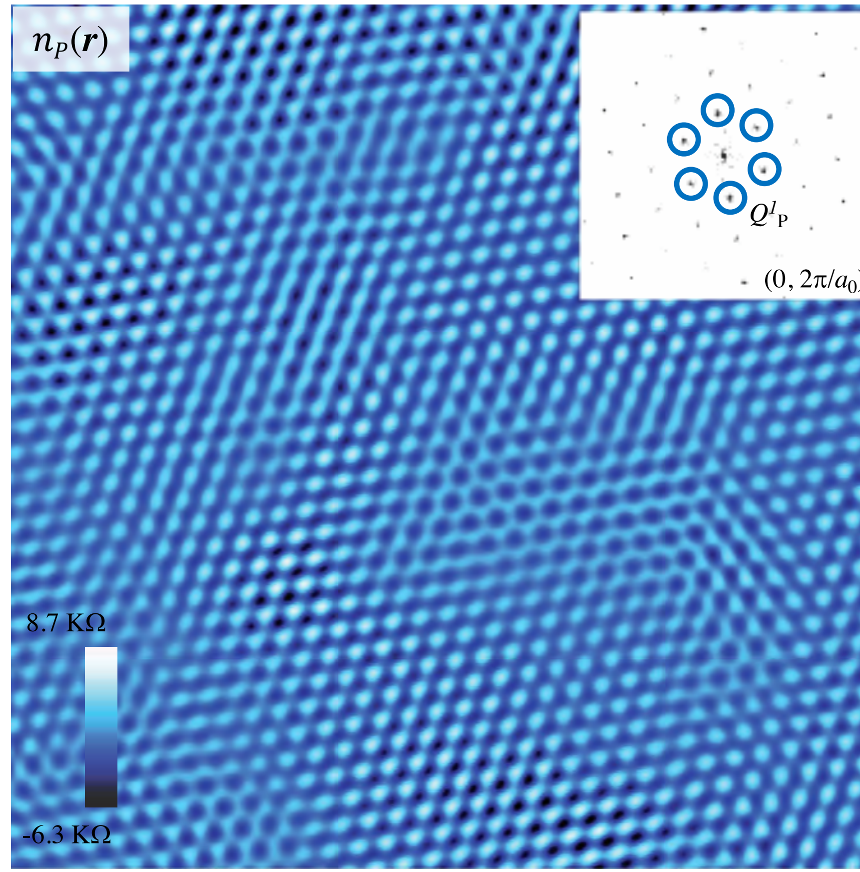
CHARGE DENSITY WAVE



$$\rho_C^{\boldsymbol{Q}_C}(\boldsymbol{r}) = \rho e^{i\boldsymbol{Q}_C\cdot\boldsymbol{r}} + \rho^* e^{-i\boldsymbol{Q}_C\cdot\boldsymbol{r}}$$

T = 280mK

PAIR DENSITY WAVE



$$\Delta_P^{\boldsymbol{Q}_P}(\boldsymbol{r}) = \left[\Delta e^{i\boldsymbol{Q}_P\cdot\boldsymbol{r}} + \Delta^* e^{-i\boldsymbol{Q}_P\cdot\boldsymbol{r}}\right] e^{i\phi}$$

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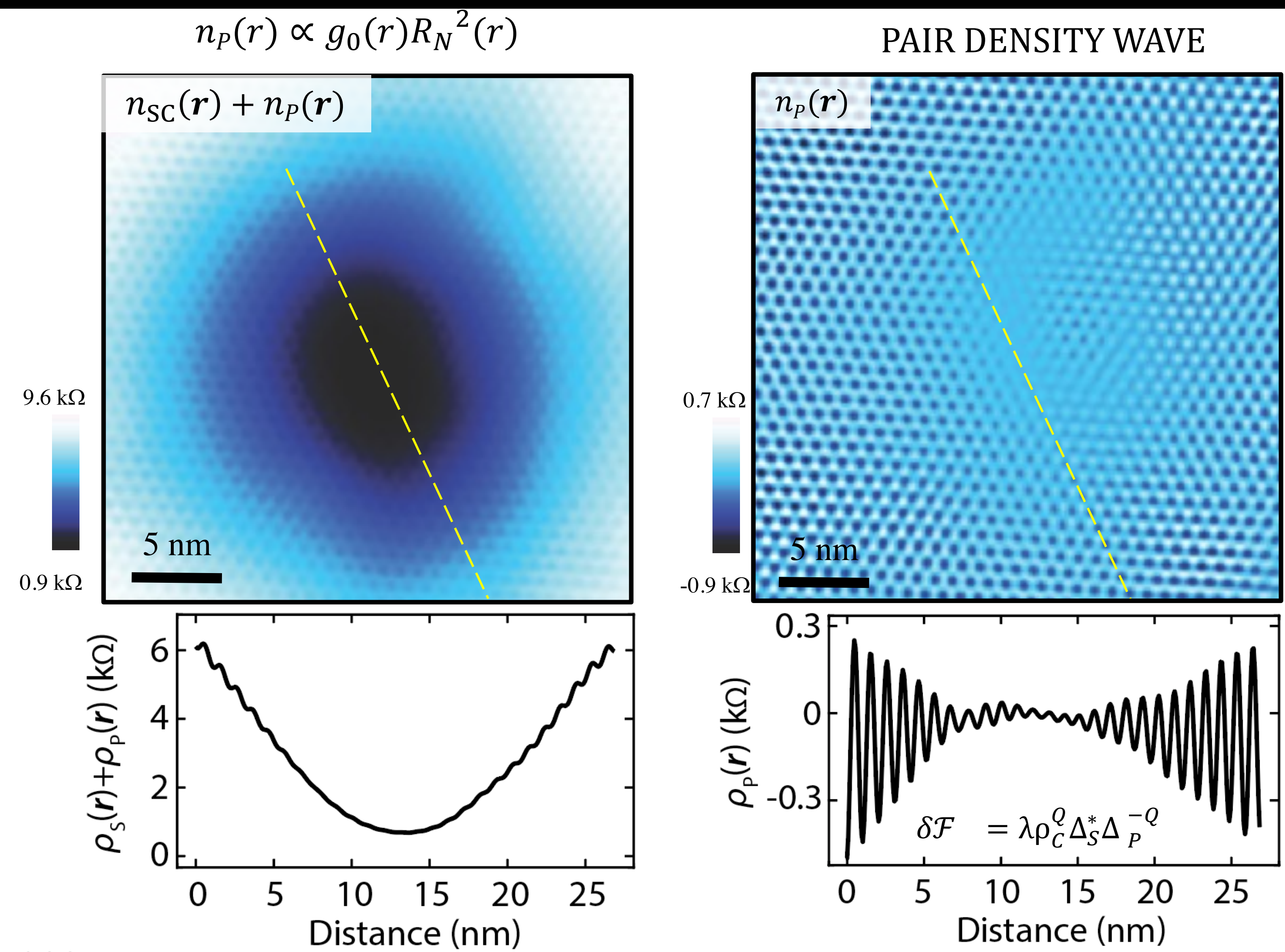
PDW STATE COUPLES SUPERCONDUCTOR



T = 280mK

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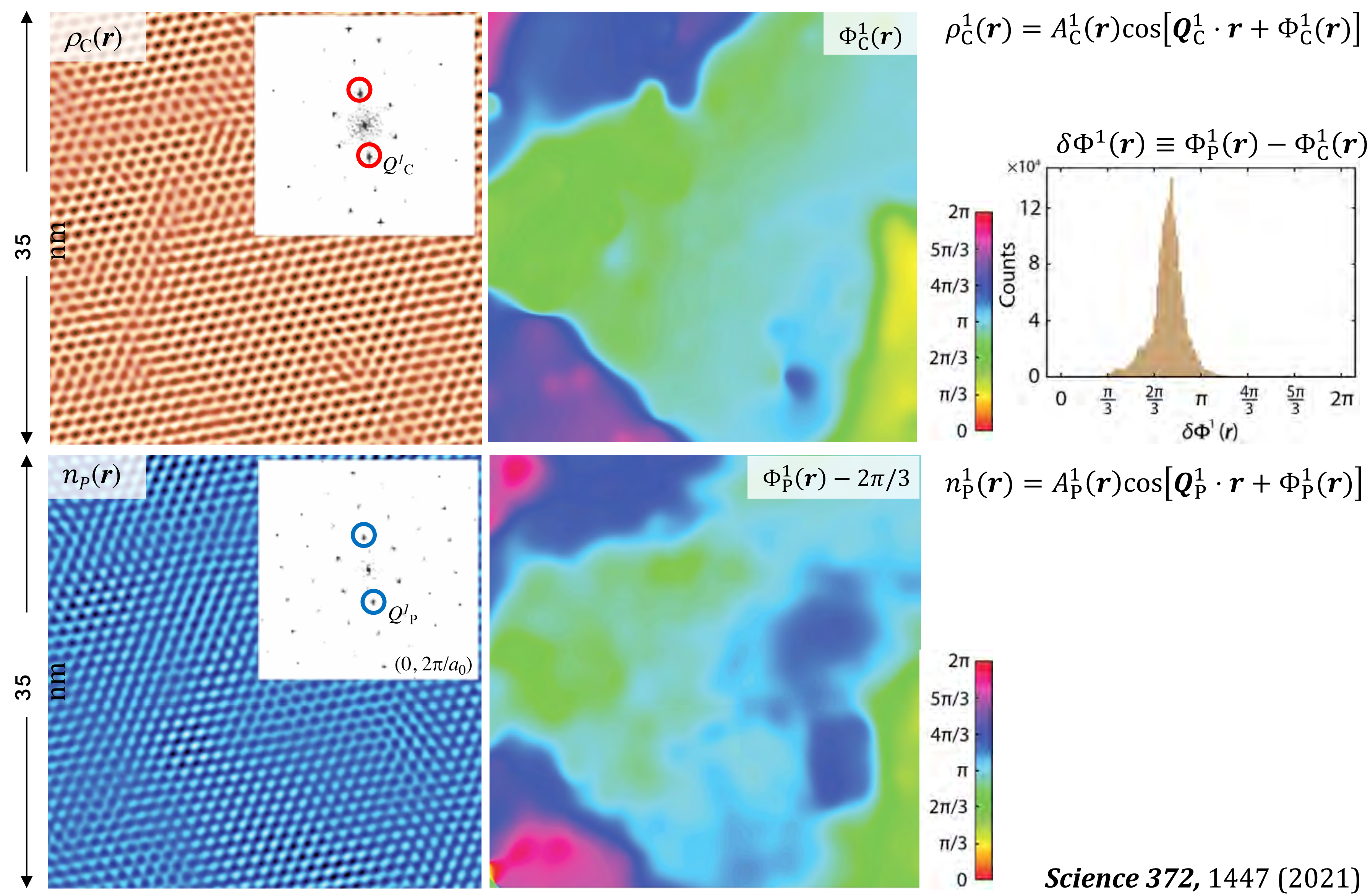
CDW:PDW PHASE SHIFT = 2π/3



$$\rho^1_C(\boldsymbol{r}) = A^1_C(\boldsymbol{r})\cos[\boldsymbol{Q}^1_C \cdot \boldsymbol{r} + \Phi^1_C(\boldsymbol{r})]$$

$$\delta\Phi^1(\boldsymbol{r}) \equiv \Phi^1_P(\boldsymbol{r}) - \Phi^1_C(\boldsymbol{r})$$

$$n^1_P(\boldsymbol{r}) = A^1_P(\boldsymbol{r})\cos[\boldsymbol{Q}^1_P \cdot \boldsymbol{r} + \Phi^1_P(\boldsymbol{r})]$$

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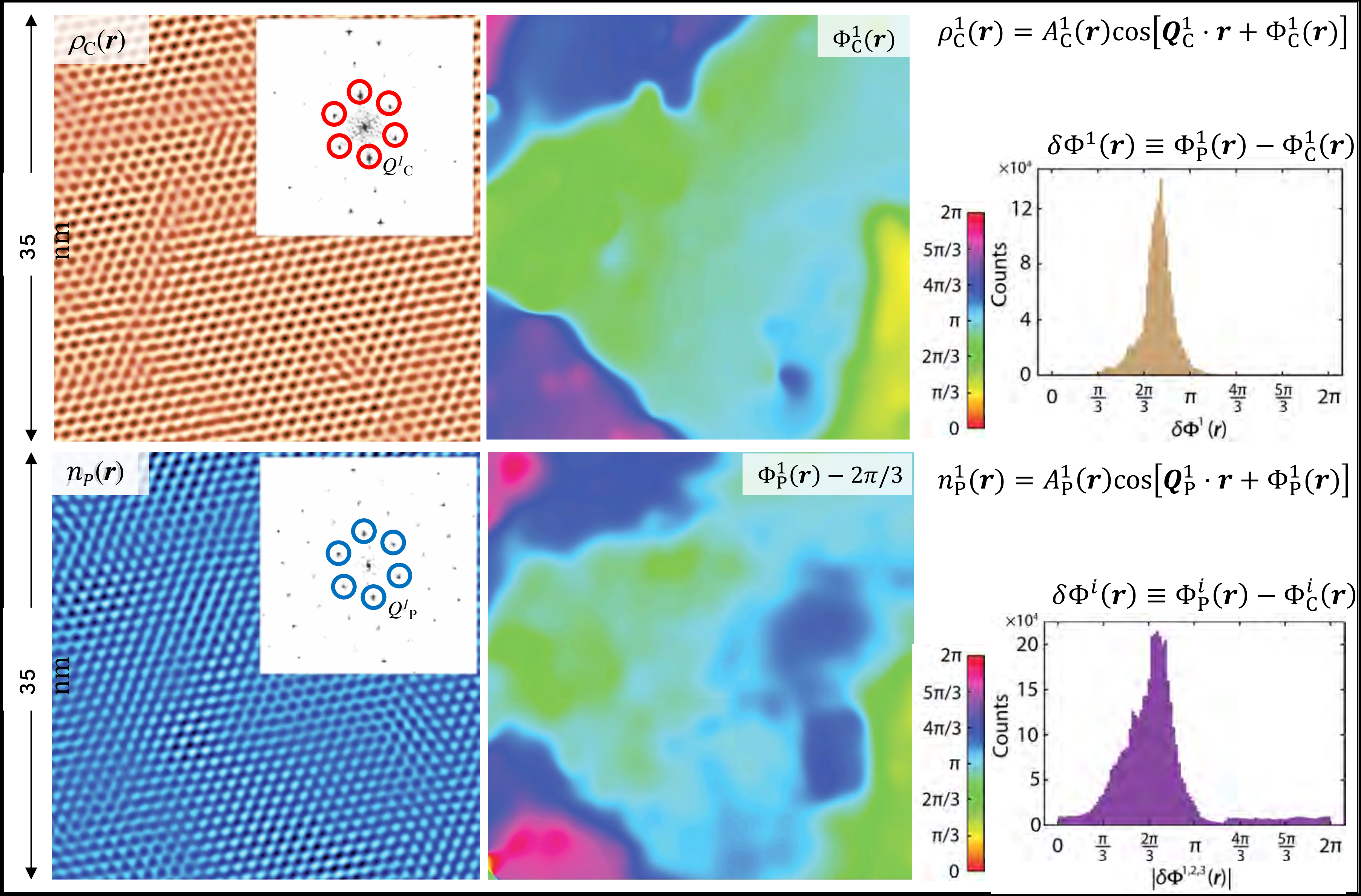
CDW:PDW PHASE SHIFT = 2π/3 UNIVERSALLY



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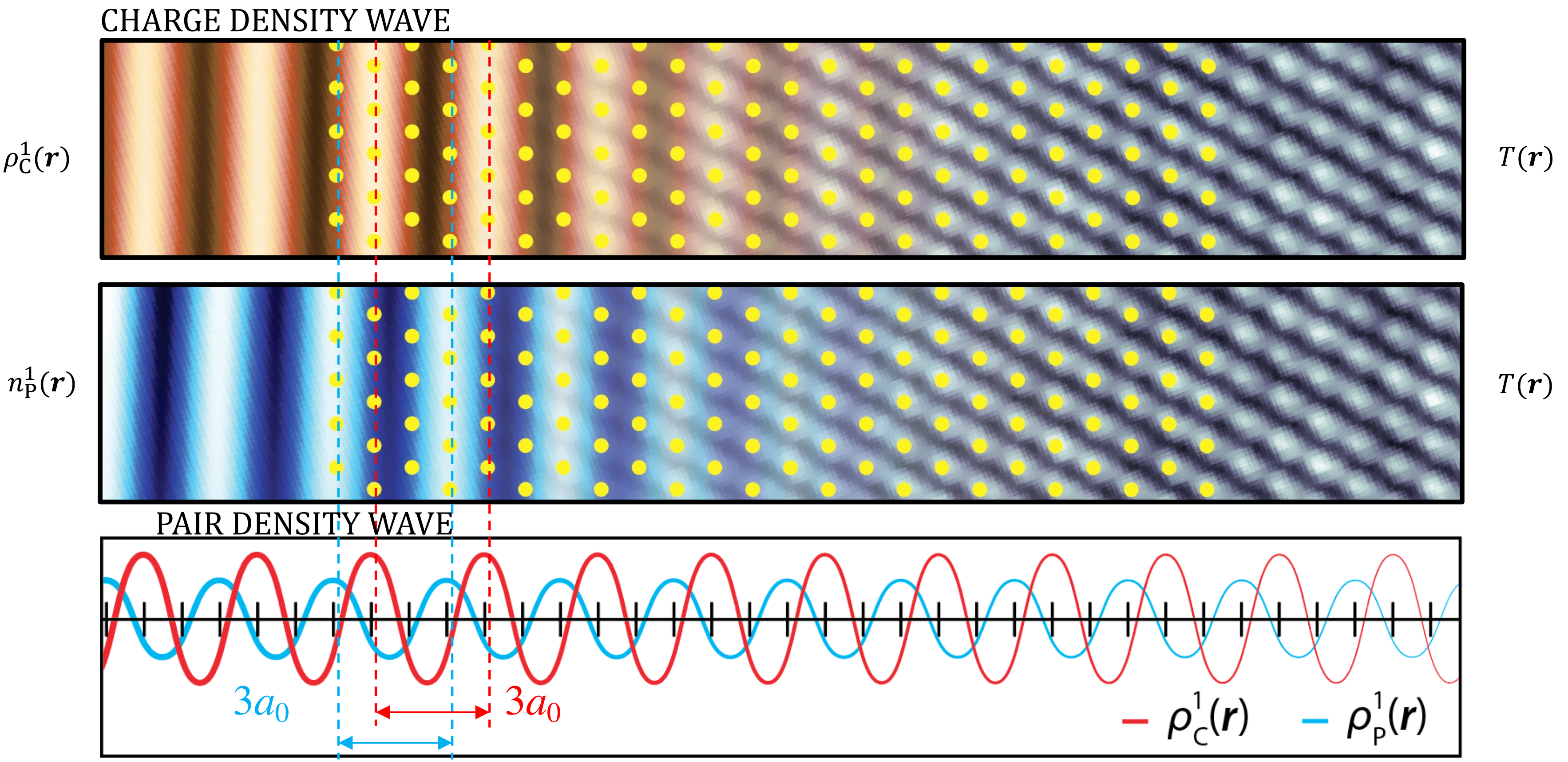
CDW:PDW INTERSTATE DISCOMMENSURATION a_0



T = 280mK

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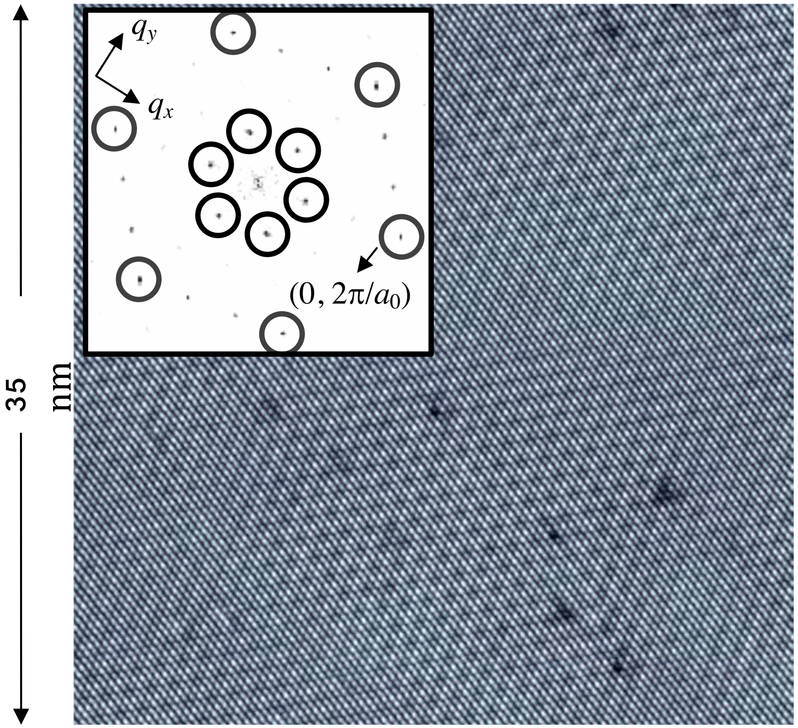
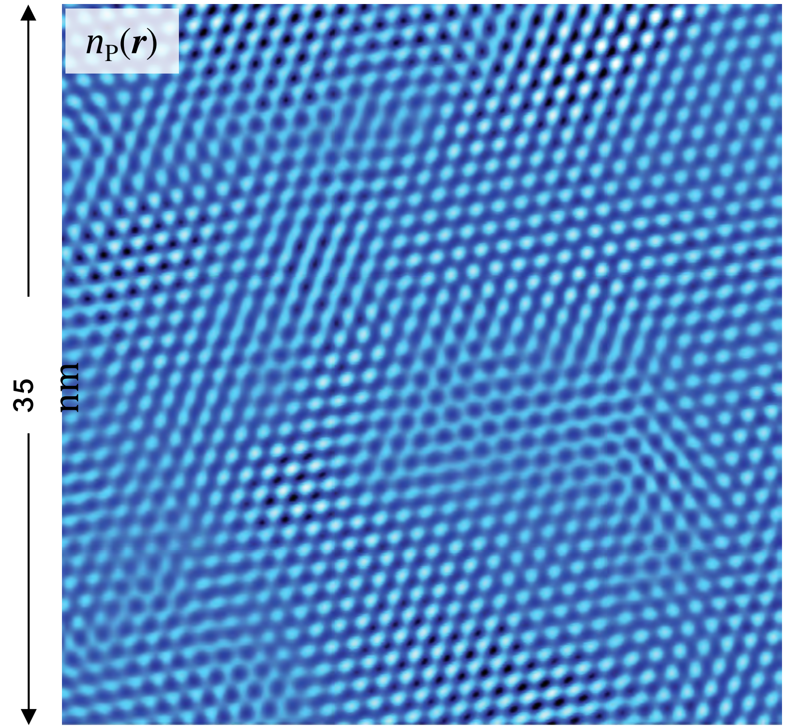
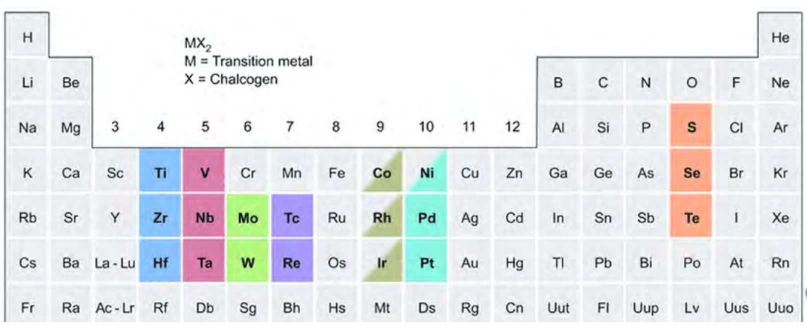
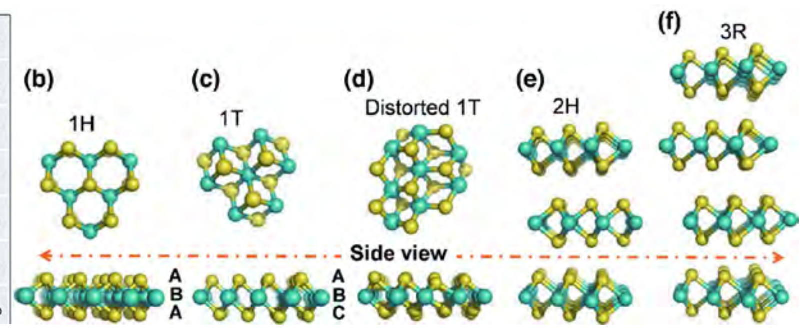
SJTM @ TMD: ABUNDANT NEW PHYSICS









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